International Journal of Advanced Technology in Engineering and Science www.ijates.com
Volume No.02, Special Issue No. 01, September 2014 ISSN (online): 2348 – 7550

INFLUENCE OF INDUCED STRAIN ACTUATION ON STATIC DEFLECTION AND FREE VIBRATION RESPONSE OF PIEZO-LAMINATED PLATES

Mary Roshni Roy

Department of Structural (Civil) Engineering, Amal Jyothi College of Engineering,
Kottayam, Kerala, (India)

ABSTRACT

Smart structures technology is one of the most rapidly growing technologies and its applications extend to various fields. Piezoelectric effect is the most common smart effect studied due to its wide range of applications in the engineering field and also due to its advantages over other smart effects. Smart structures are built with piezoelectric patches surface-bonded or as embedded sheet actuators on laminated composite beams or plates. A strain actuation is induced in the plate and this induced strain controls its bending, extension and twisting. This work presents piezo-laminated plates with induced strain actuation. The formulation for the analysis is based on Kirchoff's hypothesis. The work theoretically validates the implementation of a multilayered three-dimensional model based on the analogy between thermal strains and piezoelectric strains. To assess the piezoelectric-thermal analogy for different loading conditions, the numerical results obtained from this model are compared to the results obtained from a finite element reference model based on a three-dimensional piezoelectric formulation. The static deflection for various loading conditions and free vibration frequencies are obtained and are verified whether in good agreement with those obtained from Finite Element Methods.

Keywords: Free Vibration, Piezoelectric Effect, Smart Structure, Static Deflection, Thermal Analogy

I. INTRODUCTION

Smart materials possess adaptive capabilities in response to external stimuli, such as loads or environment, with inherent 'intelligence'. The stimuli could be pressure, temperature, electric and magnetic fields, chemicals or nuclear radiation. The changeable physical properties could be shape, stiffness, viscosity or damping. The kind of smart material is generally programmed by material composition, special processing, and introduction of defects or by modifying the micro-structure, so as to adapt to the various levels of stimuli in a controlled fashion. Optical fibres, piezo-electric polymers and ceramics, electro-rheological fluids, magneto-strictive materials and shape memory alloys are some of the smart materials. [1]

Smart materials can be either active or passive. Active smart materials possess the capacity to modify their geometric or material properties under the application of electric, thermal or magnetic fields, thereby acquiring an inherent capacity to transduce energy. Smart materials that lack the inherent capacity to transduce energy are called passive smart materials. Fibre optic material is a passive smart material whereas piezoelectric materials, shape memory alloys(SMAs), electro-rheological(ER) fluids are all active smart materials. Piezoelectric materials are the most common type of smart materials and have a wide range of applications. They are used in accelerometers, strain sensors, emitters and receptors of stress waves, vibration dampers, underwater acoustic absorption, robotics, smart skins for submarines etc. The most recent advancements in the field of smart materials and structures are:

- Materials which can restrain the propagation of cracks by automatically producing comprehensive stresses around them (Damage arrest).
- Materials which can discriminate whether the loading is static or shock and can generate a large force against shock stresses (Shock absorbers).
- Materials possessing self-repairing capabilities, which can heal damages in due course of time (Self-healing materials).
- Materials which are usable up to ultra-high temperatures by suitably changing composition through transformation (Thermal mitigation). [1]

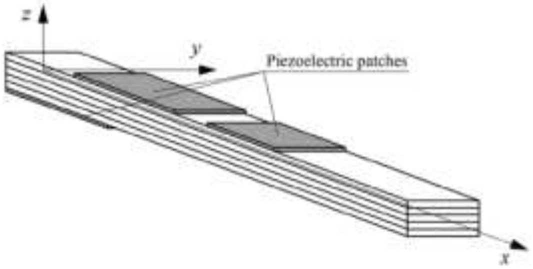


Fig 1: A laminated plate with surface bonded piezoelectric actuator and sensor

A smart system/ structure can be hence defined as "A system or material which has built-in or intrinsic sensor, actuator or control mechanism whereby it is capable of sensing a stimulus, responding to it in predetermined manner and extend, in a short appropriate time, and reverting to its original state as soon as the stimulus is removed." [2] Based on level of sophistication, smart structures are classified as:

- Sensory Structures
- Adaptive Structures
- Controlled Structures
- Active Structures
- Intelligent Structures

