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OUTAGE PROBABILITY ANALYSIS OF FREE SPACE OPTICS FOR WEAK AND STRONG TURBULENCE CHANNELS

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ABSTRACT

Free space optical (FSO) communication systems have shown to be a viable alternative to the fibre optics technology in several applications. Now a day, the most cost effective method for providing high data rate is free space optical (FSO) communication systems. But its application is limited to short range applications. We propose a single user diversity FSO scheme. We first present the outage probability analysis for the considered system over weak and strong atmospheric turbulence channels. The outage probability is analysed on the basis of these statistics. For this work, we investigate an architecture that uses single transmitter and single coherent receiver to overcome these turbulence-induced outages. In Free Space Optical Communication (FSOC) links, weather effect has a significant impact on the quality of laser beam propagation through the atmosphere. Recently free-space optical (FSO) communication systems have received considerable attention due to the high capacity ability it offers for the last-mile applications.

Keywords:-Fading Free Space Optical (FSO) Communication Systems, Gamma Gamma Distribution, Log Normal Distribution, Turbulence.

I. INTRODUCTION

Free space optical communication through the atmosphere has the potential to provide secure, low-cost, rapidly deployable, dynamic, data transmission at very high rates. [1] However, the deleterious effects of turbulence can severely limit the utility of such a system, causing outages of up to 100 ms. FSO technology is surprisingly simple. It's based on connectivity between FSO-based optical wireless units, each consisting of an optical transceiver with a transmitter and a receiver to provide full-duplex (bi-directional) capability [2].Several laser FSO communication systems were developed in the 1980s for secure ship-to-ship communication and ground-to-aircraft applications [3]. This FSO technology approach has a number of advantages: Requires no RF spectrum licensing, is easily upgradeable, and its open interfaces support equipment from a variety of vendors, which helps enterprises and service providers protect their investment in embedded telecommunications infrastructures, requires no security software upgrades, is immune to radio frequency interference or saturation, can be deployed behind windows, eliminating the need for costly rooftop rights [4][5]. In figure 1, the pulse generator generates pulses (information signal) to be transmitted through the FSO link. The pulse generators can be sine wave, triangular wave, saw up/down, and Return toZero/Non T return to Zero (RZ/NRZ) pulse, Gaussian pulse, and impulse or M-ary generator [6]. Optical carrier wave can be produced by using a LASER or LED. These pulse shape modulate the optical signal generated by laser. Further modulation of information pulse with carrier optical signal is done utilizing optical modulator. Optical modulator can be Mach-Zehnder, AM, FM or PM. Modulated signal is transmitted on FSO link then optical demodulator is used to detect electrical signal from optical signal. Demodulator to be employed can be PIN, APD, spatial PIN or spatial APD. Later demodulated signal is utilized by receiver can be in form of sound or broadband data.

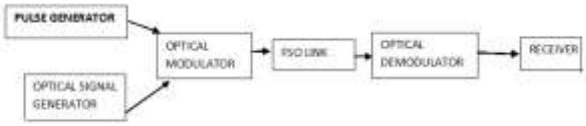

Fig.1 Free-Space Optics Major Subsystems

II. SYSTEM AND CHANNEL MODEL

We consider a FSO communication system with single users as depicted in Fig. 2 and in which the central node is equipped with single apertures and the users are only equipped with single apertures.[7]A multi-hop free space optical (FSO) communication system with intensity modulation/direct direction links-hops using on-off keying is considered[7].In this work we are considering the channel model where both the turbulence-induced fading and path-loss are included. For one link between the transmitter and receiver.

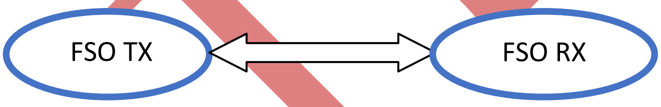

Fig 2. Single User For K=N=1 FSO System.

