VARIOUS FUZZY MODELING TECHNIQUES AND FUZZY CONTROLLER DESIGN FOR NONLINEAR SYSTEM

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ABSTRACT

Complex nonlinear systems can be represented by Takagi–Sugeno (TS) fuzzy systems with each local model. In this paper, we discuss the TS fuzzy modeling methods with linear and bilinear local models. This paper describes how to obtain a fuzzy bilinear model for a class of nonlinear systems. Also a nonlinear time delay System is represented with fuzzy blending of local linear models. A fuzzy state feedback controller is designed to stabilize such systems. A simulation example is given to verify the designed controller.

Keywords: Fuzzy Bilinear System (FBS), Fuzzy Control, Nonlinear System, Takagi–Sugeno (T–S) Fuzzy Model.

I. INTRODUCTION

All physical systems are nonlinear in nature. Sometimes it is possible to describe the operation of physical system by a linear model, such as ordinary differential equations. But analyzing the behavior of any physical system, the linearized model is inadequate or inaccurate. We know the powerful analysis tool for linear systems, founded on the basis of the superposition principle. As we move from linear to nonlinear systems, we are face with different situation. The superposition principle does not hold any longer, and analysis tools involve more advanced mathematics. Because of the powerful tools we know for linear systems, the first step in analyzing a nonlinear system is usually to linearize it. There are two basic limitations of linearization. First it is an approximation in the neighborhood of an operating point. Second thing is that the dynamics of nonlinear systems are much richer than the dynamics of linear systems. Hence the linearization will not be sufficient; we must develop tools for the analysis of nonlinear system. The first step in analyzing a nonlinear system is usually linearizing about some nominal operating point and analyzing the linear model. It can only predict the "local" behavior of the nonlinear system in the vicinity of that point. It cannot predict the "nonlocal" behavior far from the operating point and certainly not the "global" behavior throughout the state space. Second, the dynamics of nonlinear systems are much richer than the dynamics of linear systems. Dynamical systems with time delay are common in chemical processes, microwave oscillators, nuclear reactors, long transmission lines in pneumatic, hydraulic, or rolling mill systems [1]-[4] (and the references therein). It is important to have new stability

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analysis and controller design methods for nonlinear time-delay systems. A typical approach for the analysis and synthesis of nonlinear system with time delay is the local linearization approach. First a linearization model on the nominal operating point is gotten and then a linear feedback control is designed for this linear model. Fuzzy logic control is another approach to obtain nonlinear control systems, especially in the presence of incomplete knowledge of the plant or even of the precise control action appropriate to a given situation. First the nonlinear plant is represented by a dynamic fuzzy model. In this type of fuzzy model, local dynamics in different state-space regions are represented by linear models. The overall model of the system is achieved by fuzzy "blending" of these fuzzy models. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller, which is nonlinear in general, is again a fuzzy blending of each individual linear controller. In this paper, we extend the above procedure to the nonlinear systems with time delay. But it is impossible to approximate some equations by a linear model then we have to go for bilinear model. As stated, a bilinear system is different from a linear one and is considered a type of nonlinear system. In recent years, bilinear systems and controls have been widely applied to a wide variety of fields, for example, bioengineering, biochemistry, nuclear engineering and socio-economics. A bilinear system exists between nonlinear and linear systems, and its dynamic is simpler than that of nonlinear systems. In fact, "the bilinear systems provide much better approximation of the original nonlinear systems than the linear systems," and this is why we are investigating the T–S fuzzy bilinear system. It is well known that fuzzy control has been attracting increasing attention in the study and application of the stabilization of nonlinear systems [8], [9].

II. TAKAGI-SUGENO FUZZY MODEL

2.1. Introduction

It is similar to the Mamdani method in many respects. The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator, are exactly the same. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant. A typical rule in a Sugeno fuzzy model has the form

If $Input\ 1 = x$ and $Input\ 2 = y$, then $Output$ is $z = ax + by + c$ (1)

For a zero-order Sugeno model, the output level z is a constant (a=b =0). The output level z_i of each rule is weighted by the firing strength w_i of the rule. For example, for an AND rule with $Input\ 1 = x$ and $Input\ 2 = y$, the firing strength is

$$w_i = AndMethod(F_1(x), F_2(y)) \quad (2)$$

where $F_{1,2}(.)$ are the membership functions for $Input\ 1$ and $Input\ 2$.