Structural vibrations are controlled by modifying the mass, stiffness and damping of the structure. This increases the overall mass of the structure and is found to be unsuitable for controlling low frequency vibrations. This method does not suit applications where weight restrictions are present and low frequency vibrations are encountered. For such applications, smart structures are being developed which are light weight and attenuate low frequency vibrations. A structure in which external source of energy is used to control structural vibrations is called a 'smart structure' and the technique is called active vibration control(AVC). A smart structure essentially consists of sensors to capture the dynamics of the structure, a processor to manipulate the sensor signal, actuators to obey the order of processor and a source of energy to actuate the actuators. [3]

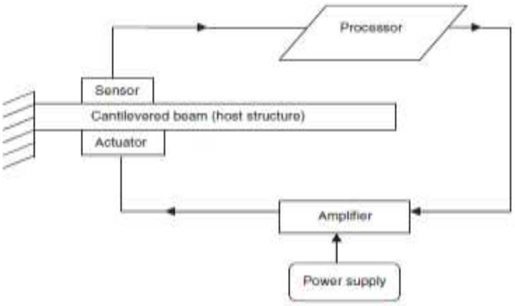


Fig 2: Schematic of a smart structure

Many materials such as piezoelectric materials, shape memory alloys, electro-strictive materials, electro-magneto-strictive materials, electro and magneto rheological fluids etc can be used for suppressing vibrations. Piezoelectric sensors/ actuators are more widely used because of its excellent electromechanical properties, fast response, easy fabrication, design flexibility, low weight, low cost, large operating bandwidth, low power consumption, generation of no magnetic field while converting electrical energy into mechanical energy etc. Piezoelectric materials generate strains when an electric signal is applied on them and vice versa. So they are used as sensors and actuators for structural vibrations. They are used in the form of distributed layers, surface bonded patches, embedded patches, cylindrical stacks, active fiber composite patches etc. Surface mounted or embedded piezoelectric patches can control a structure better than a distributed one because the influence of each patch on the structural response can be individually controlled.

The development of durable and cost effective high performance construction materials is important for the economic well-being of a country. The use of smart materials is encouraged for the optimal performance and safe design of buildings and other infrastructures particularly those under the threat of earthquake and other natural hazards. There is a wide range of structural applications for smart materials. SMAs can repeatedly absorb large amounts of strain energy under loading without permanent deformation. They have an extraordinary fatigue resistance and great durability and reliability. SMAs can be used for the active structural vibration control. When smart materials are used in composites, they can be monitored externally throughout the life of the structure to relate the internal material condition by measuring stress, moisture, voids, cracks, discontinuities via a remote sensor.[1]

SMAs can be used as self-stressing fibres and thus can be applied for retrofitting. Self-stressing fibres are the ones in which reinforcement is placed into the composite in a non-stressed state. A pre-stressing force is introduced into the system without the use of large mechanical actuators, by providing SMAs. Self-healing behavior of smart structures is one of its major structural applications. Use of piezo-transducers, surface bonded to the structure or embedded in the walls of the structure can be used for structural health monitoring and local damage detection.

Smart materials offer several advantages over conventional materials. They include:

- Improved strength
- Better stiffness
- Fatigues and impact resistance
- Corrosion and wear resistance
- Longer life
- Damping

- Cost advantage
- Weight reduction

Smart materials also possess certain drawbacks when compared to conventional materials which include:

- Complexity in the structure
- Difficulty in repairing
- Reuse and disposal is difficult
- Availability

The design process of smart structures involves three main phases, namely, structural design, optimal size and location of actuators/sensors and controller design. Performance of AVC depends on the proper placement of piezoelectric sensors and actuators. The positioning of piezoelectric patches should be in such a way that the structure should not be unstable. Optimization techniques are used in AVC to find the optimal sensor/actuator locations in smart structures. There are five optimizing criteria based on which the locations are fixed which are briefly explained:

A. Maximizing Modal Forces/Moments Applied by Piezoelectric Actuators

Piezoelectric actuators are desired to strain the host structure in a direction opposite to the strains developing in the host structure. So, the piezoelectric actuators should be placed in the regions of high average strains and away from the areas of zero strain. If an electric field is applied across piezoelectric actuators in the same direction, the host structure will be deformed in extension mode. If the field is applied across piezoelectric actuators in the opposite direction, the host structure will be deformed in bending mode.