The channel with distance d_1 can be modeled as [8]

$$h_1 = L(d_1)\, \tilde{h}_1 \qquad (1)$$

Where h_1 =Turbulence fading, $\tilde{h}_1$=Atmospheric fading, $L(d_1)$ =path loss,

$L(d_1)$ denotes the path loss and can be written as [9]:

$$L(d_1) = \frac{D_R^2}{(D_T + \theta_T d_1)^2} e^{-vd_1} \qquad (2)$$

D_R =Receiver Aperture diameter, D_T = Transmitter Aperture diameter, v= Weather dependent attenuation coefficient, θ_T = optical beam divergence angle, γ=Instantaneous SNR, $\tilde{\gamma}$ =Avg. SNR

The instantaneous SNR can be defined as [8],[9]:

$$\gamma_1 = \tilde{\gamma}_1 \left|\tilde{h}_1\right|^2 \qquad (3)$$

In communication system, two types of distribution exists log normal and gamma gamma distribution.

2.1 Log Normal (For Weak Turbulence) Distribution and Condition

The conceptualization of the log-normal distribution is straightforward. It results from modelling the field of the optical wave at any point in the medium along the propagation path as the product of field components scattered from a large number of independent scatterers. By applying the central limit theorem, the logarithm of this product, i.e. the log-amplitude of the optical intensity, will obey a normal (Gaussian) distribution. Mathematically, the log-normal distribution follows from the Rytov method, i.e. Rytov approximation, for solving Maxwell's wave equation in a random medium. The log-normal channel is classified as weak turbulence, which is characterized by a scintillation index less than 0.75. In general, the scintillation index is a complicated function of the beam parameters, propagation distance, heights of the transmitter and the receiver, and the fluctuations in the index of refraction [40]. In fact, the main source of scintillation is due to fluctuations (due to temperature variations) in the index of refraction, which is commonly known as optical turbulence. The log-normal model is also valid for propagation distances less than 100 m.[10]

$\underline{x}_1$ =Normally distributed with mean μx_1 and variance $\sigma_{x_1}^2$

The fading coefficient conserves power, we have $E\left[\left|\widetilde{h_1}\right|\right]^2=1$, which implies $\mu x_1 = -\sigma_{x_1}^2$ [11]

Probability density function PDF of γ_1[11]

$$f_{1,1}^{LN}(\gamma)=\frac{1}{2\gamma_{\gamma}\sqrt{2\pi\sigma_{x_1}^2}}\exp\left(-\frac{\ln\left(\frac{\gamma}{\gamma_1}\right)+2\sigma_{x_1}^2)}{8\sigma_{x_1}^2}\right), \quad (4)$$

Corresponding CDF for this distribution can be written as:

$$F_{\gamma_1}^{LN}(\gamma)=1-Q\left(\frac{\ln\left(\frac{\gamma}{\gamma_1}\right)+2\sigma_{x_1}^2)}{2\sigma_{x_1}}\right), \quad (5)$$

Where, h_1 = Turbulence fading, $\widetilde{h_1}$=Atmospheric fading

Q is Gaussian Q- function defined as

$$Q(y) = {}^{1}/_{\sqrt{2\pi}}\int_y^{\infty} \exp\left(-{}^{t^2}/_{2}\right)dt. \quad (6)$$

Log normal Variance can be given by [12]

$$\sigma_{x_1}^2=\exp\left[\frac{0.49\delta^2}{(1+0.18L^2+0.56\delta^{12/5})^{7/6}}+\frac{0.51\delta^2}{(1+0.9L^2+0.62L^2\delta^{12/5})^{5/6}}\right] \quad (7)$$

Where Rytov variance = $\delta^2 = 1.23C_n^2(2\pi/\lambda)^{7/6}d_1^{11/6}$, C_n^2Refractive index Structure constant given by [12]

$$L = \sqrt{2\pi D_R^2/(4d_1\lambda)} \quad (8)$$

The Rytov approximation has been used to describe atmospheric turbulence and the log-normal turbulence model has been derived. The Rytov approximation predicts that the Rytov parameter increases without limit with the index of refraction structure parameter and/or the path length. However, based on experimental results as reported in, this prediction holds only in the weak turbulence regime when $\sigma_x^2<0.3$. As the turbulence strength increases beyond the weak regime, due to a combination of increased path length and/or increased, C_n^2, the turbulent eddies result in multiple scatterings that are not accounted for by Rytov in his approximation.