The final output of the system is the weighted average of all rule outputs, computed as

$$Final\ Output = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i} \quad (3)$$

where N is the number of rules.

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A Sugeno rule operates as shown in the following diagram

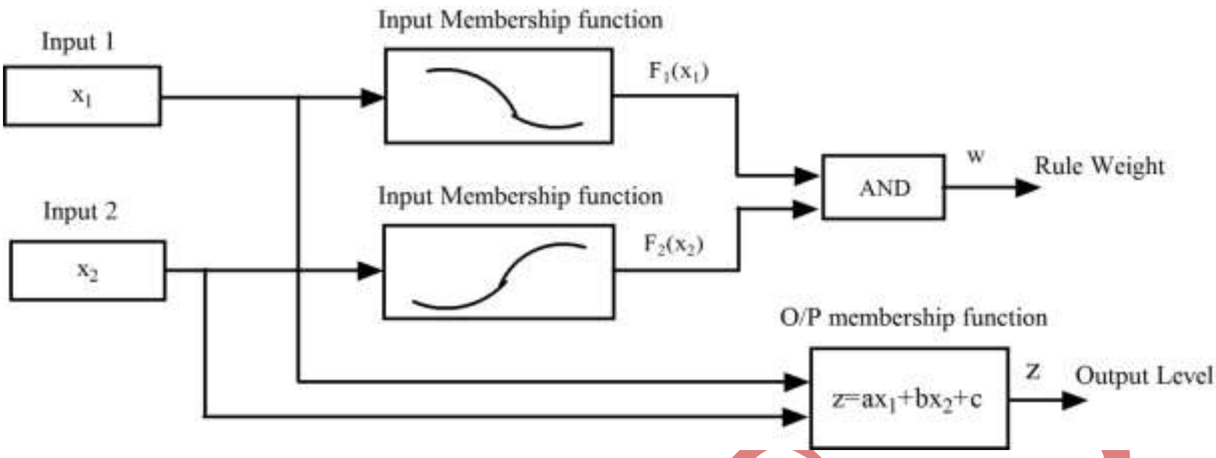

Fig. 1. Sugeno Rule Operation

Takagi and Sugeno proposed an effective way to represent a fuzzy model of a nonlinear dynamic system. It uses a linear input/output (I/O) relation as its consequence of individual plant rules. A TS fuzzy time-delay model is composed of r plant rules that can be represented as follows.

IF $\theta_1(t)$ *is* μ_{i1} *and ... and* $\theta_p(t)$ *is* μ_{ip} *THEN*

$$\dot{x}(t) = A_{1i}x(t) + A_{2i}x(t - r(t)) + B_i u(t)$$
$$y(t) = C_{1i}x(t) + C_{2i}x(t - r(t))$$
$$x(t) = \psi(t), \quad t \in [-\tau_0, 0],$$
$$i = 1, \dots, r \qquad (4)$$

where μ_{ij} is the fuzzy set, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector and $y(t) \in \mathbb{R}^q$ is the output vector, and $A_{1i}, A_{2i}, B_i, C_{1i},$ *and* C_{2i} are some constant matrices of compatible dimensions, r is the number of IF-THEN rules, and $\theta(t) = [\theta_1(t)\ \theta_2(t) \dots \theta_p(t)]$ are the premise variables. It is assumed that the premise variables do not depend on the input variables explicitly. $r(t) \leq \tau_0$ is the bounded time-varying delay in the state and it is assumed that

$$\dot{r}(t) \leq \beta < 1 \qquad (5)$$

The derivative of the time-varying delay function is continuous and bounded, which is a natural supplementary condition. $\psi(t) \in C_{n,\tau_0}$ is a vector-valued initial continuous function. Given a pair of$(x(t), u(t))$, the final outputs of the fuzzy Systems are inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=0}^{r} w_i(\theta(t))[A_{1i}x(t) + A_{2i}x(t - r(t)) + B_i u(t)]}{\sum_{i=1}^{r} w_i(\theta(t))} \qquad (6)$$

