DEVELOPMENT AND APPLICATION OF MULTIOBJECTIVE FUZZY WASTE LOAD ALLOCATION MODEL

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ABSTRACT

Water quality management though a very important part of overall water resources management programs is also unfortunately given the last priority among all the other objectives like water supply for irrigation, drinking purpose, hydropower, flood mitigation etc. In the present paper a multiobjective waste load allocation (MOWLA) model has been developed. Though there are many MOWLA models with cost minimization and DO maximization, there are very few models which have adequately addressed the issues of equity in waste load allocation models. The present paper includes equity as one of the objective and shows that by introducing equity in the objective function the solution obtained is more fair when compared to the overall cost minimization solution. However the decision maker has different satisfaction levels to different solutions. To address this uncertainty or ambiguity in a decision maker's (DM) satisfaction level, fuzzy membership functions are introduced for each objective function. thus by introducing fuzziness in the objective functions, a modeler can give a wide range solutions from which the DM can select the most satisfying solution for himself.

Keywords: ***Water Quality Management, Waste Load Allocation, Multi Objective Programming, Fuzzy Logic***

I. INTRODUCTION

Waste load allocation refers to the problem where it is required to maintain the water quality of a stream to a desirable level by allowing the polluters to treat their effluent to an optimum level before discharging it into the stream. The model however should be economically, technologically and socially feasible. Traditionally most of the waste load allocation models have been analyzed as a single objective optimization, with treatment cost minimization as the objective function. However in most of the cases the least cost treatment plan always lead to satisfaction of water quality standards to a very stringent levels and thereby increasing the risk of violating water quality standards (measured in terms of available dissolved oxygen) more frequently. These factors lead many modelers to realize that the treatment cost cannot be a sole criterion for analyzing a waste load allocation system. This lead to the development of multiobjective waste load allocation models where along with treatment cost minimization, minimization of DO deficit in a reach or maximization of BOD load was also considered as an objective function. Since both the objectives are conflicting in nature, the model should be solved using available multiobjective optimization techniques. Various multiobjective optimization techniques are available in the literature, ranging from complex evolutionary algorithms to simple weighted average methods. The choice of a proper solution technique depends on the type, complexity and size of the problem. For a simple multiobjective problem, having not more than three objectives and whose objective functions are also linear (or not complex), traditional MO (multiobjective) methods like goal programming, compromise methods, interactive methods etc., will be sufficient. However for large complex problems which have more than three objectives, which are highly non-linear and have large number of constraints, traditional MO methods may become very tedious to solve and also they may not be able to find all the non-dominated solutions. For such type of problems evolutionary algorithms become a must. After this brief introduction to waste load allocation problem and MO problems, the next section will be devoted to a brief overview on the literature. There is a great amount of literature available on the application of optimization techniques to waste load allocation models (also referred to as water quality modeling). The earliest models were mostly uni-objective and used simple linear programming techniques (Loucks et. al., (1967)[1], ReVelle et. al., (1968)[2], Graves et. al., (1972)[3]), dynamic programming (Leibmann and Lynn, (1966)[4], Cardwell and Ellis, (1993)[5]) and geometric programming (Ecker, 1973[6]), to solve the models. However, due to brevity of space availability, the literature review in this section will be mostly concentrated on the application of fuzzy logic to water quality modeling. One of the earliest works which showed the application of fuzzy logic in the field of water quality modeling was by Jowitt and Lumbers (1982)[7]. In their paper they discussed the ways in which fuzzy logic can be used to address the issues of vagueness and ambiguity in defining the water quality standards (measured in terms of available dissolved oxygen). Hathorn and Tung (1989)[8], developed a bi-objective (the objectives were maximization of waste discharge and minimizing equity difference among various users) waste load allocation model using fuzzy linear programming. They considered both the objective functions as fuzzy. However all the other water quality and river parameters were deterministic. Julien (1994)[9] applied fuzzy sets and possibility theories for the representation of imprecise information in water quality management problems. He showed that possibilistic programming can be used as an alternative to stochastic programming where the parameters are modeled as fuzzy variable instead of as random variables. In both the models mentioned previously, the objective function was linear. Chang and Chen (1996)[10] developed a nonlinear fuzzy programming model for water pollution control in a river system and used genetic algorithms to solve optimization problem. In their model they modeled both objective function (which was nonlinear) and constraint as fuzzy variables. Thus by introducing nonlinearity in the model they were able to develop a more realistic picture of water quality management (since most of the water quality problems are highly non linear). Lee and Wen (1996)[11] developed a fuzzy GP (goal programming) approach for water quality modeling in a river basin. Several computational models, including equal weight and unequal weight, and non-preemptive and preemptive priority models were proposed by them for the solution of FGP problems. Sasikumar and Mujumdar (1998)[12] developed a fuzzy optimization model for water quality management of a river system. They defined the fuzzy membership functions for satisficing the goals of pollution control authority (PCA) (achieving the water quality at each check point to a desirable level) and dischargers (make the fractional removal level as close as possible to aspiration level). They solved the FLP using max-min and max-bias formulations. However all other river and water quality parameters were considered deterministic. Sasikumar and Mujumdar (2002)[13] again modified the previously developed FWLAM (fuzzy waste load allocation model) by considering seasonal variation in the river discharge and specifying seasonal fractional removal levels for each discharger. Gosh and