2.1.1 Weak Turbulence Condition

From, [13], Log normal distributed model, the Q (‘) function can be

$$Q(x) \approx \frac{1}{12}\exp\left(-\frac{x^2}{2}\right)+\frac{1}{4}\exp\left(-\frac{2x^2}{3}\right) \quad x\geq 0 \tag{9}$$

The CDF for log normal case can be found based on equation (9) and (5)

$$F_{\gamma 1,1}^{LN}(\gamma) \approx 2-\frac{1}{12}\exp\left(-3\,\frac{(\ln(\gamma+\mu)^2}{24\sigma_x^2}\right)-\exp\left(-2\,\frac{(\ln(\gamma+\mu)^2}{24\sigma_x^2}\right) \tag{3.10}$$

Similarly, combing (4), (9), we get PDF:

$$f_{\gamma 1,1}^{LN}(\gamma) \approx \frac{1}{24\,\sigma_x\sqrt{2\pi\gamma}}\exp\left(\frac{(\ln(\gamma+\mu)^2}{8\sigma_x^2}\right) \tag{3.11}$$

Since equation (9) is only applicable to x >0.ie. Equation (10) and (11) is applicable for low SNR regime. To make our results applicable for the whole SNR range, from equation. (5) we get

$$F_{\gamma,1}^{LN}(\gamma)=Q\left(-\frac{\ln(\gamma)+\mu}{2\sigma_x}\right) \quad ; \qquad 0<\gamma<\gamma e^{-2\sigma_x^2} \tag{12}$$

Then, using (9) again, we obtain the CDF and PDF for γ_1

$$F_{\gamma,1}^{LN}(\gamma) \approx -\frac{1}{12}\exp\left(-\frac{(\ln(\gamma+\mu)^2}{8\sigma_x^2}\right)-\frac{1}{4}\exp\left(-\frac{(\ln(\gamma)+\mu)^2}{4\sigma_x^2}\right) \tag{13}$$

$$f_{\gamma 1,1}^{LN}(\gamma) \approx \frac{1}{24\,\sigma_x\sqrt{2\pi\gamma}}\exp\left(\frac{(\ln(\gamma+\mu)^2}{8\sigma_x^2}\right) \tag{14}$$

X = is a random variable with standard Normal distribution.

2.2 Gamma Gamma (Strong Turbulence) Distribution And Condition

For strong atmospheric condition, the channel fading can be modelled using the well-known Gamma-gamma distribution [14][12], Pdf of $|\bar{h}_1|$ can be given by[12][11]

$$f_{|\bar{h}_1|}^{GG}(x)=\frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)}x^{\frac{\alpha+\beta}{4}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta x}\right), \tag{15}$$

where Γ (°) is a gamma function α and β are re the fading parameters related to the effective atmospheric conditions of the link and depend on the Rytov variance, and $K_v(x)$ is the modified Bessel function of the second kind of order v. More specifically, from [41], the values of α and β can be expressed as

Where $\alpha = \exp\left[\left(0.49\delta^2/\left(1+0.18L^2+\ 0.56\delta^{12/5}\right)^{7/6}\right)-1\right]^{-1}$

And $\beta = \exp\left[\left(0.51\delta^2/\left(1+0.9L^2+\ 0.62\delta^{12/5}\right)^{5/6}\right)-1\right]^{-1}$

Pdf of γ_1 can be given by [11]

$$f_{\gamma_1}^{GG}(\gamma) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)}\gamma^{\frac{\alpha+\beta}{4}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}_1}}}\right) \tag{16}$$

And corresponding Cumulative distribution function, CDF

$$F_{\gamma_1}^{GG}(\gamma) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)}G_{1,3}^{2,1}\left[\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}_1}}\Big|_{\alpha,\beta,0}^{1}\right] \tag{17}$$

$G_{1,3}^{2,1}$is the Meijer's G-function [15] . Let denote the order statistics obtained by arranging the random variables assume that in an increasing order of magnitude. For tractable analysis, we are independent and identically distributed, i.e., for all users. The CDF and PDF for single user having single aperture denoted by K, hence, PDF and CDF $\gamma_{N,K}$ are given as [16]

$$F_{\gamma_{1,1}}(\gamma) = [F_{\gamma_1}(\gamma)] \tag{18}$$

$$f_{\gamma_{1,1}}(\gamma) = f_{\gamma_1}(\gamma), \tag{19}$$

Where $F_{\gamma_1}(\gamma)$ and $f_{\gamma_1}(\gamma)$ have been given (16), (17), respectively.