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$$y(t) = \frac{\sum_{i=0}^{r} w_i(\theta(t))\left[C_{1i}x(t) + C_{2i}x(t - r(t))\right]}{\sum_{i=1}^{r} w_i(\theta(t))} \qquad (7)$$

2.2 Fuzzy Bilinear Model

In recent years, bilinear systems and controls have been widely applies to variety of fields, for example, bioengineering, biochemistry, nuclear engineering and socio-economic. A bilinear system exists between nonlinear and linear systems, and its dynamic is simpler than that of nonlinear systems. Also bilinear systems are described by ordinary differential equations that are linear in state and linear in control, but not jointly linear in both processes are very common in man and his environment.By examination of a modelling problem, we describe how to transform a nonlinear system into a bilinear one via Taylor's series expansion. Consider the nonlinear equation

$$\dot{x} = f(x(t), u(t)) \qquad (8)$$

where $x \in R^n, u \in R, f \in R^n$. Above System is assumed to be continuously differentiable with$f(x_e, u_e) = 0$ where $(x_e, u_e)$is the equilibrium point. By Taylor's series expansion equation (8) can decompose into

$$f(x,u) = \sum_{n-1}^{N-1} \frac{1}{n!}\left[(x - x_e)\frac{\partial}{\partial x} + (u - u_e)\frac{\partial}{\partial u}\right]^n f(x,u)\Big|_{(x_e,u_e)} + RN \qquad (9)$$

where RN denotes the n^{th} higher order Taylor remainder term.
As N=3, equation (9) becomes

$$f(x,u) = \sum_{n=1}^{2} \frac{1}{n!}\left[(x - x_e)\frac{\partial}{\partial x} + (u - u_e)\frac{\partial}{\partial u}\right]^n f(x,u)_{(x_e,u_e)} + R3 \qquad (10)$$

If the Hessian term of the $f(x,u)$ is zero or sufficiently small and R3 is very close to zero, then

$$f(x,u) \approx \left[\frac{\partial f}{\partial x}\Big|_{(x_e,u_e)}(x - x_e) + \frac{\partial f}{\partial u}\Big|_{(x_e,u_e)}(u - u_e)\right]$$
$$+\frac{1}{2}\left[\frac{\partial^2 f}{\partial x \partial u}\Big|_{(x_e,u_e)}(x - x_e)(u - u_e) + \frac{\partial^2 f}{\partial u \partial x}\Big|_{(x_e,u_e)}(x - x_e)(u - u_e)\right] \qquad (11)$$

where $x_\delta = x - x_e$ and $u_\delta = u - u_e$. Then the bilinear approximation of equation (8) can be described by,

$$\dot{x_\delta} = f(x,u) \approx \left[\frac{\partial f}{\partial x}\Big|_{(x_e,u_e)} x_\delta + \frac{\partial f}{\partial u}\Big|_{(x_e,u_e)} u_\delta\right] + \left[\frac{\partial^2 f}{\partial x \partial u}\Big|_{(x_e,u_e)} x_\delta u_\delta\right] \qquad (12)$$

$$\dot{x_\delta} = Ax_\delta + Bu_\delta + Nx_\delta u_\delta$$

where

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$$A = \frac{\partial f}{\partial x}\Big|_{(x_e,u_e)} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}\right|_{(x_e,u_e)} \quad (13)$$

$$B = \frac{\partial f}{\partial u}\Big|_{(x_e,u_e)} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_2}{\partial u} \cdots & \frac{\partial f_n}{\partial u} \end{bmatrix}\right|_{(x_e,u_e)} \quad (14)$$

$$N = \frac{\partial^2 f}{\partial x \partial u}\Big|_{(x_e,u_e)} = \left.\begin{bmatrix} \frac{\partial^2 f_1}{\partial x \partial u} & \frac{\partial^2 f_1}{\partial x \partial u} & \cdots & \frac{\partial^2 f_1}{\partial x \partial u} \\ & \vdots & \ddots & \vdots \\ \frac{\partial^2 f_n}{\partial x_1 \partial u} & \frac{\partial^2 f_n}{\partial x_2 \partial u} & \cdots & \frac{\partial^2 f_n}{\partial x_n \partial u} \end{bmatrix}\right|_{(x_e,u_e)} \quad (15)$$

