VOLUMETRIC ACCURACY ANALYSIS BASED ON GENERALIZED POSITIONING ERROR MODEL IN THREE-AXIS CNC MACHINE TOOL

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ABSTRACT

The three-axis machine tools produce an inaccuracy at the tool tip which is caused by kinematics parameter deviation resulting mainly from manufacturing error and assembly error. Here, all linear axes are theoretically perpendicular (dot product, $\cos 90^o = 0$*) to each other and directed along the X, Y, Z coordinate, but in working machines, the axes are nearly perpendicular (*$\cos 89.9^o \neq 0$*) because of the reasons mentioned above. This kind of error can be taken into consideration only by the precise description of the actual kinematics of the machine tool. This paper attempts to develop a generalized error model for the effects of positioning errors of the components of the kinematic chain of a machine in the work space and the results obtained by this model have been verified experimentally. The mathematical model of the volumetric error, based on positioning error component, has been derived and the effect of error component on the volumetric accuracy at the cutting point has been examined. Volumetric error has been studied further for machining a Carrier and an improvement in quality of Carrier has been obtained by error compensation.*

Key Words*: *Kinematics; Positioning Error; Volumetric Error; Error Model; Error Measurement.

I. INTRODUCTION

Errors in position and orientation in multi-axis machine are the result of errors in the individual links of the machine and the interactions between them. Errors originate from the numerous factors such as geometrical errors of the link parameters, thermally induced errors, static and dynamic deflections, kinematic errors related to the relative motion of joints etc.

Dimensional accuracy has become one of the most important parameters in determining the quality of a machined component. The higher accuracies have to be economically achieved and maintained on finished components. In order to achieve better product, its performance and reliability, there is a continuous striving for higher precision in manufacturing machines. Presently, the machines might be considered fairly evolved and no substantial increase in attainable accuracy can be expected from economically, viable hardware changes.

Therefore, accepting the current technology, it has become necessary to precisely understand the accumulated errors and the effects of error components.

For a three-axis horizontal machining centre (Fig.1), the precision of the cut part is determined by the moving accuracy of an open kinematic chain consisting of three movable slides and one rotational axis. That means, the positioning accuracy at the cutting edge should be defined by the relative moving error between the cutting tool and workpiece . Volumetric error/accuracy represents the overall error/accuracy of a machine tool. Therefore, it has become one of the important indices to represent the quality of such machines.

Fig. 1: Horizontal Machining Centre

According to Hocken [1], error is the difference between the actual and the anticipated response of the machine to a command issued, according to the machine's accepted protocol. Furthermore, accuracy is defined as the maximum translational or rotational error between any two points in the machine's work volume [2]. Several factors, which may produce the errors on the work piece, are: geometric-kinematics errors of the machine tool; thermally induced error on the machine tool; static and dynamic loading error; error due to clamping; and spindle errors of the machine tool. Considerable work on the general area of machine tool accuracy is reported in literature [3-10]. Most of the authors (3-10) have studied the working accuracy of a CNC machine tool taking into consideration the influence of geometrical errors and / or thermal deformations. Berman and Sen [11] have discussed the techniques to measure the parametric errors of CNC CMM in the form of linear, angular, straightness, squareness, flatness of the machine bed and the diagonal errors by a laser calibration system. Hsu and Wang [12] proposed the method of compensation based on the model that considers the tool orientation error related to the motion of machine rotation axes, and it further calculates the error compensation for rotation axes and linear axes separately. Weidong et. al. [13] have established the B-spline mathematical model to represent the component error friction, and the least-squares fitting method to measure data point . Finally, based on the component error extraction method, numerical error compensation experiments were conducted.

The objective of the present paper is to develop a generalized positioning error model associated with three axes CNC machine tool as well as to examine the effect of the error component on the volumetric accuracy at the final cutting point . Further, an improvement in the quality has been noticed by compensating the volumetric error of the machine tool in machining a Carrier , used in automobile industries .

II. PARAMETRIC ERROR MODELING FOR THREE-AXIS MACHINING CENTRE

A kinematic model on the basis of the characterization of errors, takes into consideration the deviations in motions and alignment of the structural members of a machine. This coupled with schemes, for tracking changes in model parameter and introducing compensations, forms a comprehensive system for compensating effects of the errors on a machine's accuracy. The error model developed are discussed in following steps.