Mujumdar (2005)[14] developed a risk minimization model to minimize the risk of low water quality along a river in the face of conflict among various stake holders. Probabilistic Global Search Laussane (PGSL), a global search algorithm developed recently, was used for solving the resulting non-linear optimization problem. From the literature reviewed so far it appears that there is a scope in WLA problems where the issues of both fuzziness and randomness in model formulation can be addressed and incorporated. Saadatpour and Afshar (2007) [15] developed a simulation-optimization model for waste load allocation problem. For simulating the effects of pollutant load at a downstream point they used QUAL 2E and results obtained thus by were coupled with GA (genetic algorithm) optimization model. Singh et. al. (2007)[16] applied interactive fuzzy multiobjective linear programming model for finding BOD removal rates for each discharger located along river Yamuna.

II. DEFINING OBJECTIVES AND CONSTRAINTS

This section provides a brief overview of the methodology involved in the model development. The waste load allocation model was solved as a bi-objective optimization model. The objectives considered were, minimization of overall treatment level and maximization of equity. The objective functions $Z_1(x)$ and $Z_2(x)$ are defined below.

Objective 1:- Minimization of overall treatment level, which is given by:

$$\min Z_1(x) = \sum_{i=1}^{n} x_i$$

Where x_i is the treatment level for an individual discharger i and n is the total number of dischargers.

Objective 2:- Maximization of equity for each discharger. Before formulating the objective function for equity, a brief overview on the concept of equity would be appropriate in understanding the equity objective function. Johnson (1967)[17], defined equity as a criterion where equals would be treated equally and non-equals would be treated differently i.e. dischargers who are discharging nearly same amount of effluent should be given same treatment levels, and all others should be given different treatment levels. Equity can also be defined based on the criterion that:

"The treatment level for each discharger should be equivalent to the product of the total effluent share of an individual discharger and the overall treatment removal for the system."

Following above definition the objective function for equity can be given as:

$$\max Z_2(x) = \sum_{i=1}^{n} a_i$$

$$where\ a_i = 1 - e_i$$

$$and\ e_i = \left| x_i - X\frac{m_i}{M} \right|$$

where m_i is the effluent BOD (after treatment) for an individual discharger, M is the total effluent BOD load (after treatment) entering into the river and is given by $\sum_{i=1}^{n} m_i$ and X is the overall treatment level of the system given by $\sum_{i=1}^{n} x_i$ .The constraints for the optimization problem are as given below:

2.1 Constraints for satisfying the maximum allowable DO deficit at a checkpoint (as a function of treatment levels):

$$\sum_{i=1}^{k} B_{im} x_i \le d_i \; ; k = 1 \ldots . n \quad (5)$$

A_{im} is a constant at a checkpoint m, for reach i, and it is a function of initial BOD of the river

B_{im} is a constant for a checkpoint m, within a reach i, and it is a function of effluent BOD entering the river reach.

The above constants are obtained by using Streeter-Phelps (1925) [18] equations and they show the effect on downstream DO deficit due to release of a unit point load at any upstream point.

2.2 Constraints on maximum and minimum treatment levels:

$x_{min} \le x_i \le x_{max}$

where x_{min} and x_{max} are the minimum and maximum treatment levels within which the treatment levels for each discharger would be fixed. For the present study, x_{min} has been taken as 0.3 and x_{max} as 0.9, except for D6 (sixth discharger) for whom x_{max} has been taken as 0.95, by virtue of his releasing nearly half of the total effluent load.