Suppose $(x_e, u_e) = (0,0)$ then equation (8) becomes

$$\dot{x} = Ax + Bu + Nxu \quad (16)$$

This is the bilinear approximation of equation (8) at zero equilibrium point. It is well known that a linear approximation of equation (8) is

$$\dot{x} = Ax + Bu \quad (17)$$

The linear model can be considered as a special class of the bilinear system [7].

$$f(x,u) = F(x) + G(x,u) + Nxu \quad (18)$$

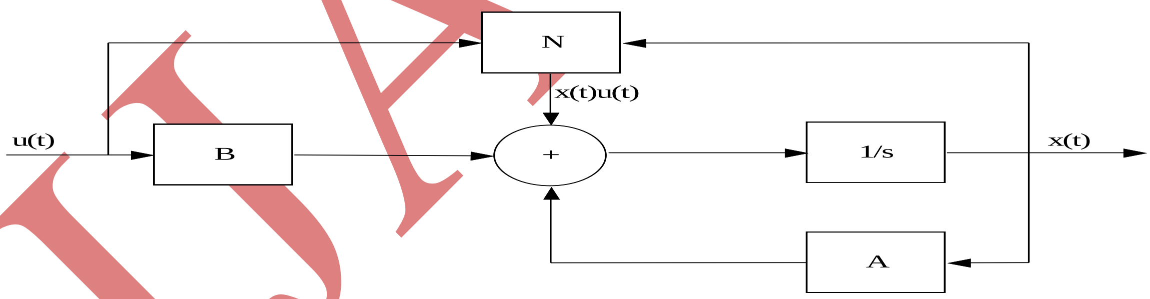

Fig. 2. General Form Of Bilinear System

III. FUZZY LOGIC CONTROLLER

Based on the parallel distributed compensation (PDC), we consider the following fuzzy control law for the fuzzy model (4)

IF $\theta_1(t)$ is μ_{i1} and ... and $\theta_p(t)$ is μ_{ip} THEN $u(t) = -F_i x(t), \quad i = 1, 2, \ldots, r.$ (19)

The overall state feedback fuzzy control law is represented by

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$$u(t) = -\frac{\sum_{i=0}^{r} w_i(\theta(t)) F_i x(t)}{\sum_{i=1}^{r} w_i(\theta(t))} \qquad (20)$$

3.1. Nonlinear Time Delay System Example

Consider a nonlinear time delay system given by equations

$$\dot{x}_1(t) = f_1(x) + \left(\frac{1}{\lambda} - 1\right) x_1(t-\tau) \qquad (21)$$

$$\dot{x}_2(t) = f_2(x) + \left(\frac{1}{\lambda} - 1\right) x_2(t-\tau) + \beta u \qquad (22)$$

$$f_1(x) = \frac{-1}{\lambda} x_1(t) + D_a(1 - x_1(t)) exp\left(\frac{x_2(t)}{1 + x_2(t)/\gamma_0}\right) \qquad (23)$$

$$f_2(x) = -\left(\frac{1}{\lambda} + \beta\right) x_2(t) \quad + HD_a(1 - x_1(t)) exp\left(\frac{x_2(t)}{1 + x_2(t)/\gamma_0}\right) \qquad (24)$$

$$x_i(t) = \theta_i(t) \quad \text{For,} \quad t \in [-\tau, 0], \qquad i = 1,2.$$