B. Maximizing deflection of the Host Structure

When an external voltage is applied on the surface bonded piezoelectric actuator, it produces transverse deflections in the host structure. Transverse deflection of the host structure is a function of actuator placement. So, transverse deflection of the host structure can be used as a criterion for optimal placement of actuators.

C. Maximizing Degree of Controllability

The smart structure should be controllable for effective active vibration control. Controllability is a function of both the system dynamics and location and number of actuators. Matrix B of the system is determined by actuator locations on the smart structure. A standard check for the controllability of a system is a rank test of the global matrix.

D. Maximizing Degree of Observability

A closed loop system is completely observable if, examination of the system output determines information about each of the state variables. If one state variable cannot be observed in this way, the system is said to be unobservable. Observability is a function of both system dynamics and location and number of sensors. The output influence matrix C is determined by the position of sensors on the smart structure.

E. Minimizing Spillover Effects

A smart flexible structure is discretized into finite number of elements for vibration analysis and control. Only first few low frequency modes of interest are considered which results in a reduced model. A reduced model may excite residual modes. These residual modes appear in sensor output but are not included in the control design. This closed-loop interaction with low damping of residual modes, results in spillover effects, which should be minimized. [3]

These optimization criteria should be followed for optimal placement of actuators and sensors in a smart structure.

In this work, numerical results are obtained using Whitney's formulations for static deflection and free vibration derived using Classical Laminated Plate Theory with thermal analogy for deriving equivalent Piezoelectric strain. The results are compared with Finite Element Method done in Feast software.

FEAST (**F**inite **E**lement **A**nalysis of **ST**ructures) is the Indian Space Research Organisation (ISRO) structural analysis solver software based on Finite Element Method (FEM) realized by Structures group of Vikram Sarabhai Space Centre (VSSC). Initiated in 1977, the first version for the linear analysis of metallic structures was released in 1981.

II WORKING OF A SMART STRUCTURE

Based on the properties of the Piezoelectric material Lead Zirconate Titanate (PZT) the detection of damages, the assessment of their severity level in non-accessible RC members and the monitoring of the possible damage evolution with time are possible. A PZT sensor can produce electrical charges when subjected to a strain field and conversely mechanical strain when an electrical field is applied.

The Structural Health Monitoring (SHM) and damage detection techniques have been developed based on the coupling properties of the piezoelectric materials. The impedance-based SHM approach utilizes the electromechanical impedance of these materials that is directly related with the mechanical impedance of the host structural members, a property that is directly affected by the presence of any structural damage. Thus the impedance extracts and its inverse, the admittance, constitute the properties on which the PZT approach is based for the SHM of reinforced concrete structures. The produced effects by the structural damages on the PZT admittance signals are vertical enlargement or/and lateral shifting of the baseline signals of the initially healthy structure. These effects are the main damage indicators for damage detection.

To experimentally show the working of a smart structure, a RC model with piezoelectric patches is created in Feast software. The frequency response of healthy condition is first recorded, followed by a frequency response of a damaged condition which is applied to the model in the form of a hole and delaminate in the patches. The increase in admittance indicates damage and can be rectified by using suitable dampers in damage detected areas or by increasing the stiffness. In actual conditions, the admittance versus frequency signals are displayed by the sensors which gives signals once the actuators on the piezoelectric patches get actuated.

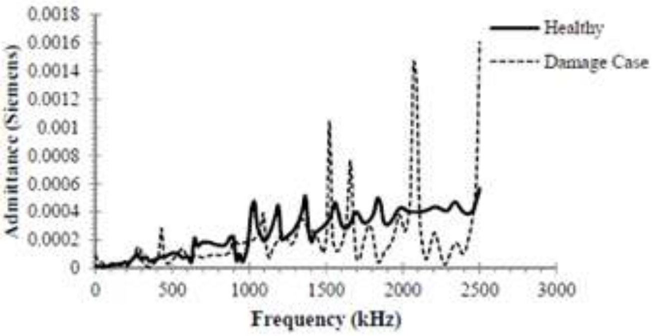


Fig 3: Frequency Response of healthy and damaged cases

III PIEZO-LAMINATED PLATE

Smart structures are constructed of thin composite laminates with surface embedded or surface bonded piezoelectric layers as induced strain actuators. In this work, strain given to the piezoelectric layer is in the form of thermal strain. Consider a piezo-laminated plate as shown in figure. Dimensions of the plate are *a* and *b* which are the plate length in x-direction and plate width in y-direction respectively. *h* is the total thickness of the plate in z-direction. It consists of two piezoelectric layers embedded on the top and bottom surfaces of the plate. The plate consists of four graphite-epoxy layers well stacked between the piezoelectric layers.

