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APPLICATION OF FUZZY INFORMATION MEASURE TO CODING THEORY

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ABSTRACT

The objective of the present communication is to develop new measure L_α^β, called average code word length of order α and type β and establish its relationship with a result of generalized fuzzy information measures. Using L_α^β, Noiseless coding theorems connected with fuzzy information measure have been proved.

Keywords: Fuzzy Information Measure, Codeword Length, Uniquely Decipherable Codes, Kraft's Inequality, Noiseless Channel.

I. INTRODUCTION

Information theory is a relatively new branch of Mathematics that was made mathematically rigorous only in the 1940s. Information theory deals with the study of problems concerning any system. This includes information processing, information storage, information retrieval and decision making. , information theory studies all theoretical problems connected with the transmission of information over communication channels. This includes the study of uncertainty (information) measures and various practical and economical methods of coding information for transmission. The first studies in this direction were undertaken by Nyquist [9] in 1924 and 1928 [10] and by Hartley in 1928 [27], who recognized the logarithmic nature of the measure of information. In 1948, Shannon [5] published a remarkable paper on the properties of information sources and of the communication channels used to transmit the outputs of these sources. Around the same time Wiener [22] also considered the communication situation and came up, independently, with results similar to those of Shannon. In the past fifty years the literature on information theory has grown quite voluminous and apart from communication theory it has found deep applications in many social, physical and biological sciences, for example, economics, statistics, accounting, language, psychology, ecology, pattern recognition, computer sciences, fuzzy sets, etc. A key feature of Shannon information theory is the term *"information"* that can often be given a mathematical meaning as a numerically measurable quantity, on the basis of a probabilistic model, in such a way that the solutions of many important problems of information storage and the transmission can be formulated in terms of this measure of the amount of information. As pointed out by Renyi [3] in his fundamental paper on generalized information measures, in other sort of problems other quantities may serve just as well, or even better, as measures of information. This should be supported either by their operational significance or by a set of natural postulates characterizing them, or, preferably, by both. Thus the idea of generalized entropies arises in the literature. It started with Renyi [3] who characterized scalar parametric entropy as entropy of order r, which includes Shannon entropy as a limiting case. Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. In 1978, Zadeh [16] first created the theory of fuzzy, which is related to fuzzy set theory. His study showed that the importance of the theory of fuzzy is based on the fact that much of the information on which human decisions is possibilistic rather than probabilistic in nature. Fuzzy set theory is being recognized as an important problem modeling and solution technique. The use of fuzzy set theory as a methodology for modeling and analyzing decision systems is of particular interest to researchers. In 1976, Zimmermann [11] first introduced fuzzy set theory into an ordinary linear programming problem with fuzzy objective and constraints. Zadeh [16] introduced the concept of fuzzy sets in which imprecise knowledge can be used to define an event.To explain the concept of fuzzy entropy in general, Kapur [12] considered the following vector:

$$(\mu_A(x_1), \mu_A(x_2), \ldots\ldots \quad \ldots\ldots \ \mu_A(x_n))$$

Where $\mu_A(x_i)$ gives the perception of the grade of membership of the i^{th} element of set A. Thus if $\mu_A(x_i)$ = 0 then the i^{th} element certainty does not belong to set A and if $\mu_A(x_i)$ = 1, it definitely belongs to set A. If $\mu_A(x_i)$ =0.5, there is maximum certainty whether x_i belongs to set A or not. The above vector in which every element lies between 0 and 1 and has the interpretation given above, is called fuzzy vector and the set A is called a fuzzy set. If every element of the set is 0 or 1, there is no uncertainty about it and the set is said to be a crisp set.Thus there are 2^n crisp sets with n elements and infinity many sets with n elements.If Some elements are 0 or 1 and the others lie between 0 or 1 the set will still said to be a fuzzy set. With the i^{th} element, we associate a fuzzy uncertainty f ($\mu_A(x_i)$), where f(x) has following properties:

I. f(x) = 0 when x = 0 or 1
II. f(x) increases as x goes from 0 to 0.5
III. f(x) decreases as x goes from 0.5 to1.0
IV. f(x)=f(1-x)

If the n elements are independent, the total fuzzy uncertainty is given by,

$$H(A) = \sum_{i=1}^{n} f(\mu_A(x_i))$$

The fuzzy uncertainty is called fuzzy entropy.