The state $x_1(t); 0 \leq x_1(t) \leq 1$ and $x_2(t)$ is the dimensionless variable. Clearly, we must restrict $\lambda \neq 0$, or the right-hand side of both (21) and (22) become invalid. Constants H, β, D_a, γ_0 and τ are all positive. We assume that only $x_2(t)$ can be measured on line,

$$y(t) = [0 \quad 1] x(t). \qquad (25)$$

3.2 Linearizing The System Around Operating Point

The system described by (21) and (22) is a classical nonlinear system with time delay. In this section, we will consider designing a simple fuzzy control law to stabilize the system. In the following, we use the analytical approach of [7] to model the nonlinear delay system (21) and (22). Consider the nonlinear delay system described by (21) and (22), whose linearization around the stationary point is given by

$$\dot{x}(t) = A_1(x^s, u^s)(x(t) - x^s) + A_2(x(t-\tau) - x^s) + B(u - u^s) \qquad (26)$$

Where $x_s = [x_1^s \quad x_2^s]^T$

$$A_1(x^s, u^s) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_2}{\partial x} \end{bmatrix}^T (x^s)$$

$$A_2(x^s, u^s) = \begin{bmatrix} \frac{1}{\lambda} - 1 & 0 \\ 0 & \frac{1}{\lambda} - 1 \end{bmatrix}$$

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$$B(x^s, u^s) = \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

We present the following fuzzy control law for any an expected operating point (x_d, u_d), which is a stationary point of the nonlinear system [6].

Rule 1: If $x_2(t)$ is low (i.e., $x_2(t)$ is about 0.8862)

THEN $\delta\dot{x}(t) = A_1^1\delta x(t) + A_2^1\delta x(t-\tau) + B^1\delta u(t)$; $\delta u(t) = -F_1\delta x(t)$

Rule 2: If $x_2(t)$ is Middle (i.e., $x_2(t)$ is about 2.7520)

THEN $\delta\dot{x}(t) = A_1^2\delta x(t) + A_2^2\delta x(t-\tau) + B^2\delta u(t)$; $\delta u(t) = -F_2\delta x(t)$

Rule 2: If $x_2(t)$ is High (i.e., $x_2(t)$ is about 4.7052)

THEN $\delta\dot{x}(t) = A_1^3\delta x(t) + A_2^3\delta x(t-\tau) + B^3\delta u(t)$; $\delta u(t) = -F_3\delta x(t)$

Where $\delta x(t) = x(t) - x_d, \delta x(t-\tau) = x(t-\tau) - x_d, \delta u(t) = u(t) - u_d$ and F_1, F_2, F_3 are to be designed.

We choose the fuzzy membership functions as shown in Figure.3.