Laminated plates undergo large amplitude vibrations and deflections when they are subjected to large dynamic loading conditions.

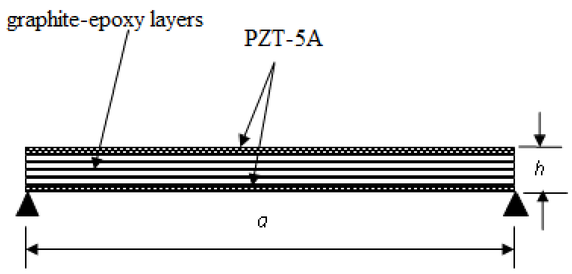


Fig 4 Simply supported piezo-laminated plate

The piezo-laminated plate considered consists of two distinctly different materials exhibiting two different behaviours. The composite layers contribute to the stiffness characteristic and the piezo layer act as active elements responding to external excitation owing to their piezoelectric characteristics. The heterogeneity in a composite material is not only because of the bi-phase composition, but also because of laminations. This leads to a distinctly different stress-strain behavior in laminates.

A. *System Equations*

Using the assumptions of Classical Laminated Plate Theory, the strain displacement and equations are motion are derived. From these standard equations the static deflection and free vibration formulations are developed.

The constitutive relation for any ply of generic coupled piezo-laminated beam with surface bonded or embedded induced strain actuator is:

$$\boldsymbol{\sigma} = \mathbf{Q_T}(\boldsymbol{\varepsilon} - \boldsymbol{\Lambda}) = \mathbf{Q_T}\boldsymbol{\varepsilon} - \boldsymbol{\sigma_\Lambda} \qquad (1)$$

where, $\mathbf{Q_T}$ - transformed reduced stiffness matrix,

$\boldsymbol{\varepsilon}$ - strain, $\boldsymbol{\Lambda}$- Actuation strain.

The total strain is expressed using the Kirchhoff's hypothesis as:

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{\circ}+z\boldsymbol{\kappa} \qquad (2)$$

where, the components of stress ($\boldsymbol{\sigma}$), strain ($\boldsymbol{\varepsilon}^{\circ}$), curvature changes ($\boldsymbol{\kappa}$), actuation strain ($\boldsymbol{\Lambda}$) and equivalent stress $\boldsymbol{\sigma_\Lambda}$ due to actuation are:

$$\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_z]^T\,; \quad \boldsymbol{\varepsilon}^{\circ} = [\varepsilon_x^{\circ}, \varepsilon_y^{\circ}, \varepsilon_{xy}^{\circ}]^T;$$

$$\boldsymbol{\kappa} = [\kappa_x, \kappa_y, \kappa_{xy}]^T; \ \boldsymbol{\Lambda} = [\Lambda_x, \Lambda_y, \Lambda_{xy}]^T \text{and}$$

$$\mathbf{Q}_T\boldsymbol{\Lambda} = [\sigma_{\Lambda x}, \sigma_{\Lambda y}, \sigma_{\Lambda xy}]^T \qquad (3)$$

The induced strain actuation is modeled by considering the constitutive equations for piezoelectric materials, which are poling direction dependant. The planar isotropy of the poled ceramics is expressed by their piezoelectric constants, such that piezoelectric charge constant $d_{31}=d_{32}$. The applied static electric field within the piezoelectric actuator is assumed to be constant as the thickness of the layer is relatively small.

In order to include the effects of expansional strains, the following Duhamel-Neumann form of Hooke’s law is followed

$$\in_i = \sum_{j=1}^{6} S_{ij}\sigma_j - \bar{\epsilon} \qquad (i= 1,2,\ldots\ldots 6) \qquad (4)$$

Where S_{ij} are the anisotropic compliances and $\bar{\in}$ are generalized expansional strains.

