ROLE OF 3D-INERTIAL ALFVÉN WAVE IN TURBULENCE SCALING AND FORMATION OF LOCALIZED STRUCTURE IN LOW BETA PLASMAS LIKE AURORAL REGION

M. L. Rinawa[1] and R. P. Sharma[2]

[1, 2]*Centre for Energy Studies, Indian Institute of Technology Delhi, Delhi, (India)*

ABSTRACT

In the present paper, we have investigated nonlinear interaction of 3D- inertial Alfvén wave (3D-IAW) and perpendicularly propagating magnetosonic wave (PMSW) for low β-plasma $\left(\beta \ll m_e / m_i\right)$ like auroral plasma. We have developed the set of dimensionless equations in presence of ponderomotive nonlinearity due to 3D-IAW in the dynamics of PMSW. Numerical simulation has been carried out to study the effect of nonlinear coupling between waves (3D-IAW and PMSW) which results in formation of localized structure, applicable for low β-plasma like auroral plasma. The result illustrates that localized structure, becomes more and more complex with progressive time. Relevance of the obtained results from the simulation is consistent with the observations reported from FAST spacecraft.

Keywords: Low Beta Plasma, 3D-Inertial Alfvén Wave, Magnetosonic Wave.

I. INTRODUCTION

Alfvén waves (AWs) are ubiquitous in space plasmas. These waves were discovered for the first time theoretically by Hannes Alfvén (*Alfvén*, 1942), experimentally by Lundquist and observed by *Chaston et al.* (2008). AWs are low frequency, electromagnetic, magnetohydrodynamic (MHD) waves propagating along low beta plasma leads to self localization resulting into formation of magnetic intensity filaments. The restoring force and inertia for AWs is provided by pressure of the magnetic field and ion mass respectively. From the magnetohydrodynamic (MHD) equations, the dispersion relation for the low-frequency AW is obtained (*Alfvén*, 1942; *Cramer*, 2001), which do not have dispersion. In cold magnetoplasmas, dispersion comes (*Damino et al.*, 2009) due to the finite frequency $\left(\omega_0 / \omega_{ci}\right)$ and electron inertial force along magnetic field. AW becomes dispersive due to finite ion gyro radius effect and the gradient of the electron pressure in a warm magnetoplasma. When AWs generate a large perpendicular wave number transverse to the ambient magnetic field (*Hollweg*, 1999), in low-β plasmas (i.e. $\beta \ll m_e / m_i$, where β is thermal to magnetic pressure ratio) is known as inertial Alfvén wave (IAW), which plays an important role in the auroral region. The AW dispersion relation may result ultimately acceleration and heating of the plasma particles (Goertz, 1984), wave-particle interactions (*Hasegawa and Chen*, 1976). Specifically, it is presumption that the dispersion in AW provides a power to auroral activities (*Chaston et al.*, 2008) and causes the solar coronal heating (*Tomczyk et al* , 2007). The existence and crucial role of IAW has been reported from sounding rockets as well as spacecraft, including S3-3, FAST (*Boehm et al.*, 1990; *Aikio et al.*, 1996). In space and astrophysical plasma, due to the finite frequency ($\omega_0 < \omega_{ci}$), finite ion gyroradius and electron inertia rather than pressure was dominated effect, AW becomes dispersive which may result in the formation of nonlinear localized structures such as envelope soliton, fold as well as vortices (*Petviashvili and Pokhotelov*, 1992), Solitary waves plasma in atmoshphere and the formation of nonlinear Alfvénic turbulence (*Sundkvist et al.*, 2005) in the Earth's auroral ionosphere. Observations reported from FAST spacecraft pointed out that auroral ionosphere can also be powered by the transverse turbulent cascade from larger to smaller wave lengths (*Chaston et al., 2008*). Resulting turbulent spectra derived from these observations follows Kolmogorov (1941) power law i.e. $\sim k^{-5/3}$ (the inertial range) for $k_x < (0.1/\lambda_e)$, here λ_e is the electron inertial length. Below this length, spectrum deviates until a break point is reached (at quasi-steady state). Here break point occurs at $k_x \rho_i \sim 1$ and ρ_i is the ion gyro radius. After this break point spectrum becomes steeper with the scaling of $\sim k_x^{-3.5}$. The main objective of this paper is to evaluate nonlinear coupling of 3D-IAW with perpendicularly PMSW to study the formation of localized structure applicable to auroral region (Earth's ionosphere). For this purpose, using bi-fluid approach the coupled dynamics of 3D-IAW and PMSW in the presence of ponderomotive force has been developed. Numerical simulation results have been carried out to study the nonlinear stage of modulational instability. The paper is further organized as follows: dynamics of 3D-IAW and PMSW is presented in sections-2 and 3 respectively. Numerical simulation results are presented in section-4. Results are discussed in section-5 and finally, the last section comprises of the conclusion.