III. STUDY AREA DESCRIPTION

Tambraparni river system is one of the seventeen river system in the state of Tamil Nadu. The river for its major part flows through the district of Tirunelveli and a small part of its catchment also lies in the district of Tuticorin. The total length of the river from its origin at Agastya Malai, in Western Ghats to its fall in Bay of Bengal at Palaya Kayal, is about 120 Km (74.56 miles). Tambraparani River Basin lies between geographic co-ordinates N. lat.8º 26’ 45” to 9º 12’ 00” and E. long 77º 09’ 00” to 78º 08’30” and falls within the survey of India Degree sheets 58G, 58 H and 58 L. Along its course it is joined by numerous major and minor tributaries, the most notable among them being, Servalar, Manimutthar, Gadana, Pachaiyar, and Chittar. The slope of the river is towards southeast, with an average slope of about 4.5 m/Km.

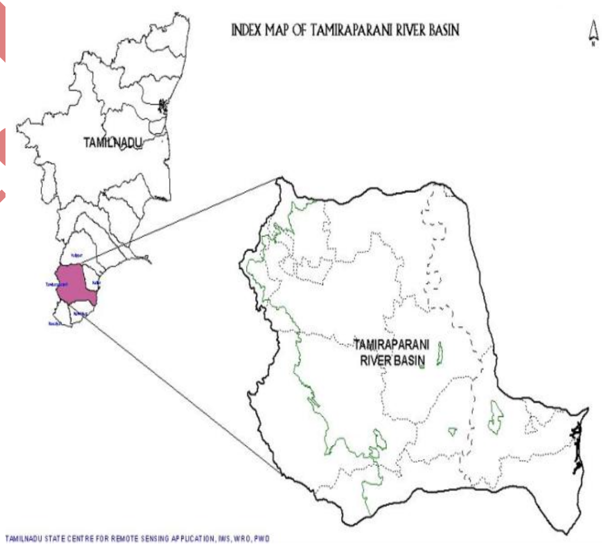

Figure 1 Layout Map of Tambraparni River Baisn

3.1 Environmental Status of Tambraparni River

The major sources of pollution in the river are industrial effluent, domestic sewage and agricultural runoff. However there are very few major industries in the river basin, the major ones being Madhura Coats at V.S.Puram, and Sun Paper Mills at Cheranmahadevi. The major share of pollution in the river is due to uncontrolled disposal of domestic sewage and non point source pollution from agricultural runoff. There are no existing treatment plants for any of the major or minor towns in the river basin. This makes the effect of domestic sewage on the river water quality even more pronounced. Most of the small towns and villages in the basin area have no drainage (sewerage) facilities. The only towns which have partial sewerage system are Tirunelveli, V.S.Puram, Ambasamudram, Cheranmahadevi, Palayamkottai, Melapalayam and Srivaikundam. However The Government of India has sanctioned about Rs. 700 millions to implement underground sewerage system in Tirunelveli Corporation area (Microlevel Status Report for Tambraparni River Basin, 2003[19]). The DO in the river varies from a lowest value of 1.7 mg/l during the month of March to a highest value of 6.8 mg/l during the monsoon seasons of August and September whereas the BOD varies from 0.13 mg/l to 10.2 mg/l. Figure 2 below shows various towns located along the river which are discharging their domestic sewage directly into the river.

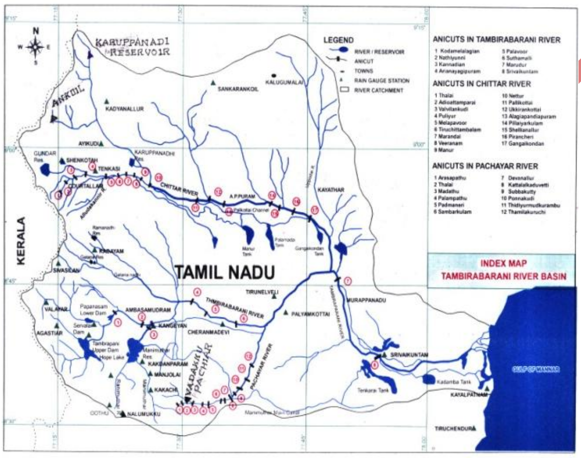

Figure 2 Index Map for Tambraparni River

VI. MODEL APPLICATION TO THE CASE STUDY

1. In the development of models for water quality certain assumptions need to be made and they are:
2. The population details obtained from Tamil Nadu Census Board were as upto the year 2001. However the present study is carried out for a projected population as on year 2010 with an assumption of a linear growth rate of 2% for all towns.
3. The daily per capita water consumption is taken as 90 lpcd and the waste water volume that reaches the river is assumed to be 80% of 90 lpcd.
4. The BOD load per person is taken to be about 0.08 kg/day.
5. The analysis has been done for the peak effluent flow which is given by (Fair et al., 1966[20]):

$$Q_{max} = 3.2 * (Q_{avg})^{\left(\frac{5}{6}\right)}$$

Table 1 below gives the summary of the population detail, BOD load (kg/d), and effluent discharge for each town and Table 2 gives the particulars for the river.