2.1 Transformation Model of a Single Joint-link Combination

Links are considered to be rigid body connected either by revolute or prismatic joints. A coordinate frame may be assigned to each link and the relationship between succeeding links may be established using transformations for each of the four variables , namely , d , x , a and t . Using the transformations, nominal relative relationship between adjacent links can be expressed as

$$T_{i-1,i} = T_{z,d} \text{ x } T_{z,x} \text{ x } T_{x,a} \text{ x } T_{x,t}$$

$$= \begin{bmatrix} \cos\ x & -\cos\ t\sin\ x & \sin\ t\sin\ x & a\cos\ x \\ \sin\ x & \cos\ t\cos\ x & -\sin\ t\cos\ x & a\sin\ x \\ 0 & \sin\ t & \cos\ t & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \dots (1)$$

2.2 Kinematics of Three-Axis Machining Centre

Kinematics of three axes machining centre are shown in Fig. 2, following the D-H representation rules; and its corresponding D-H parameters are listed in Table 1.

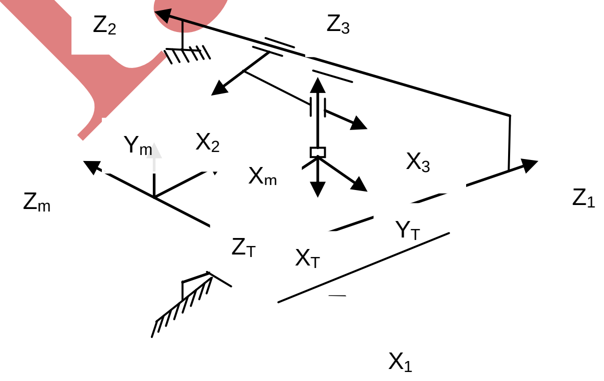

Fig. 2: Kinematic Diagram of Three – Axis Machine Tool

Table 1: D-H Parameters for Three-Axis Machining Centre

Link/par.	M	1	2	3	4	5
d_i	d_m	X^*	Z^*	Y^*	0	L
a_i	a_m	a_1	a_2	a_3	0	
x_i	-90^o	-90^o	-90^o	-90^o	θ_4^*	
t_i	-90^o	90^o	-90^o	90^o	90^o	

*** indicate the variables**

2.3 Transformation of Coordinates

Suppose P_p is the position of the cutter in programming coordinate system; P_m is the position of the cutter in the machine coordinate system; P_t is the position of the cutter in the tool coordinate system; $T_{p,m}$ is the transformation matrix between programming coordinate system and the machine coordinate system; and $T_{m,t}$ is the transformation matrix between the machine coordinate system and the tool coordinate system. It follows that :

$$P_p = T_{p,m} \times P_m$$

$$P_m = T_{m,t} \times P_t$$

$$P_p = T_{p,m} \times T_{m,t} \times P_t$$

$$P_p = T_{p,t} \times P_t \qquad \ldots(2)$$

Assuming that the transformation between the programming and machine coordinate system is purely translational, then :

$$T_{p,m} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_z \\ 0 & 0 & 1 & t_y \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(3)$$

Substituting the D-H parameters, listed in Table 1, into equation (1), the transformation matrices along each link/joint can be established as follows (note: use of "cos (x)" = " Cx" and " sin (x)" = "Sx") :

$$T_{m,1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -a_m \\ 0 & -1 & 0 & d_m \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(4)$$

$$T_{1,2} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -a_1 \\ 0 & 1 & 0 & X \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(5)$$

$$T_{2,3} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -a_2 \\ 0 & -1 & 0 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(6)$$

$$T_{3,4} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -a_3 \\ 0 & 1 & 0 & Y \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(7)$$

$$T_{4,t} = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(8)$$

$T_{p,4} = T_{p,m}$ X $T_{m,1}$ X $T_{1,2}$ X $T_{2,3}$ X $T_{3,4}$

$T_{p,t} = T_{p,4}$ X $T_{4,t}$

$$T_{p,t} = \begin{bmatrix} 0 & 1 & 0 & \{-a_2 + X\} + t_x \\ C\theta_4 & 0 & S\theta_4 & \{-a_m + a_3 + Z\} + t_z \\ S\theta_4 & 0 & -C\theta_4 & \{a_1 + d_m + Y\} + t_y \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(9)$$

and

$$\begin{bmatrix} x_p \\ z_p \\ y_p \end{bmatrix} = \begin{bmatrix} T_{p,t} \end{bmatrix} \begin{bmatrix} x_t \\ z_t \\ y_t \\ 1 \end{bmatrix} \qquad \ldots(10)$$