II. DYNAMICS OF 3D-IAW

Let us consider the dynamics of 3D-IAW. The ambient magnetic field is along the $z-$axis. i.e. $\vec{B}_0 = B_0\hat{z}$, where B_0 is the background magnetic field. The wave is assumed to be propagating in the $x-y-z$ plane i.e. $\vec{k} = k_r\hat{r} + k_z\hat{z}$; here $\hat{r} = \hat{x} + \hat{y}$. The wave electric field and perturbed magnetic fields of the 3D-IAWs are given by $\vec{E}_A = -\vec{\nabla}\phi_A - \hat{z}c^{-1}\partial_t A_z$ and $\vec{B}_\perp = \vec{\nabla}A_z \times \hat{z}$, respectively, where ϕ_A is the scalar potential and A_z the parallel (to $B_0\hat{z}$) component of vector potential.

The perpendicular components of electron and ion fluid velocities for 3D-IAW (low β - plasma $\beta \ll m_e / m_i$) are given as

$$\vec{\upsilon}_{e\perp} \approx \frac{c}{B_0}\left(\hat{z} \times \vec{\nabla}_\perp \phi\right) - \frac{c}{B_0}\frac{\gamma_e k_B T_e}{e}\hat{z} \times \vec{\nabla}_\perp \left(\frac{n_{e1}}{n_0}\right) \quad (1)$$

$$\vec{\upsilon}_{i\perp} \approx \frac{c\omega_{ci}^{2}}{(1-\alpha)B_0}\left[\hat{z}\times\left\{\vec{\nabla}_{\perp}\phi + \frac{T_i}{en_0}\vec{\nabla}_{\perp}\left(\frac{n_{i1}}{n_0}\right)\right\}\right] + i\frac{c\omega_0}{(1-\alpha)\omega_{ci}B_0}\left\{\nabla_{\perp}\phi\frac{T_i}{en_0}\nabla_{\perp}\left(\frac{n_{i1}}{n_0}\right)\right\} \quad (2)$$

The parallel component of electron fluid velocity is

$$\upsilon_{ez} \approx -\frac{e}{i\omega_0 m_e}\left(\nabla_z\phi + \frac{1}{c}\frac{\partial A_z}{\partial t}\right) \quad (3)$$

Taking parallel component of Ampere's law, we get

$$J_z = -\frac{c}{4\pi}\nabla_{\perp}^2\tilde{A}_z \quad (4)$$

where $\alpha = \frac{\omega_0^{2}}{\omega_{ci}^{2}}$, $\nabla_{\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, Taking time derivative of equation (4) and substituting the value of current density, J_z using equation (3) and taking a perturbation in density as $n_{e1} \approx n_0 + \tilde{n}_e$, one can get the equation as below,

$$\frac{\partial^2\phi}{\partial z\partial t} = \frac{c}{\omega_{pe}^{2}}\left(1-\frac{\tilde{n}_e}{n_0}\right)\frac{\partial^4\tilde{A}_z}{\partial t^2\partial r^2} \quad (5)$$

where $\tilde{n}_e(\ll n_0)$ is a small electron number density perturbation n_0 is the unperturbed plasma number density and $\tilde{n}_e$ is the number density change due to the presence of magnetosonic wave, ω_{pe} is the electron plasma frequency and c is the velocity of light.