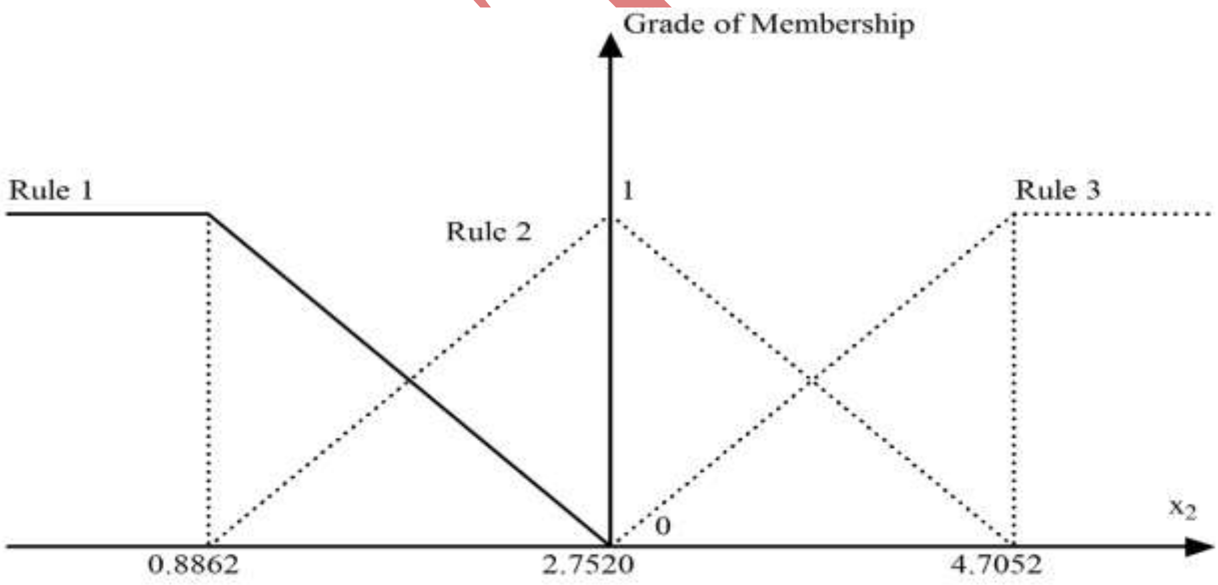

Fig.3. Membership Functions Of Fuzzy Model

IV. CONCLUSION

This paper has presented design methods for modeling and control of a class of nonlinear systems based on TS fuzzy modeling and control approach. Initially, TS-fuzzy modeling techniques for nonlinear system are discussed. The design methodology is illustrated by application to the stabilization problem of nonlinear retarded system. The system responses for the different state conditions are obtained. The open loop and closed loop responses are shown in figure 4. It can be observed that the closed loop response reaches to the desired states.

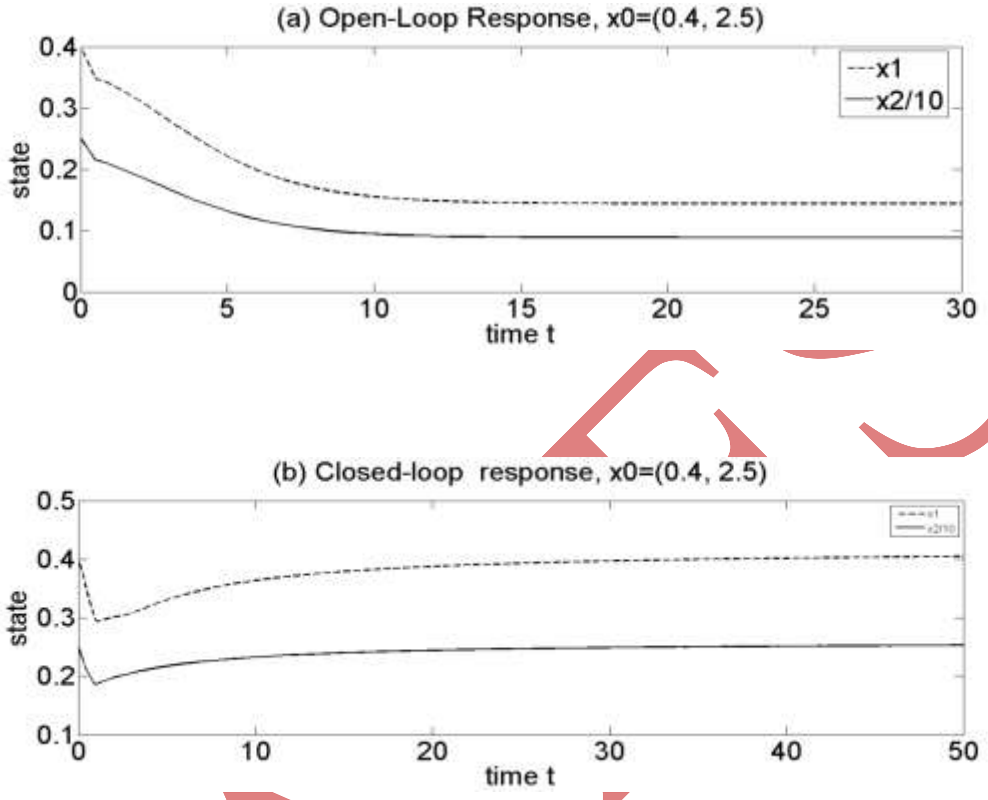

Fig.4. Different state responses under operating point (0.4472, 2.7520).

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