Klir and Folger [8] stated that the term fuzzy entropy was apparently due to clarity of product terms in the following two expressions:

$$H(A) = \sum_{i=1}^{n} \mu_A(x_i) \ln \mu_A(x_i) - \sum_{i=1}^{n} (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i))$$

After this development, a large number of measures of fuzzy entropy were discussed, characterized and generalized by various authors.

In coding theory, Error-correcting codes play an important role in many areas of science and engineering. Error-correcting codes constitute one of the key ingredients in achieving the high degree of reliability required in modern data transmission and storage systems. We come across the problem of efficient coding of messages to be sent over a noiseless channel and attempt to maximize the number of messages that can be sent through a channel in a given time. Therefore, we find the minimum value of a mean codeword length subject to a given constraint on code- word lengths. Let us assume that the messages to be transmitted are generated by a random variable X and each value x_i, i = 1, 2, ..., n of X must be represented by a finite sequence of symbols chosen from the set { a_1, a_2, ..., a_D }. This set is called code alphabet or set of code characters and sequence assigned to each i x , i = 1, 2,....., n is called code word. Let n_i be the length of code word associated with x_i satisfying Kraft's [13] inequality given by the following mathematical expression:

$$D^{-l_1} + D^{-l_2} + \cdots + D^{-l_n} \leq 1$$

Where, D is the size of alphabet. In calculating the long run efficiency of communications, we choose codes to minimize average code word length, given by

$$L = \sum_{i=!}^{n} n_i p_i$$

Where p_i is the probability of occurrence of x_i .For uniquely decipherable codes, Shannon's[4] noiseless coding theorem states that

$$\frac{H(P)}{\log D} \leq L < \frac{H(P)}{\log D} + 1$$

Determines the lower bounds on L in terms of Shannon's entropy H(P).

Campbell [17] for the first time introduced the idea of exponentiated mean codeword length for uniquely decipherable codes and proved a noiseless coding theorem.

He considered a special exponentiated mean of order α given by

$$L_\alpha = \frac{\alpha}{1-\alpha} log_D \left[\sum_{i=1}^{n} p_i D^{(1-\alpha)n_i/\alpha} \right]$$

And showed that its lower bound lies between $R_\alpha(P)$ and $R_\alpha(P)$ + 1

Where $R_\alpha(P) = (1-\alpha)^{-1} log_D [\sum_{i=1}^{n} p_i^\alpha]$; $\alpha > 0, \alpha \neq 1$.

The above is Renyi's [3] measure of entropy of order α.As $\alpha \to 1$, it is easily shown that $L_\alpha \to L$ and $R_\alpha(P)$ approaches H(P).

It may be seen that the mean codeword length had been generalized parametrically and their bounds had been studied in terms of generalized measures of entropies. Here we give another generalization and study its bounds in terms of generalized fuzzy information measures of order α and type β.

Generalized coding theorems by considering different information measure under the condition of unique decipherability were investigated by several authors.

Arun Choudhary and Satish Kumar[1] proved some noiseless coding theorem on generalized R- Norm entropy.

Also , Arun Choudhary and Satish Kumar [2] proposed some coding theorems on generalized havrda-charvat and tsalli's entropy.

M.A.K. Baig and Mohd Javid Dar [19] & [20] introduced some Coding theorems on Fuzzy entropy Function Depending Upon Parameter R and V . Further, Fuzzy coding theorem on generalized fuzzy cost measure.

Parkash and P. K. Sharma [23] & [24] proved some noiseless coding theorems corresponding to fuzzy entropies and introduced a new class of fuzzy coding theorems.

Guiasu and Picard [6] defined the weighted average length for a uniquely decipherable code as:

$$\bar{L} = \sum_{i=1}^{n} \frac{u_i n_i p_i}{\sum_{i=1}^{n} u_i p_i}$$

Longo [18] interpreted this as the average cost of transmitting letters x_i with probability p_i and utility u_i and gave some practical interpretation of this length. Lower and upper bounds for the cost function in terms of weighted entropy have also been derived.

Longo [18] gave lower bound for useful mean codeword length in terms of quantitative-qualitative measure of entropy introduced by Belis and Guiasu [4]. Guiasu and Picard [6] proved a noiseless coding theorem by obtaining lower bounds for similar useful mean codeword length. Gurdial and Pessoa [7] tried to extend the theorem by finding lower bounds for useful mean codeword lengths of order α in terms of useful measures of information of order α .