On the other hand, assuming quasi-neutrality approximation ($n \approx \tilde{n}_e \approx \tilde{n}_i$) and by substituting equations (2) and (3) into the conservation law of current density $(\vec{\nabla}.\vec{J} = 0)$ we have

$$\frac{\partial\phi}{\partial t} = -\frac{V_A^{2}}{c\omega_{ci}^{2}}\left(\omega_{ci}^{2} + \frac{\partial^2}{\partial t^2}\right)\left(1-\frac{\tilde{n}_e}{n_0}\right)\frac{\partial\tilde{A}_z}{\partial z} \quad (6)$$

Taking a time derivative of Faraday's law,

$$\frac{\partial^2\nabla_{\perp}\tilde{A}_z}{\partial t^2} = c\frac{\partial^2\nabla_z\phi}{\partial t} - c\frac{\partial^2\nabla_{\perp}\phi}{\partial t\partial z} \quad (7)$$

Using equations (5) and (6), substituting in equation (7), we get the dynamical equation for 3D-IAW

$$\frac{\partial^2\tilde{A}_z}{\partial t^2} = \lambda_e^{2}\frac{\partial^4\tilde{A}_z}{\partial^2 r\partial t^2} + V_A^{2}\left(1-\frac{n}{n_0}\right)\frac{\partial^2\tilde{A}_z}{\partial z^2} \quad (8)$$

Here we take without finite frequency ($\alpha = 0$), in equation (8), $V_A = (B_0^2/4\pi n_0 m_i)^{1/2}$ is the Alfvén speed, ω_{ci} =ion cyclotron frequency and $\lambda_e = \frac{c}{\omega_{pe}}$ is the electron inertial length. Equation (8) gives the dispersion relation as follows

$$\frac{\omega_0^{\,2}}{V_A^{\,2} k_{0z}^{\,2}} = \frac{1}{\left(1+\lambda_e^{\,2} k_{0r}^{\,2}\right)} \tag{9}$$

where $\lambda_i = \dfrac{c}{\omega_{pi}}$ is an ion inertial length.

Considering plane wave solution of equation (8) as follows

$$\tilde{A}_z = A_z(x, y, z, , t)\, e^{i(k_{0r}\hat{r} + k_{0z}\hat{z} - \omega_0 t)}$$

Using equation above in equation (8), following equation has been obtained for the case, when $\partial_z A_z \ll k_{0z} A_z$,

$$i\frac{2\omega_0\left(1+\lambda_e^{2}k_{0r}^{\,2}+\tau\right)}{V_A^{\,2}k_{0z}^{\,2}}\frac{\partial A_z}{\partial t} - i2k_{0r}\frac{\omega_0^{\,2}\lambda_e^{\,2}}{V_A^{\,2}k_{0z}^{\,2}}\frac{\partial A_z}{\partial r} - \frac{\omega_0^{\,2}\lambda_e^{\,2}}{V_A^{\,2}k_{0z}^{\,2}}\frac{\partial^2 A_z}{\partial r^2} + i\frac{2}{k_{0z}}\frac{\partial A_z}{\partial z} + \frac{n}{n_0}A_z = 0 \tag{10}$$

where $\tau = \dfrac{V_A^{\,2}k_{0z}^{\,2}}{\omega_{ci}^{\,2}}$, k_{0r} (k_{0z}) is the component of the wave vector perpendicular (parallel) to $B_0\hat{z}$ and ω_0 is the frequency of the 3D-IAW.

III. DYNAMICS OF PMSW

Assuming the dynamics of low frequency PMSW along the $x-$ axis and electric field polarized in ‘y’ direction i.e. $\vec{k} = k_x\hat{x}$ and $\vec{E} = E\hat{y}$ .The background magnetic field is along the $z-$ axis i.e. $\vec{B}_0 = B_0\hat{z}$, where B_0 is the ambient magnetic field.