2.2.1 Strong Turbulence Condition

Here we will focus on gamma gamma distribution and find out the corresponding CDF. By using [17, Eq. (19)], the CDF in (17) can be rewritten as

$$F_{\gamma}^{GG}(\gamma)=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\frac{\pi}{2\sin((\alpha-\beta)\pi)}\sum_{s=0}^{\infty}\left[\frac{2}{(s+\beta)\Gamma(\beta-\alpha+1+s)s!}\left(\frac{\alpha\beta}{\sqrt{\bar{\gamma}}}\right)^{s+\beta}\gamma^{\frac{s+\beta}{2}}-\left(\frac{\alpha\beta}{\sqrt{\bar{\gamma}}}\right)^{s+\beta}\gamma^{\frac{s+\beta}{2}}\right] \tag{20}$$

It should be noted that $\beta-\alpha$ is not an integer in (3.20). Substituting (3.20) into (3.18), we get the CDF of the SNR of a gamma-gamma modeled FSO system can be written as:

$$F_{\gamma_{1,1}}^{GG}(\gamma) = \sum_{j=0}^{M(1-j)}\sum_{p=0}^{M(1-j)}\sum_{q=0}^{Mj}\binom{1}{j}(-1)^{j}\eta_p\eta_q\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\cdot\left\{\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}}}\right\} \tag{21}$$

η_p, η_q are the coefficient of $(\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}}})^p$ and $(\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}}})^q$ in the expansion of

$$\sum_{s=0}^{M}\left[\frac{2}{(s+\beta)\Gamma(\beta-\alpha+1+s)s!}\left(\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)^{s}\right]^{1-j}$$

and

$$\sum_{s=0}^{M}\left[\frac{2}{(s+\beta)\Gamma(\beta-\alpha+1+s)s!}\left(\alpha\beta\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)^{s}\right]^{j}$$

respectively.

III. PERFORMANCE ANALYSIS

Performance Analysis of free space optics can be done in term of BER rate, outage probability, average capacity etc. But we discussing in terms of outage probability for strong and weak turbulence.

3.1 Outage Performance Analysis

The Outage Probability is defined as the probability that the end-to end output SNR falls below a specified threshold SNRγth .This threshold is a minimum value of the SNR above which the quality of service is satisfactory [18]. The outage performance of the system degrades as the number of hops increases, this being similar behavior as in RF multi-hop systems. The outage probability is the average proportion of time that the instantaneous BER, Pi, is greater than some prescribed BER threshold, P*[19]

$$P_{out}(P^*) = P_r(P_i > P^*) \qquad (22)$$

Atmospheric turbulence results in a very slowly-varying fading in FSO systems. The channel coherence time is about 1-100 ms, therefore fading remains constant over hundreds of thousand up to millions of consecutive bits for typical transmission rates [20].It is defined as the probability that the instantaneous SNR γ falls below a given threshold SNRγth. Recall that the outage probability is related to the derived CDF expressions. Therefore,the exact outage probability can be simply obtained by replacing γ with γth in (18)

$$P_{out} = \left[\left[F_{\gamma,1}(\gamma_{th})\right]\right] \qquad (23)$$

Where the exact expressions F $\gamma_{th}(\gamma_{th})$ can be readily obtained from (5) and (17). We also can use the approximate formulas (10), (13), and (21) to evaluate the outage probability performance.

The outage probability is inversely proportional to the FSO system performance. As the outage probability increases system performance decreases.