when the tool length is 1 unit

$$\begin{bmatrix} x_p \\ z_p \\ y_p \end{bmatrix} = \begin{bmatrix} i \\ k \\ j \end{bmatrix} = \begin{bmatrix} T_{p,t} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \ldots(11)$$

where, $[i,k,j]^T$ is the specified direction cosine. By simplifying the above equation, we get

$$\begin{bmatrix} k \\ j \end{bmatrix} = \begin{bmatrix} S\theta_4 \\ -C\theta_4 \end{bmatrix} \qquad \ldots(12)$$

$$\theta_4 = \tan^{-1}(-k/j) \qquad \ldots(13)$$

When the tool length is L, and considering the programmed position as being located at the tip of the tool.

$$\begin{bmatrix} x_p \\ z_p \\ y_p \end{bmatrix} = \left[T_{p,t}\right] \begin{bmatrix} 0 \\ 0 \\ L \\ 1 \end{bmatrix} = \begin{bmatrix} -a_2 + X + t_x \\ LS\,\theta_4 - a_m + a_3 + Z + t_z \\ -LC\,\theta_4 + a_1 + d_m + Y + t_y \end{bmatrix} \qquad \ldots(14)$$

The values of X, Z and Y can be written as:

$$\begin{bmatrix} x_p - (-a_2 + t_x) \\ z_p - (LS\,\theta_4 - a_m + a_3 + t_z) \\ y_p - (-LC\,\theta_4 + a_1 + d_m + t_y) \end{bmatrix} = \begin{bmatrix} X \\ Z \\ Y \end{bmatrix} \qquad \ldots(15)$$

2.4 Error Model for Three-Axis Machine Tools

The joint axes orientations vary with the degree of misalignment. If we find the value of i and k in the reference coordinate system, then j is given by

$$j = \sqrt{1 - i^2 - k^2}$$

The location of the actual joint varies with the location of the plane.

In the zero-position-modeling, the machine zero position and the coordinate system are defined before defining the unit vector of each axis. The unit vector from Fig.2 has been tabulated in Table 2, may be defined as follows: [i,k,j] are the direction cosines of individual axis in the reference coordinate system. The rows 1-4 are for the four axes of the machine. The above model (Table 2) is for the ideal case. According to Table 2, the X-axis should pass through the point P_1 [P_{1x}, P_{1z}, P_{1y}] with a direction cosine [1,0,0]. If P_{1r} is the real point [$P_1 \neq P_{1r}$] through which the X-axis passes, then

$$\Delta P_1 = 1\,i + (P_{z1r} - P_{z1})\,k + (P_{y1r} - P_{y1})\,j$$

where, $\Delta P_{1z} = (P_{z1r} - P_{z1})$ and $\Delta P_{1y} = (P_{y1r} - P_{y1})$ are small deviations in the position of the origin in the base Z- and Y- directions, respectively. The actual direction cosine of joint one is given by

$$\left[\sqrt{1 - k_1^2 - j_1^2}\ ,\quad k_1\ ,\quad j_1\right]$$

The results have been obtained, by extending the same idea to all the joints, and tabulated in Table 3. The unknown direction and position values can be found by the calibration process.

Table 2: Position of Frame in Zero Position (Ideal Case)

Coor. /Link	i	k	J	P_x	P_z	P_y
1	1	0	0	p_{1x}	p_{1z}	p_{1y}
2	0	1	0	p_{2x}	p_{2z}	p_{2y}
3	0	0	1	p_{3x}	p_{3z}	p_{3y}
4	0	1	0	p_{4x}	p_{4z}	p_{4y}

Table 3: Exact Parameters for a Three-Axis Machine Tool for Zero Reference Model

Coor ./ Link	i	k	j	p_x	p_z	p_y
1	$\sqrt{1-k_1^2-j_1^2}$	k_1	j_1	p_{1x}	Δp_{1z}	Δp_{1y}
2	i_2	$\sqrt{1-i_2^2-j_2^2}$	j_2	Δp_{2x}	p_{2z}	Δp_{2y}
3	i_3	k_3	$\sqrt{1-i_3^2-k_3^2}$	Δp_{3x}	Δp_{3z}	p_{3y}
4	i_4	$\sqrt{1-i_4^2-j_4^2}$	j_4	Δp_{4x}	p_{4z}	Δp_{4y}

Table 3 describes the real machine geometry with either arbitrary rotation or a displacement axis. $[P_x, P_z, P_y]^T$ is the transformation at the zero position. $[P_{xr}, P_{zr} P_{yr}]^T$ is the actual location (offsets) of the axis. If we know the direction cosine of the actual axis and the offsets, we can find the tool tip and the orientation of the tool axis vector for a given displacement of joints.