The dynamical equation for PMSW is as follows

(i) The equation of motion:

$$\frac{\partial \vec{v}_j}{\partial t} = \frac{q_j}{m_j}\vec{E} + \frac{q_j}{cm_j}(\vec{v}_j \times \vec{B}_0) - \frac{\gamma_j kT_j}{m_j}\vec{\nabla}\frac{n_j}{n_0} + \vec{F}_j , \tag{11}$$

(ii) The continuity equation:

$$\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{v}) = 0 , \tag{12}$$

(iii) Faraday’s law:

$$(\vec{\nabla}\times\vec{E}) = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{13}$$

where $\vec{v}_j$ is the velocity of species j = i, e (i = ions, e = electrons), m_j and T_j are the masses and temperature of ions and electrons respectively, c is speed of light in vacuum and $\vec{F}_j = -[m_j(\vec{\upsilon}_j.\vec{\nabla})\vec{\upsilon}_j - \frac{q_j}{c}(\vec{\upsilon}_j \times \vec{B}_0)]$ is the ponderomotive force due to 3D-IAW. Putting the values of $\vec{v}_j$ in wave equation and taking 'y' component of that, one can have

$$\frac{\partial^2 E_y}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}-\frac{1}{V_A^2}\frac{\partial^2 E_y}{\partial t^2}=\frac{4\pi n_0 T_e}{cB_0}\frac{\partial^2}{\partial x\partial t}\left(\frac{n_e}{n_0}\right)+\frac{4\pi n_0 e}{c^2}\left[-\frac{\omega}{i\omega_{cj}}\frac{F_{jx}}{m_j}-\frac{\omega^2}{i\omega_{cj}^2}\frac{F_{jy}}{m_j}\right] \quad (14)$$

The electron continuity equation yields

$$\frac{\partial}{\partial t}\left(\frac{n_e}{n_0}\right)=-\frac{\partial}{\partial x}\left(\frac{cE_y}{B_0}\right). \quad (15)$$

Dynamical equation for PMSW by substituting equation (15) into equation (14), can be obtained as

$$\left[(1+\beta)\frac{\partial^2}{\partial x^2}-\frac{1}{v_A^2}\frac{\partial^2}{\partial t^2}\right]\left(\frac{n}{n_0}\right)=-\frac{4\pi n_0 e k_x}{cB_0}\left(-\frac{F_{ix}}{i\omega_{ci}m_i}-\frac{\omega}{\omega_{ci}^2}\frac{F_{iy}}{m_i}-\frac{F_{ex}}{i\omega_{ce}m_i}+\frac{\omega}{\omega_{ce}^2}\frac{F_{ey}}{m_i}\right) \quad (16)$$

$$\vec{F}_j=-m_j\left(\upsilon_{jx}^*\frac{\partial}{\partial x}+\upsilon_{jy}^*\frac{\partial}{\partial y}+\upsilon_{jz}^*\frac{\partial}{\partial z}\right)\vec{\upsilon}_e+\frac{q_j}{c}\left(\vec{\upsilon}_{j\perp}\times\vec{B}_\perp\right) \quad (17)$$

After substituting the value of velocity components in the equation (17), we obtained components of ponderomotive force due to 3D-IAW as below

$$F_{ex}=\left(\frac{\omega_o}{B_0}\right)^2\left[-\frac{m_e}{4}\frac{(1+\varepsilon)^2k_{r0}^2}{2k_{z0}^2}+\frac{m_i}{8}\frac{(1+\varepsilon)k_{r0}^2}{(1-\alpha)k_{z0}^2}\right]\frac{\partial}{\partial x}|A_z|^2,$$

And $$F_{ey}=\frac{m_e}{4}\left(\frac{\omega_0}{B_0}\right)^2\frac{(1+\varepsilon)^2k_{r0}^2}{2k_{z0}^2}\frac{\partial}{\partial x}|A_z|^2 \quad (18)$$

$$F_{ix}=\frac{m_i}{4}\left(\frac{\omega_o}{B_0}\right)^2\left[-\frac{(1+\varepsilon)^2(1+\alpha)k_{r0}^2}{(1-\alpha)^2k_{z0}^2}+\frac{\varepsilon}{\alpha}\right]\frac{\partial}{\partial x}|A_z|^2,$$