Some other pioneer who extended their results towards the development of coding theory are Korada and Urbanke [15], Szpankowski [28], Merhav [21] etc. Recently, Kapur [14] has established relationships between probabilistic entropy and coding. But there are many situations where probabilistic measures of entropy do not work and to tackle such situations, instead of taking the idea of probability, the idea of fuzziness can be explored.

In the next section, we have considered the fuzzy distributions and developed a new mean codeword lengths by proving noiseless coding theorems.

In this paper we study noiseless coding theorem by considering a fuzzy information measure depending on two parameters.

II. FUZZY NOISELESS CODING THEOREM

In this section, we consider the following generalized parametric measure of fuzzy entropy:

$$H_\alpha^\beta(A) = \frac{1}{(1-\alpha)\beta} \sum_{i=1}^{n} \left(\{ \mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))} \}^\beta - 2^\beta \right)$$

,where $\alpha > 0, \alpha \neq 1, \beta \neq 0.$ (2.1)

Under the assumption, $0^{0.\alpha} = 1$ we study the following properties:

1. $H_\alpha^\beta(A) \geq 0$
2. When $\mu_A(x_i) = 0$, $H_\alpha^\beta(A) = 0$
3. When $\mu_A(x_i) = \frac{1}{2}$,

$$H_\alpha^\beta(A) = \frac{n.2^\beta}{(1-\alpha).\beta}\left(\frac{1-2^{\frac{\alpha\beta}{2}}}{2^{\frac{\alpha\beta}{2}}}\right)$$

Hence, $H_\alpha^\beta(A)$ is an increasing function of $\mu_A(x_i)$ for $0 \le \mu_A(x_i) \le \frac{1}{2}$

4. When $\mu_A(x_i) = 1$, $H_\alpha^\beta(A) = 0.$ Hence $H_\alpha^\beta(A)$ is an decreasing function of $\mu_A(x_i)$ for $\frac{1}{2} \le \mu_A(x_i) \le 1$

5. $H_\alpha^\beta(A)$ does not change when $\mu_A(x_i)$ is changed to (1 - $\mu_A(x_i)$)

Under the above conditions, the generalized measure proposed in (2.1) is a valid measure of fuzzy entropy.

For this fuzzy measure of information of order α and type β we obtained the following mean codeword length L_α^β :

$$L_\alpha^\beta = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{1+\frac{(1-\beta)}{\beta}\left[\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\right]D^{n_i\frac{\beta}{1-\beta}}\right\} - 2^\beta\right]$$

,

where $\alpha > 0 >, \alpha \neq 1, \beta \neq 0$.

Now, corresponding to the proposed fuzzy information measure a noiseless coding theorem has been proved.

Theorem 2.1 : For all uniquely decipherable codes ,

$$H_\alpha^\beta(A) \le L_\alpha^\beta \qquad (2.2)$$

Where,

$$L_\alpha^\beta = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{1+\frac{(1-\beta)}{\beta}\left[\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\right]D^{n_i\frac{\beta}{1-\beta}}\right\} - 2^\beta\right]$$

,

where $\alpha > 0 >, \alpha \neq 1, \beta \neq 0$. (2.3)

Proof : By Holder 's inequality, we have

$$x_i y_i \ge \frac{x_i^p}{p} + \frac{y_i^q}{q} \quad , 0 < p < 1, q < 0 \text{ or } 0 < q < 1, p < 0. \qquad (2.4)$$

Set $x_i = [f(\mu_A(x_i).(\mu_B(x_i)]^{\frac{-1}{t}} D^{-n_i}$

$y_i = [f(\mu_A(x_i).(\mu_B(x_i)]^{\frac{1}{t}}$, $-1 < t < 0.$

Take $p = -t$, $q = \frac{t}{1+\frac{1}{t}}$.