2.5 Representation of Volumetric Error

When three-directional movement in three-axis machine tool is considered, the volumetric error at the cutting point can be obtained by combining the error components of each axis which are described in the previous section. That is,

Volumetric error in X-direction ,

$$V\Delta X = \Delta p_{1x} + \Delta p_{2x} + \Delta p_{3x} + \Delta p_{4x}$$

Volumetric error in Z-direction ,

$$V\Delta Z = \Delta p_{1z} + \Delta p_{2z} + \Delta p_{3z} + \Delta p_{4z}$$

Volumetric error in Y-direction ,

$$V\Delta Y = \Delta p_{1y} + \Delta p_{2y} + \Delta p_{3y} + \Delta p_{4y}$$

III. EXPERIMENTAL ERROR MEASUREMENT

3.1 Calibration of Machine Tool

The calibration of the machine tool geometry has been done, using 1-D ball array [14], with micron dial indicator. The construction design of the 1-D ball array has been shown schematically in Fig. 3. A series of balls with the same diameter and small sphericity errors are fixed on a rigid bar. The manufacturing process of this type of 1-D ball array is very simple. A series of cone holes are bored by a jig boring machine. Then the balls are glued in these holes.

The distance between the balls of 1-D ball array are calibrated by a specially designed device consisting of a laser interferometer, similar to that described by Rademacher [15]. The geometric errors of the machine tool along its motion axis has been measured by direct comparison of the readings from CRT display of the machine tool with the calibrated data of the ball array, when the later is aligned along the motion axis.

Fig.3: Structure of 1-D Ball Array

The measuring principle is shown in Fig. 4. The micron dial with magnetic stand has been mounted on the spindle box and a 1-D ball array has been fixed on the working table.

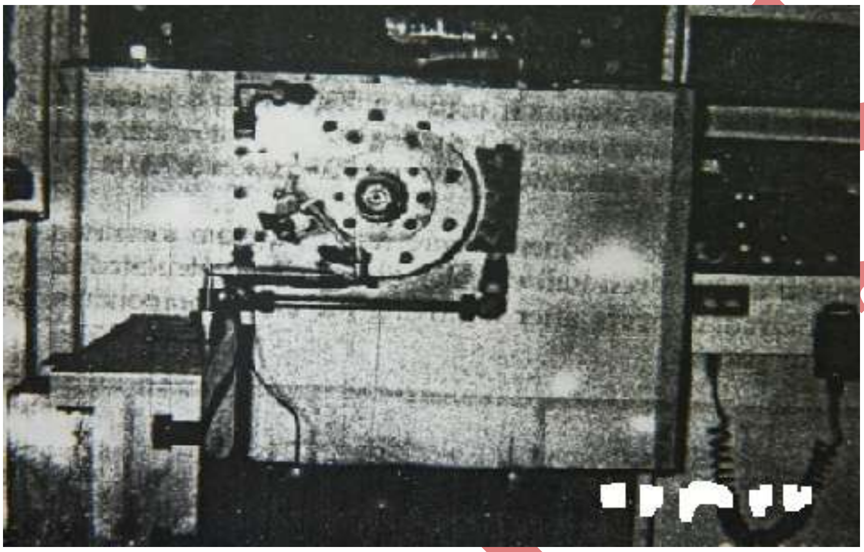

Fig. 4: Measurement Set – Up

3.2 Machine Specification

For the verification of errors and its calculation, three-axis Horizontal Machining Centre (Trade Name : HMC-450), is used , detail of which is given as below:

Specification of the Machine

Machine type	Control type	Axis Movement	Feed Movement
HMC-450	FANUC 0MA	X = 630 mm	1-4000 mm/min
		Z = 600 mm	
		Y = 600 mm	
		θ_4 = +/- 360^O	

3.3 Positioning Error Calculation

Positioning error for the movement along each linear axis and its effect on the other axes has been calculated. The errors of the machine are shown in Fig. 5.