Table 1 Population, BOD Load and Effluent Flow Rate Details for Each Town

Discharger	Town	Population	BOD Load (kg/d)	Qavg (m^3/sec)	Qmax (m^3/sec)
D1	V. S. Puram	97258	7781	0.0605	0.309
D2	Ambasamudram	49376	3950	0.041	0.223
D3	Cheranmahadevi	129813	10385	0.0286	0.165
D4	Melapalayam	99525	7962	0.083	0.402
D5	Palayamkottai	142250	11380	0.1185	0.541
D6	Thirunelveli	492370	39389	0.41	1.522
D7	Thatchanallur	97353	7788	0.0811	0.394
D8	Srivaikundam	25312	2025	0.0211	0.128

Table 2 Hydraulic Particulars and Streeter-Phelps Equation Parameters for Each Reach

Reach	Distance (Km)	River Discharge (m^3/sec)	Velocity (m/sec)	Travel time within the reach (days)	Temperature (^oC)	Deoxy-genation Coefficient kd, (d^{-1})	Reaeration Coefficient, ka (d^{-1})	Saturation DO (mg/l)
1	9	6.51	0.258	0.258	27.61	0.315	0.542	7.800
2	17	11.01	0.306	0.306	27.77	0.317	0.546	7.580
3	13	8.63	0.277	0.277	26.81	0.303	0.521	8.025
4	3	11.96	0.303	0.303	26.81	0.303	0.521	8.025
5	3	11.96	0.303	0.303	26.74	0.301	0.520	8.030
6	24	11.96	0.303	0.303	26.7	0.301	0.519	8.040
7	14	7.44	0.267	0.267	26.11	0.292	0.504	8.125
8	19	7.44	0.267	0.267	26.68	0.301	0.518	8.040

V. MODEL DEVELOPMENT

The objective functions as defined in section (number) are conflicting and cannot be solved using traditional LP techniques. Also by virtue of their being conflicting, there is an element of vagueness or ambiguity involved on part of the decision maker in prioritizing the objective functions. Hence to address the vagueness or ambiguity of the decision maker vis-à-vis the objective functions, a fuzzy membership function will be elicited for each objective function. After eliciting a membership function for each objective function, Bellman and Zadeh's

(1970)[21] min-max function can be used to convert the fuzzy multiobjective program into a fuzzy LP model, which can be then solved using any of the traditional LP optimization techniques. Before proceeding to solve the model we have to first develop membership functions for each objective functions. To develop membership function for an objective function, the model is first solved as a single objective optimization model for the given set of constraints, by both maximizing and minimizing the objective function for which the membership function is to be developed. During this operation other objective functions are kept as constraints. Thus for each objective function, we can find max and min values by following the above procedure. Once the max and min values for each objective functions are obtained, the membership functions for each objective function can be given as below (assuming that the variation of objective function from max to min is linear):

If the objective function is minimization, then the membership function is given as:

$$\mu_i(x) = \begin{Bmatrix} 1 \; for \; z_i(x) \le z_i^{min} \\ 1 - \dfrac{z_i(x) - z_i^{max}}{z_i^{max} - z_i^{min}} \; for \; z_i^{min} \le z_i(x) \le z_i^{max} \\ 0 \; for \; z_i(x) \ge z_i^{max} \end{Bmatrix}$$

However if the objective function is maximizing, the membership function is given by:

$$\mu_i(x) = \begin{Bmatrix} 0 \; for \; z_i(x) \le z_i^{min} \\ \dfrac{z_i(x) - z_i^{min}}{z_i^{max} - z_i^{min}} \; for \; z_i^{min} \le z_i(x) \le z_i^{max} \\ 1 \; for \; z_i(x) \ge z_i^{max} \end{Bmatrix}$$