Then equation (2.4) becomes

$$[f(\mu_A(x_i).(\mu_B(x_i)]^{\frac{-1}{t}} D^{-n_i}\ [f(\mu_A(x_i).(\mu_B(x_i)]^{\frac{1}{t}}$$

$$\ge \frac{\left\{[f(\mu_A(x_i).(\mu_B(x_i)]^{\frac{-1}{t}}\right\}^{-t} D^{n_i t}}{-t} + \frac{\left\{[f(\mu_A(x_i).(\mu_B(x_i)]^{\frac{1}{t}}\right\}^{\frac{t}{1+\frac{1}{t}}}}{\frac{t}{1+\frac{1}{t}}}$$

Or

$$D^{-n_i} \geq \frac{-[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i t}}{t} + \frac{\left(1+\frac{1}{t}\right)\{f(\mu_A(x_i),(\mu_B(x_i)\}^{\frac{1}{1+\frac{1}{t}}}}{t}$$

By Kraft's inequality,

$$\sum_{j=1}^{n} D^{-n_j} \leq 1$$

Or

$$\frac{\left(1+\frac{1}{t}\right)\{f(\mu_A(x_i),(\mu_B(x_i)\}^{\frac{1}{1+\frac{1}{t}}}}{t} \leq 1 + \frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i t}}{t}$$

Take $t = \frac{\beta}{1-\beta}$,we have

$$\frac{(1-\beta)\{f(\mu_A(x_i),(\mu_B(x_i)\}^{\beta}}{\beta^2} \leq 1 + \frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}$$

Or

$$[f(\mu_A(x_i),(\mu_B(x_i)]^{\beta} \leq \frac{\beta^2}{1-\beta}\{1\{+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\}$$

Subtracting 2^{β} from both sides, we get

$$[f(\mu_A(x_i),(\mu_B(x_i)]^{\beta} - 2^{\beta} \leq \left[\frac{\beta^2}{1-\beta}\left\{+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\right\} - 2^{\beta}\right]$$

Taking summation on both sides,

$$\sum_{i=1}^{n}\{[f(\mu_A(x_i),(\mu_B(x_i)]^{\beta} - 2^{\beta}\} \leq \sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\right\} - 2^{\beta}\right]$$

Let $f(\mu_A(x_i),(\mu_B(x_i)) = \mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}$ and multiplying both sides by $\frac{1}{(1-\alpha)\beta}$, we get,

$$\frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\{[\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]^{\beta} - 2^{\beta}\}$$

$$\leq \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{+\frac{[\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\right\} - 2^{\beta}\right]$$

That is,

$H_{\alpha}^{\beta}(A) \leq L_{\alpha}^{\beta}$ which proves the result.

Theorem 2.2: For all uniquely decipherable codes, we have the following inequality:

$H_\alpha^\beta(A,U) \le L_\alpha^\beta(U)$ (2.5)

where

$$L_\alpha^\beta(U) = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{1+\frac{(1-\beta)}{\beta}\left[u_i\{\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\}\right]D^{n_i\frac{\beta}{1-\beta}}\right\} - 2^\beta\right]$$

, is a new mean codeword length where $\alpha > 0 >, \alpha \ne 1, \beta \ne 0$. (2.6)

Proof: Consider a quantitative –qualitative measure of information of order α and type:

$$H_\alpha^\beta(A,U) = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\{u_i^\beta[\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]^\beta - 2^\beta\}$$

For this quantitative–qualitative measure of information of order α and type β we obtain the following new mean codeword length of order α and type β:

$$L_\alpha^\beta(U) = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{1+\frac{(1-\beta)}{\beta}\left[u_i\{\mu_A^{\alpha\mu_A(x_i)}(x_i) + (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\}\right]D^{n_i\frac{\beta}{1-\beta}}\right\} - 2^\beta\right]$$

Now, By Holder's inequality, we have

$$x_i y_i \ge \frac{x_i^p}{p} + \frac{y_i^q}{q}\ , 0 < p < 1, q < 0 \ or\ 0 < q < 1, p < 0.$$ (2.7)

Set $x_i = [f(\mu_A(x_i), (\mu_B(x_i)]^{\frac{-1}{t}} D^{-n_i}$

$y_i = [f(\mu_A(x_i), (\mu_B(x_i)]^{\frac{1}{t}}\ , -1 < t < 0.$

Take $p = -t,\ q = \frac{t}{1+\frac{1}{t}}$.