IV. FIGURES AND TABLES

Here, the outage probability for the free space optics has been for both log normal and gamma gamma distribution has been derived. Here the results are different for both strong turbulence channel and weak atmospheric channels. I have plot outage probability curves for the weak turbulence (log-normal) and strong turbulence (gamma gamma) by using the exact and approximate formulas. The modulation used here is IM/DD OOK modulation scheme. The table for system parameter and outage probability parameter are given below:

Table 1. System Parameters And Outage Probability Analysis Parameter

Parameter	Value
Modulation	IM/DD OOK
Quantum Efficiency	50%
Wavelength ,λ	1550 nm
d	5km
D_R	0.1m
D_T	0.1m
C_n^2 (Strong turbulence)	2×10^{-14}
C_n^2 (Weak turbulence)	1×10^{-14}

σ	0.011/km
Rytov variance(Strong turbulence)	2.65×10^{-7}
Rytov variance(Weak turbulence)	1.99×10^{-7}
σ_x^2 (Strong turbulence)	$0.3<\sigma_x^2<5$
σ_x^2 (Weak turbulence)	0.3
Alpha	0.97
Beta	0.95

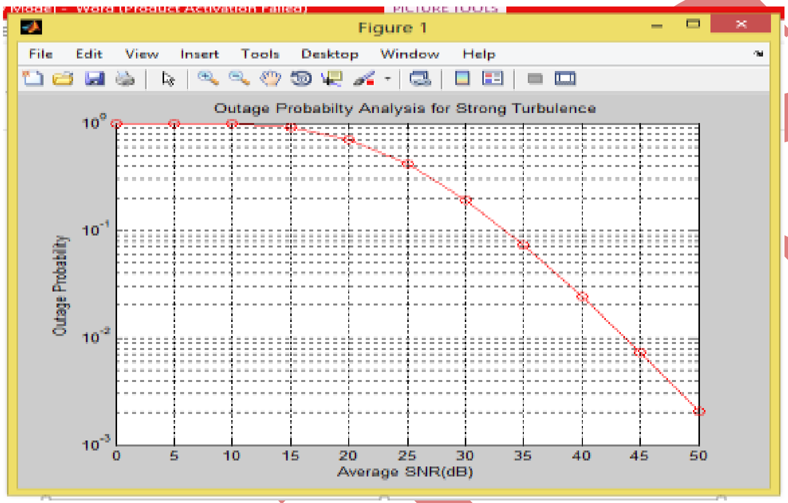

Fig.3 Outage Probability Of FSO Communication Systems Over Strong Atmospheric Turbulence Channels,

From Fig.3 we can conclude that first is that as the Average SNR (dB) increases the corresponding outage probability decreases which results in improved system performance. For the strong turbulence channel outage probability is inversely proportional to Average SNR (dB).

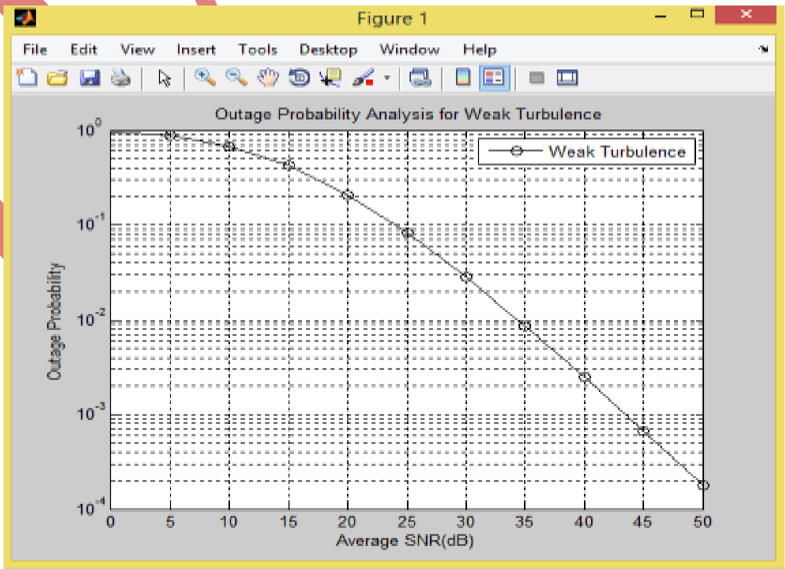

Fig. 4. Outage Probability Of FSO Communication Systems Over Weak Atmospheric Turbulence Channels,

Fig 4 shows the plot of outage probability as a function of Average SNR (dB).This plot has been plotted for the weak turbulence channels. With this figure we can conclude two things: First is that as the Average SNR (dB) increases the corresponding outage probability decreases which results in improved system performance. For the strong turbulence channel outage probability is inversely proportional to Average SNR (dB).It decreases more rapidly as compared to strong turbulence channels.