For thermal expansion,

$$\bar{\in} = \alpha_i \Delta T\ (x, y, z, t) \qquad (5)$$

The inverted form of the equation B.1 for plane stressis given by the relationship

$$\sigma_i = \sum_{j=1,2,6} Q_{ij}\ (\epsilon_j - \bar{\epsilon}_j) \qquad (i=1,2,6)\ (6)$$

Where $Q_{ij}$are the reduced stiffness for plane stress. Using contracted notation, the following transformation relations for expansional strains under the inplane rotation is obtained in the reduced form

$$\begin{bmatrix} \bar{\in}_1 \\ \bar{\in}_2 \\ \bar{\in}_6 \end{bmatrix} = \left\{ \begin{array}{c|c} m^2 & n^2 \\ n^2 & m^2 \\ 2mn & -2mn \end{array} \right\} \left\{ \begin{array}{c} \bar{\in}_L \\ \bar{\in}_T \end{array} \right\} \qquad (7)$$

Where $\in_L$ and $\in_T$ denote expansional strains parallel and transverse to the principal material axis. Thus in case of an off-axis orientation leads to an anisotropic shear coupling expansional strain $\bar{\in}_6$ relative to the x-y plane.

The laminate constitutive relation is hence obtained as

$$\begin{bmatrix} Nx \\ Ny \\ Nxy \\ Mx \\ My \\ Mxy \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \in^0{}_x \\ \in^0{}_y \\ \in^0{}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} \begin{bmatrix} \overline{Nx} \\ \overline{Ny} \\ \overline{Nxy} \\ \overline{Mx} \\ \overline{My} \\ \overline{Mxy} \end{bmatrix} \qquad (8)$$

And the expansional fore and moment resultants are defined as follows:

$$(\overline{Nx}, \overline{Mx}) = \int_{-h/2}^{h/2} (Q_{11} \in_1 + Q_{12} \in_2 + Q_{16} \in_6)(1, z)dz \qquad (9)$$

$$(\overline{Ny}, \overline{My}) = \int_{-h/2}^{h/2} (Q_{12} \in_1 + Q_{22} \in_2 + Q_{26} \in_6)(1, z)dz \qquad (10)$$

$$(\overline{Nxy}, \overline{Mxy}) = \int_{-h/2}^{h/2} (Q_{16} \in_1 + Q_{26} \in_2 + Q_{66} \in_6)(1, z)dz \qquad (11)$$

For bending of symmetric laminated plates,

$A_{16} = A_{26} = D_{16} = D_{26} = 0$, and B_{16}, B_{26} are the only non-vanishing elements of the coupling stiffness matrix Bij.

$$w = \sum_{m=1}^{M} \sum_{n=1}^{N} C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (12)$$

For a uniform load $q=q_0$=constant,

$$q_{mn} = \frac{16q_0}{\pi^2 mn} \quad \text{(m,n odd)}$$

$$q_{mn} = 0 \quad \text{(m,n even)}$$

$$C_{mn} = q_{mn}\frac{R^4 b^4}{\pi^4 D_{mn}}\left[(A_{11}m^2 + A_{66}n^2R^2)(A_{66}m^2 + A_{22}n^2R^2) - (A_{12} + A_{66})^2 m^2 n^2 R^2\right] \qquad (13)$$

Where,

$$D_{mn} = \Big\{\left[(A_{11}m^2 + A_{66}n^2R^2)(A_{66}m^2 + A_{22}n^2R^2) - (A_{12} + A_{66})^2 m^2 n^2 R^2\right]\Big[D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2R^2 + D_{22}n^4R^4 - \frac{\pi^2}{a^2}\left(\overline{N_x}m^2 + \overline{N_y}n^2R^2\right)\Big] 2m^2n^2R^2 (A_{12} + A_{66})(3(B_{16}m^2 + B_{26}n^2R^2)(B_{16}m^2 + B_{26}n^2R^2) - n^2R^2(A_{66}m^2 + A_{22}n^2R^2)(3B_{16}m^2 + B_{26}n^2R^2)^2 - m^2(A_{11}m^2 + A_{66}n^2R^2)(B_{16}m^2 + 3B_{26}n^2R^2)^2\Big\} \qquad (14)$$

For Free-vibration of unsymmetrical laminated plates,

$$\omega^2{}_{mn} = \frac{\pi^4}{\rho R^4 b^4}\Big\{D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2R^2 + D_{22}n^4R^4 - \frac{1}{J_6}\left[m(B_{16}m^2 + 3B_{26}n^2R^2)J_4 + nR(3B_{16}m^2 + B_{26}n^2R^2)J_5\right]\Big\} \qquad (15)$$