For the present problem, the membership functions for the objective function of minimizing treatment removals and maximizing the equity were obtained as:

$$\mu_1(x) = \begin{Bmatrix} 1 \; for \; z_1(x) \le 5.676 \\ 1 - \dfrac{z_1(x) - 6.15}{6.15 - 5.676} \; for \; 5.676 \le z_1(x) \le 6.15 \\ 0 \; for \; z_1(x) \ge 6.15 \end{Bmatrix}$$

The membership function for the objective function of maximizing the equity was obtained as:

$$\mu_2(x) = \begin{Bmatrix} 0 \; for \; z_2(x) \le 4.038 \\ \dfrac{z_2(x) - 4.038}{8 - 4.038} \; for \; 4.038 \le z_2(x) \le 8 \\ 1 \; for \; z_2(x) \ge 8 \end{Bmatrix}$$

The membership functions are shown below:

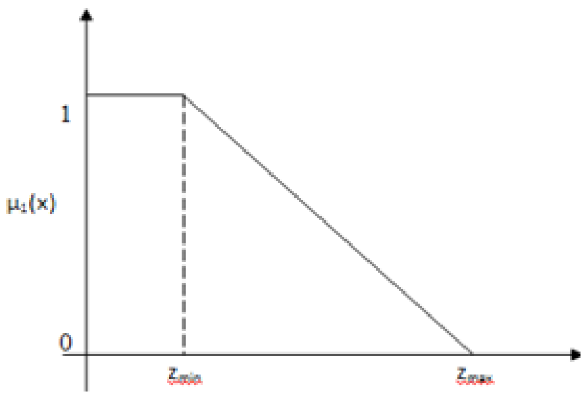

Figure 3:- Membership function μ1(x) for objective function of treatment removal levels

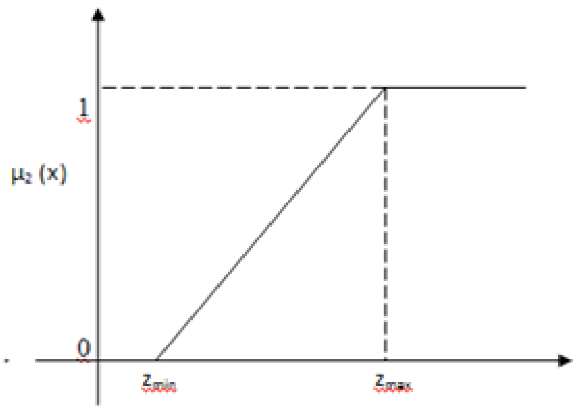

Figure 4:- Membership function $\mu_2(x)$ for the objective of equity z_2 (x)

The fuzzy decision making problem now can be written as:

$$Max[Min\{\mu_1(x), \mu_2(x)\}]$$
$$subject\ to:$$
$$A_{im} - \sum_{i=1}^{k} B_{im} x_i \le d_i;\ k = 1, \ldots\ldots, n$$
$$x_i^{\min} \le x_i \le x_i^{\max}$$

Where the terms have the same meaning as described in section (number). The above model can be converted into a fuzzy linear programming model as shown below:

$$\min \gamma$$
$$subject\ to:$$
$$\bar{\mu}_i - \mu_1(x) \le \gamma$$
$$\bar{\mu}_i - \mu_2(x) \le \gamma$$
$$A_{im} - \sum_{i=1}^{k} B_{im} x_i \le d_i;\ k = 1, \ldots\ldots, n$$
$$x_i^{\min} \le x_i \le x_i^{\max}$$
$$0 \le \gamma \le 1$$
$$0 \le \bar{\mu}_i \le 1$$

where γ is the minimum deviation that can be allowed from the reference membership function (RMF) $\bar{\mu}_i$. The above problem thus becomes a simple LP model which can be solved using any of the available programming software.

VI. STEPWISE PROCEDURE FOR SOLVING THE FMOLP MODEL

Step 1:- Define the objective functions and constraints.

Step 2:- Find the maximum and minimum value for each objective function. This is done by considering each objective function singularly and keeping other objective functions as constraints along with original set of constraint equations.

Step 3:- Once the maximum and minimum for each objective function are obtained, we can elicit a membership function for each objective function. The membership function should be non-increasing (non-decreasing) for minimization (maximization) objective function.

Step 4:- Using min-max fuzzy decision making function, the problem can be converted into a linear programming model where the objective is to minimize the maximum difference, γ between the RMF and membership function of the respective objective function.

Step5:- The model can be solved for different RMF values for each objective, whereby we will obtain a set of Pareto-optimal solution, from which the decision maker can choose an appropriate solution.