Then equation (2.6) becomes

$$[f(\mu_A(x_i), (\mu_B(x_i)]^{\frac{-1}{t}} D^{-n_i}\, [f(\mu_A(x_i), (\mu_B(x_i)]^{\frac{1}{t}}$$

$$\ge \frac{\left\{[f(\mu_A(x_i), (\mu_B(x_i)]^{\frac{-1}{t}}\right\}^{-t} D^{n_i t}}{-t} + \frac{\left\{[f(\mu_A(x_i), (\mu_B(x_i)]^{\frac{1}{t}}\right\}^{\frac{t}{1+\frac{1}{t}}}}{\frac{t}{1+\frac{1}{t}}}$$

Or

$$D^{-n_i} \ge \frac{-[f(\mu_A(x_i), (\mu_B(x_i)]D^{n_i t}}{t} + \frac{(1+\frac{1}{t})\{f(\mu_A(x_i), (\mu_B(x_i)\}^{\frac{1}{1+\frac{1}{t}}}}{t}$$

By Kraft's inequality,

$$\sum_{j=1}^{n} D^{-n_j} \le 1$$

Or

$$\frac{\left(1+\frac{1}{t}\right)\{f(\mu_A(x_i), (\mu_B(x_i)\}^{\frac{1}{1+\frac{1}{t}}}}{t} \le 1 + \frac{[f(\mu_A(x_i), (\mu_B(x_i)]D^{n_i t}}{t}$$

Take $t = \frac{\beta}{1-\beta}$,we have

$$\frac{(1-\beta)\{f(\mu_A(x_i),(\mu_B(x_i)\}^\beta}{\beta^2} \le 1+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}$$

Or

$$[f(\mu_A(x_i),(\mu_B(x_i)]^\beta \le \frac{\beta^2}{1-\beta}\{1+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\}$$

Subtracting 2^β from both sides, we get

$$[f(\mu_A(x_i),(\mu_B(x_i)]^\beta - 2^\beta \le \left[\frac{\beta^2}{1-\beta}\left\{1+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\right\}-2^\beta\right]$$

Taking summation on both sides,

$$\sum_{i=1}^{n}\{[f(\mu_A(x_i),(\mu_B(x_i)]^\beta - 2^\beta\} \le \sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{1+\frac{[f(\mu_A(x_i),(\mu_B(x_i)]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\right\}-2^\beta\right]$$

Let $f(\mu_A(x_i),(\mu_B(x_i) = u_i\{\mu_A^{\alpha\mu_A(x_i)}(x_i)+\ (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}\}$ and multiplying both

Sides by $\frac{1}{(1-\alpha)\beta}$, we get,

$$\frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\{u_i^\beta[\mu_A^{\alpha\mu_A(x_i)}(x_i)+\ (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]^\beta - 2^\beta\}$$

$$\le \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\frac{\beta^2}{1-\beta}\left\{1+\frac{u_i[\mu_A^{\alpha\mu_A(x_i)}(x_i)+\ (1-\mu_A(x_i))^{\alpha(1-\mu_A(x_i))}]D^{n_i\frac{\beta}{1-\beta}}}{\frac{\beta}{1-\beta}}\right\}-2^\beta\right]$$

That is,

$H_\alpha^\beta(A,U) \le L_\alpha^\beta(U)$ which proves the result.

III.MONOTONICITY OF THE MEAN CODEWORD LENGTH

Next, with the help of the data, we have presented the mean codeword length L_α^β graphically. For this purpose, we have computed different values of L_α^β for different values of the parameter α and β, corresponding to different fuzzy values $\mu_A(x_i)$. Next, we have presented L_α^β graphically and obtained the fig-3.1 which clearly shows that the codeword length L_α^β is monotonically decreasing function.

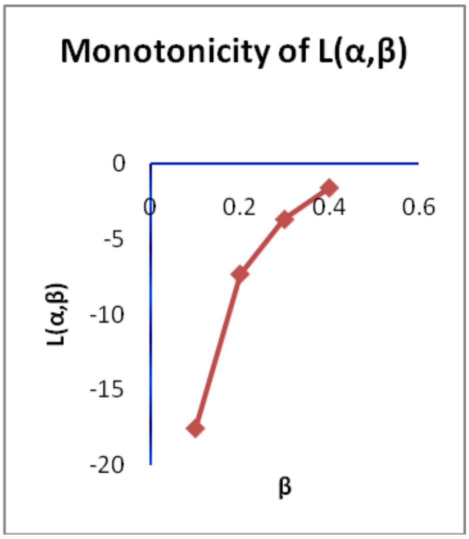

Fig-3.1

REFERENCES

[1] Arun Choudhary and Satish Kumar, "Some more noiseless coding theorem on generalized R- Norm entropy", *Journal of Mathematics Research*, vol.3, pp. 125-130, 2011.