Where,

$$J_4 = (A_{11}m^2 + A_{66}n^2R^2)(B_{16}m^2 + 3B_{26}n^2R^2) - n^2R^2(A_{12} + A_{66})(3B_{16}m^2 + B_{26}n^2R^2)$$

$$J_5 = (A_{66}m^2 + A_{22}n^2R^2)(3B_{16}m^2 + B_{26}n^2R^2) - n^2R^2(A_{12} + A_{66})(B_{16}m^2 + 3B_{26}n^2R^2)$$

$$J_6 = (A_{11}m^2 + A_{66}n^2R^2)(A_{66}m^2 + A_{22}n^2R^2) - (A_{12} + A_{66})^2 m^2 n^2 R^2 \qquad (16)$$

When coupling is neglected (Bij=0), the equation becomes

$$\omega^2{}_{mn} = \frac{\pi^4}{\rho R^4 b^4}\left[D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2R^2 + D_{22}n^4R^4\right] \qquad (17)$$

which is the frequency equation for the flexural vibration of a simply-supported homogenous orthotropic plate. Coupling reduces the fundamental vibration frequency. The fundamental frequency of orthotropic laminates always occur for m=n=1.

Thermal analogy followed in the thesis is an indirect effect.

$$\delta\emptyset_A \rightarrow PZT \rightarrow \epsilon^P$$

$$\epsilon^T = \propto \Delta T$$

$$\epsilon^P = k.\delta\emptyset_A$$

$$= -d_{31}\delta\emptyset_A \qquad (18)$$

In order to find an equivalent piezoelectric strain with thermal analogy, we equate the two strains.