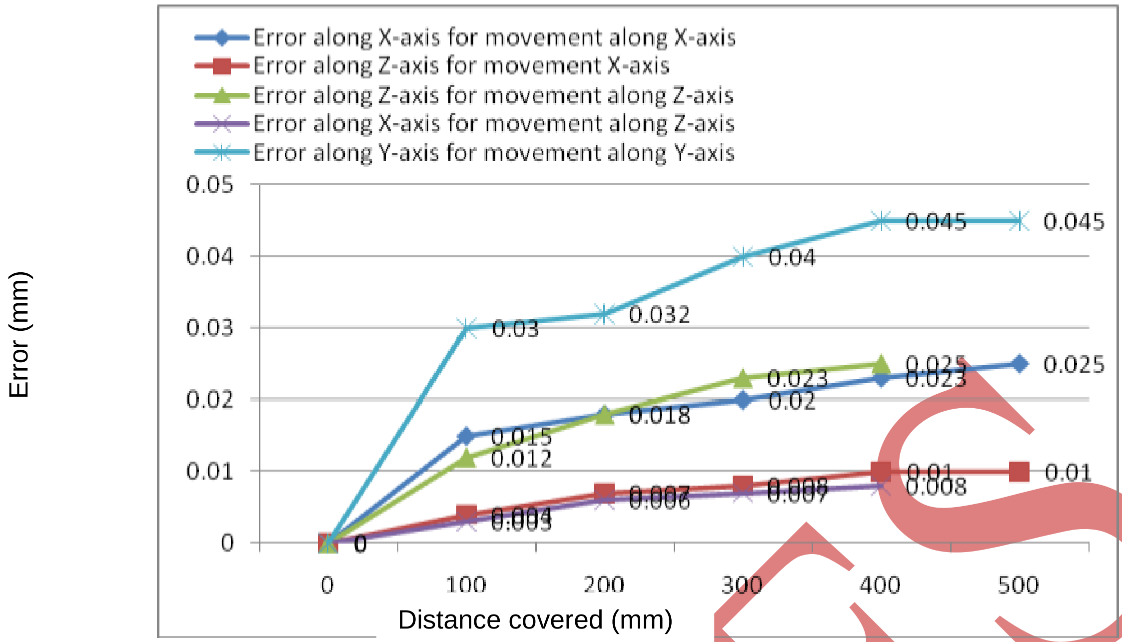

Fig.5: Positioning Error

3.4 Volumetric Error Calculation

As shown in the previous section , the volumetric accuracy is determined according to the translational and rotational errors of each axis . Namely , the X-direction component of volumetric error (VΔX) is the sum of positioning errors Δp_{1x} , Δp_{2x} , Δp_{3x} and Δp_{4x} . Similarly , the Z- and Y- directional components of volumetric error (VΔZ and VΔY) are the sum of Δp_{1z} , Δp_{2z} , Δp_{3z} and Δp_{4z} ; and Δp_{1y} , Δp_{2y} , Δp_{3y} and Δp_{4y} , respectively.
For instance , when the carriage moves 200 , 300 and 400 mm. along the X- , Z- , and Y- axis , respectively , the volumetric error at the cutting point can be obtained step by step as follows :

$$V\Delta X = 0.018 + 0.006 + 0.0$$
$$= 0.024 \text{ mm.}$$
$$V\Delta Z = 0.007 + 0.023 + 0.0$$
$$= 0.03 \text{ mm.}$$
$$V\Delta Y = 0 + 0 + 0.045$$
$$= 0.045 \text{ m}$$

The volumetric errors of the three–axis CNC machine tool for the movement along each axis has been calculated . Then the average error has been calculated and it has been pre-compensated through NC part program. The errors of the machine tool before compensation, and after compensation are graphically presented in Fig. 6.