From Table 2 We can conclude that with the increases in Average SNR (dB) there is decrease in Outage probability for both Strong and Weak Turbulence but at any instant the Outage Probability for Strong Turbulence is more than for Weak Turbulence. Hence, at any instant we can conclude that Weak Turbulence provides better System Performance as compared to Strong Turbulence.

Table 2.Comparison of Outage Probability Strong Turbulence and Weak Turbulence

Sl. No.	Average SNR(dB)	Outage Probability(Strong Turbulence)P_{out}	Outage Probability(Weak Turbulence)P_{out}
1.	5	0.9995	0.8803
2.	10	0.9882	0.6856
3.	15	0.9110	0.4265
4.	20	0.6998	0.2085
5.	25	0.4161	0.08296
6.	30	0.1930	0.02832
7.	35	0.07349	0.008686
8.	40	0.02428	0.002481
9.	45	0.007281	0.0006766
10.	50	0.002049	0.0001791

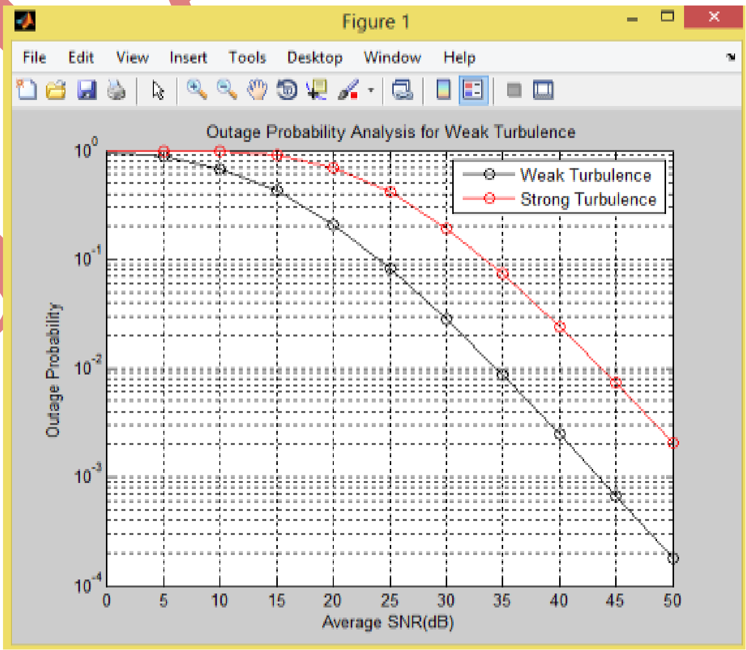

Fig.5 Comparison Of Outage Probability For Weak And Strong Turbulence

Fig 5 shows the plot of outage probability as a function of average SNR (dB).This plot has been plotted for the comparison of weak and strong turbulence channels. Outage Probability depend upon the nature of channels. With this figure we can conclude two things:

(a)First is that as the Average SNR (dB) increases the corresponding outage probability decreases which results in improved system performance for both strong and weak channels.

(b)Outage probability for weak turbulence channels decreases more rapidly as compared to strong turbulence channels. Hence, we can conclude that weak turbulence channel provides better system performance as compared to strong turbulence channels.

V. CONCLUSION

Optical communication over the turbulent atmosphere has the potential to provide reliable rapidly-reconfigurable multi-gigabit class physical links. Such systems, however, are prone to long (up to 100 ms) and deep (10 to 20 dB) fades.

We derived a comprehensive performance analysis or the FSO communication systems with single user diversity for weak and strong atmospheric turbulence in this paper. We derived some approximate expressions for the outage probability analysis for weak and strong atmospheric turbulence. Results show that the approximate analysis is quite accurate. In this chapter, a practical channel model forIM/DD FSO communications links was proposed and verified using the experimental FSO link.

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