$$\epsilon^T = \epsilon^P$$

$$\propto \Delta T = -d_{31}\delta\emptyset_A \qquad (19)$$

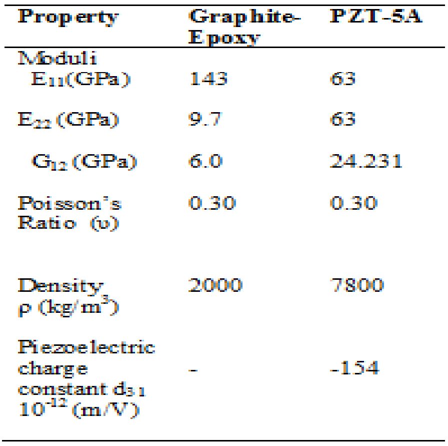
Breakdown Voltage = 75.9 MPa

Rated Stress = 20.7 MPa

$$\sigma = E\ \epsilon^P \qquad (20)$$

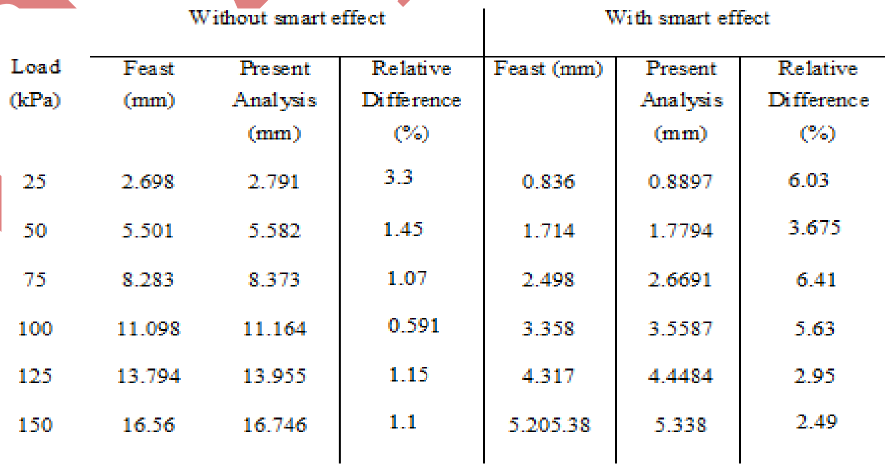
$$V = 1067V \cong 1kV$$

IV RESULTS AND DISCUSSION



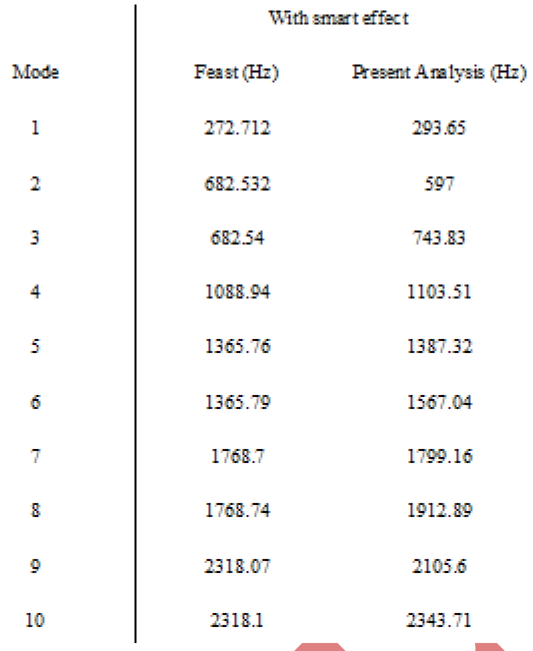
Property	Graphite-Epoxy	PZT-5A
Moduli E_{11}(GPa)	143	63
E_{22} (GPa)	9.7	63
G_{12} (GPa)	6.0	24.231
Poisson's Ratio (υ)	0.30	0.30
Density ρ (kg/m^3)	2000	7800
Piezoelectric charge constant d_{31} 10^{-12} (m/V)	-	-154

Table 1: Properties of materials used in the present analysis



Load (kPa)	Without smart effect			With smart effect		
	Feast (mm)	Present Analysis (mm)	Relative Difference (%)	Feast (mm)	Present Analysis (mm)	Relative Difference (%)
25	2.698	2.791	3.3	0.836	0.8897	6.03
50	5.501	5.582	1.45	1.714	1.7794	3.675
75	8.283	8.373	1.07	2.498	2.6691	6.41
100	11.098	11.164	0.591	3.358	3.5587	5.63
125	13.794	13.955	1.15	4.317	4.4484	2.95
150	16.56	16.746	1.1	5.205.38	5.338	2.49

Table 2: Comparison of Static deflection of piezo-laminated plates with and without smart effects



	With smart effect	
Mode	Feast (Hz)	Present Analysis (Hz)
1	272.712	293.65
2	682.532	597
3	682.54	743.83
4	1088.94	1103.51
5	1365.76	1387.32
6	1365.79	1567.04
7	1768.7	1799.16
8	1768.74	1912.89
9	2318.07	2105.6
10	2318.1	2343.71

Table 3: Comparison of Free Vibration values of present analysis and FEM of piezo-laminated plate

V CONCLUSION

The predicted results by the present formulation and finite element solutions utilizing FEAST software for static deflection and free vibration show differences due to transverse shear deformation not being considered in the formulation, assumed displacement field, Galerkin's approximation of the solution and high modular ration. The results are in good agreement and hence the piezo-electric thermal analogy adopted is a valid relation. It is clear that bending is comparatively less in case of smart structures and damages in the structure can also be rectified by various methods. Hence smart technology should be put to use in various fields as they are much more advantageous compared to conventional methods.

REFERENCES

[1] D.Y.Gao, D.L.Russell, An Extended Beam Theory for Smart Materials Applications, Applied Mathematics and Optimization 38:69-94, 1998

[2] F.L.Matthews, D.Hitchings, C.Santisi, Finite Element Modeling of Composite Materials and Structures, Woodhead Publishing Limited

[3] Vivek Gupta, Manu Sharma, Nagesh Thakur, Optimization Criteria for Optimal Placement of Piezoelectric Actuators/Sensors on a Smart Structure, Journal of Intelligent Material Systems and Structures, 21:1227, 2010

[4] Jayakumar K. ,Deepak P. ,Anil Kumar P. V, Release document FEAST^{SMT} Version9.5 , October 2013, Structural Modelling and Software Development division (SMSD), VSSC.

[5] Inderjit Chopra, Review of State of Art of Smart Structures and Integrated Systems, AIAA Journal Vol 40, No11, 2002

[6] F. Cote, P. Masson, N. Mrad, V. Cotoni, Dynamic and static modeling of piezoelectric composites, 2013.