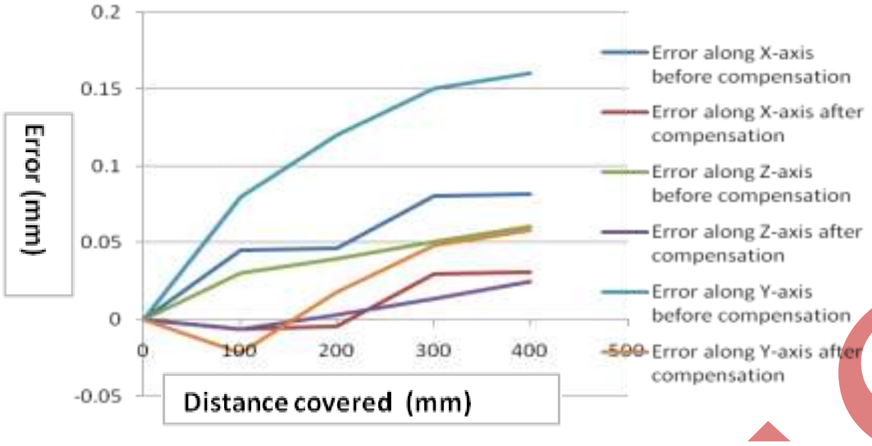

Fig. 6: Volumetric Error Before Compensation and After Compensation

3.5 Quality Improvement of Carrier

The effect of volumetric errors associated with three-axis machine tool in machining of Carrier for an automobile company , shown in Fig.7, has been studied .The quality of Carrier depends upon the machine tool errors. A practical verification has been done, by machining a Carrier under two conditions (1) without compensating the error of the machine tool , and (2) after pre-compensating the average errors in the work reference point of the machine tool. Now, both the components have been inspected on column type Coordinate Measuring Machine (CMM). The improvement in the quality of Carrier has been noticed and graphically presentation in Fig.8.

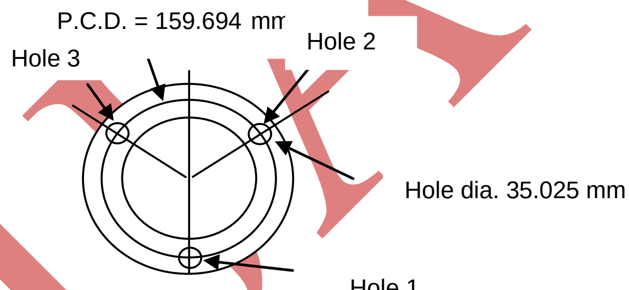

Fig.7: Carrier

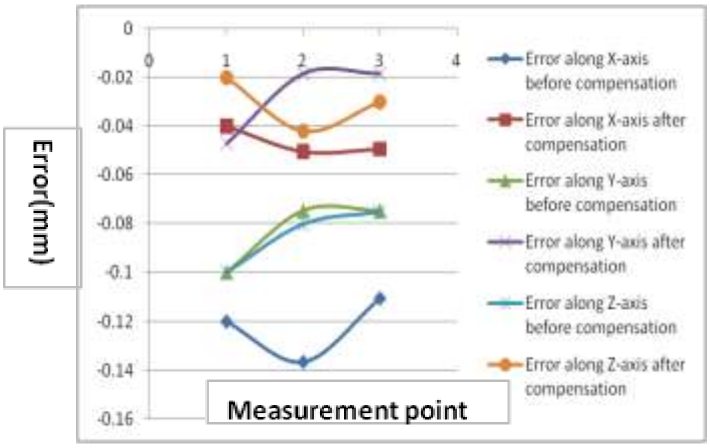

Fig.8: Error at Various Points of Carrier

IV. CONCLUSIONS

This paper describes a generalized error model at the cutter throughout the workspace due to positioning errors of individual components on a three-axis CNC machine tool. The theoretical errors associated has been practically verified and tested on the CNC machine using a 1-D ball array and micron dial indicator. The mathematical model of the volumetric has been derived by considering the interaction among the positioning error components along each axis of CNC machine tool . The effect of positioning error component on the volumetric accuracy at the cutting point has been examined. By correcting the CNC part machining program, the volumetric errors can be pre-compensated more easily before machining, and without any additional measuring device and hardware modification. The present method is more suitable and it can be applied more easily as compared to other methods available in literature.

The volumetric errors of the machine tool has been compensated, by the present error model, in machining a Carrier of an automobile industry. The quality of Carrier, after machining, has been checked up by CMM in both the cases, i.e. without compensating the errors of CNC machine and after compensating the errors of CNC machine. An improvement in the quality of Carrier has been observed by application of the present error model and error compensation .

In addition, the application of the present error model makes the production processes more flexible, automatic and controllable.

NOMENCLATURE

a_i	:	length of link
t_i	:	Twist between the axis of the joints
d_i	:	Distance between the normal a_i and a_{i-1} of the two links
x_i	:	Angle between the links measured as the angle between the normals a_i and ai_{-1} in the plane normal to the axis of the joint
i, i - 1	:	Notation for link
X	:	Movement along X-axis
Y	:	Movement along Y-axis

Z : Movement along Z-axis

θ_4 : Rotation of work piece

L : Length of tool

$T_{i-1,i}$: Homogeneous transformation matrix

$V\Delta X$: Volumetric error in X-direction

$V\Delta Y$: Volumetric error in Y-direction

$V\Delta Z$: Volumetric error in Z-direction

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