[2] Arun Choudhary and Satish Kumar, "Some coding theorems on generalized havrda-charvat and tsalli's entropy", *Tamkang Journal of Mathematics*, vol.43, pp. 437-444, 2012.

[3] A.Renyi, *On Measures of Entropy and Information*, Proc. 4th Berk. Symp. Math. Statist. and Probl, University of California Press, pp.547-561, 1961.

[4] Belis, M. and Guiasu,, "*A quantitative-qualitative measure of information in cybernetic systems"*, IEEE Transactions on Information Theory 14, pp. 593-594,1968.

[5] C.E.Shannon, *A Mathematical Theory of Communication*, Bell Syst. Tech. J, pp.379-423, 623-659, 1948.

[6] Guiasu, S. and Picard, C. F., "Borne *in ferictur de la longuerur utile de certains codes*", Comptes Rendus Mathematique Academic des Sciences Paris, pp.273, 248-251, 1971.

[7] Gurdial and Pessoa, F., "*On useful information of order* α" Journal of Combanatrics Information and System Sciences 2, pp.158-162, 1977.

[8] G.J. Klir, and T.A. Folger, *Fuzzy Sets, Uncertainty and Indetermination*, Prentice Hall, 1988.

[9] H.Nyquist, *Certain Factors Affecting Telegraph Speed*, Bell Syst. Tech.,pp. 324-325,1924.

[10] H.Nyquist, *Certain Topics in Telegraph Transmission Theory*, AIEEE Transactions, pp.617-619, 1928.

[11] H.J. Zimmermann, *Description and optimization of fuzzy systems*, International Journal of General Systems, pp. 209-215, 1976.

[12] J.N.Kapur, *Measures of Fuzzy Information*, Mathematical Sciences Trust Society, New Delhi, 1997.

[13] Kraft, "*A device for quantizing grouping and coding amplitude modulated pulses",* M.S. Thesis, Electrical Engineering Department, MIT.

[14] Kapur, J.N., "*Entropy and Coding*", Mathematical Sciences Trust Society, New Delhi, 1998.

[15] Korada, S. B. and Urbanke, R. L., "*Polar codes are optimal for lossy source coding*", IEEE Trans. Inform. Theory 56(4), pp.1751–1768, 2010.

[16] L.A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, *Fuzzy Sets and Systems*, pp. 3-28, 1978.

[17] L.L.Campbell, "*A coding theorem and renyi's entropy*", Information and control,8, pp.423-429,1965.

[18] Longo, G., "*Quantitative-Qualitative Measures of Information*", Springer-Verlag, New York, 1972.

[19] M.A.K. Baig and Mohd Javid Dar, "Some Coding theorems on Fuzzy entropy Function Depending upon Parameter R and V", *IOSR Journal of Mathematics*, vol.9, pp.119-123.

[20] M.A.K. Baig and Mohd Javid Dar, "Fuzzy coding theorem on generalized fuzzy cost measure", *Asian Journal of Fuzzy and Applied Mathematics*, vol.2, pp.28-34, 2014.

[21] Merhav, N., "*Shannon's secrecy system with informed receivers and its application to systematic coding for wiretapped channels",* IEEE Transactions on Information Theory 54,pp. 2723-2734,2008.

[22] N.Wiener, Cybernetics, MIT Press and Wiley, New York, 1948.

[23] Om Parkash and P. K. Sharma, "Noiseless coding theorems corresponding to fuzzy entropies", *Southeast Asian Bulletin of Mathematics*, vol.27, pp.1073–1080, 2004.

[24] Om Parkash and P. K. Sharma, "A new class of fuzzy coding theorems", *Caribb. J.Math. Comput.Sci.*, vol.12, pp. 1-10, 2002.

[25] R.K.Tuli, "Mean codeword lengths and their correspondence with entropy measures" , *International Journal of Engineering and Natural Sciences* , vol.4 , pp.175-180, 2010.

[26] R.P.Singh, R.Kumar and R.K.Tuteja, *Application of Holder's inequality in information theory*, Information Sciences, pp.145-154, 2003.

[27] R.T.V.Hartley, Transmission *of information*, Bell Syst. Tech. J, vol. 7, pp.535-563, 1928.

[28] Szpankowski, W., "*A one-to-one code and its anti-redundancy",* IEEE Trans. Inform. Theory 54(10), pp. 4762–4766, 2008.