VII. RESULTS AND DISCUSSION

The above optimization model was solved for different RMF values by varying it from 0 to 1 for each objective function. The treatment levels for each discharger varies considerably (except for dischargers D6 and D7) according to the way the solution is biased (i.e. biased towards minimizing treatment levels or maximizing equity). Table 3 shows a sample output of treatment levels for two cases i.e. case 1, when the solution is biased towards minimizing treatment levels and case 2, when the solution is biased towards maximizing equity.

Table 3 A Sample Output for Case 1 & Case 2

Discharger	Solution Biased Towards TL (case 1)		Solution Biased Towards Equity (case 2)	
	Treatment Level (Z_1)	Equity (Z_2)	Treatment Level (Z_1)	Equity (Z_2)
RMF	1	0.3	0.3	1
X1	0.882	0.505	0.784	1
X2	0.3	0.131	0.648	1
X3	0.9	0.538	0.858	0.827
X4	0.696	0.674	0.787	1
X5	0.9	0.58	0.841	1
X6	0.95	0.882	0.948	1
X7	0.767	1	0.784	1
X8	0.3	0.701	0.4	0.83

From the table above it can be observed that when the RMF is biased towards minimizing treatment levels, there is a compromise on equity for majority of the dischargers. Whereas when the RMF is biased towards equity, the overall treatment level of the system increases considerably. Figure 4 shows the variation in treatment levels for each discharger in each case.

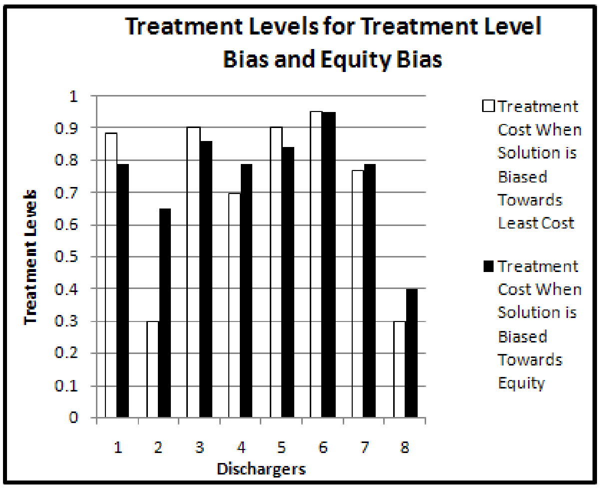

Figure 4 Treatment Levels for Each Discharger for Case 1 and Case 2

However by varying the RMF values for each objective function, a set of Pareto-optimal solution can be obtained for treatment levels and equity, from which the decision maker can select a solution which satisfies him on both treatment levels and equity.

VIII. CONCLUSION

A FMOLP model was developed for determining the treatment removal levels for each discharger in order to maintain the water quality of river Tambraparni. The objective functions for model were minimization of treatment levels for each discharger and maximization of equity among the dischargers.

When equity is not considered in the model, the treatment levels among the dischargers are distributed unevenly i.e. the treatment levels for each dischargers are not proportional to the percentage waste they are contributing into the river. Hence to overcome this type of inequity among the discharger, the objective function of maximizing equity was introduced. However maximization of equity led to increase in overall treatment level for the system, thereby increasing the overall treatment cost. To balance these two objectives, a set of compromise solutions were obtained, from which the decision maker can select a solution suitable for him.

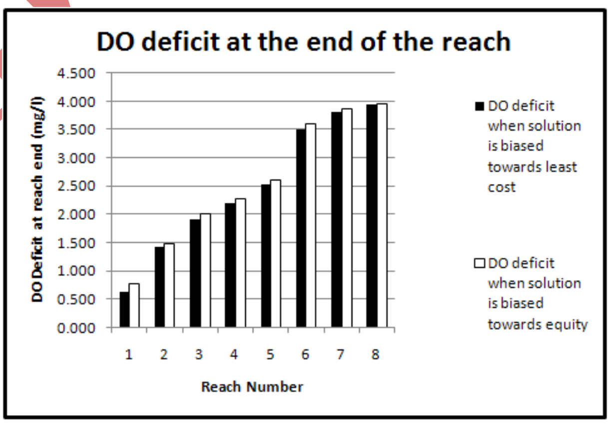

Figure 5 DO Deficit at the End of Reach

It should be however noted that just these two objective functions are not enough for bringing water quality to the desirable level. Another objective of maximizing the BOD load for a discharger can be included in the optimization model, through which the DO deficit can be reduced still further from permissible limit.

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