And $$F_{iy}=\frac{m_i}{4}\left(\frac{\omega_o}{B_0}\right)^2\left[\frac{(1+\varepsilon)^2k_{r0}^2}{2(1-\alpha)^2k_{z0}^2}\left\{1+\left(\frac{2i}{1-\alpha}\right)\left(\frac{\omega_0}{\omega_{ci}}\right)\right\}\right]\frac{\partial}{\partial x}|A_z|^2 \quad (19)$$

where $F_{ex}, F_{ey}, F_{ix}, F_{iy}$ are the components of ponderomotive force due to 3D-IAW, substituting equations (18) and (19) in equation (16) and dynamical equation for PMSW (for $\omega \ll \omega_{ci}$) as follows,

$$\left((1+\beta)\frac{\partial^2}{\partial x^2}-\frac{1}{V_A{}^2}\frac{\partial^2}{\partial t^2}\right)\left(\frac{n}{n_0}\right)=$$
$$\frac{\omega_0^2}{4V_A{}^2B_0{}^2}\left\{-\frac{\left((1+\varepsilon)^2k_{0r}^2\right)}{2k_{0z}^2}\left\{(1+\alpha)+\frac{m_e}{m_i}\right\}+\left\{\frac{\varepsilon}{\alpha}+\frac{k_{0r}^2}{k_{0z}^2}\right\}\right\}\left(\frac{\partial^2\left(|A_z|^2\right)}{\partial x^2}\right) \quad (20)$$

where $\beta=\frac{C_s{}^2}{V_A{}^2}$, $\alpha = \frac{\omega_0{}^2}{\omega_{ci}{}^2}$ and $\varepsilon=k_{0r}{}^2\lambda_e{}^2$

Equation (20) represents the dynamical equation of PMSW whose right hand side represents ponderomotive force due to 3D-IAW.

Equation (20), after normalization, and equation (10) can be written in dimensionless form as

$$\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial t^2}\right)n=-\frac{\partial^2\left|A_z\right|^2}{\partial x^2} \tag{21}$$

and

$$i\frac{\partial A_z}{\partial t}-i\xi_1\frac{\partial A_z}{\partial r}-\xi_2\frac{\partial^2 A_z}{\partial r^2}+i\frac{\partial A_z}{\partial z}+nA_z=0 \tag{22}$$

where $\xi_1=\frac{\omega_0^2}{2\sqrt{2}\times V_A^2k_{0z}^2k_{0r}\lambda_e}$, $\xi_2=\frac{\omega_0^2}{8\times V_A^2k_{0r}^2\lambda_e^2k_{0z}^2}$. The normalising parameters are $x_n\approx\lambda_e$, $z_n\approx\frac{1}{2k_{0r}^2\lambda_e^2k_{0z}}$, $t_n\approx\frac{\omega_0\left(1+\lambda_e^2k_{0r}^2+\tau\right)}{2\times V_A^2k_{0z}^2}$, $n_n\approx 4k_{0r}^2\lambda_e^2n_0$ and

$$A_{zn}\approx\left(\frac{4k_{0r}^4\lambda_e^2V_A^2(1+\beta)}{\omega_0^2\left\{-\frac{\left((1+\varepsilon)^2k_{0r}^2\right)}{2k_{0z}^2}\left\{(1+\alpha)+\frac{m_e}{m_i}\right\}+\left\{\frac{\varepsilon}{\alpha}+\frac{k_{0r}^2}{k_{0z}^2}\right\}\right\}}\right)^{\frac{1}{2}}A_{z0}$$

By considering adiabatic response of equation (21), we get

$$n=-\left|A_z\right|^2 \tag{23}$$

Combining equation (23) into equation (22), we get the following equation

$$i\frac{\partial A_z}{\partial t}-i\xi_1\frac{\partial A_z}{\partial r}-\xi_2\frac{\partial^2 A_z}{\partial r^2}+i\frac{\partial A_z}{\partial z}-\left|A_z\right|^2A_z=0 \tag{24}$$

IV. NUMERICAL SIMULATION

We have performed the numerical simulation of equations (24) using 2D pseudo-spectral method in a $\left(2\pi/\alpha_r\right)\times\left(2\pi/\alpha_z\right)$ periodic spatial domain with wave numbers of perturbation, α_r, α_z =0.2 (normalized by x_n^{-1} and z_n^{-1} respectively) and (256×256) grid points. The initial conditions of simulation are

$$A_z\left(r,z,0\right)=A_{z0}\left(1+0.1\cos\left(\alpha_r r\right)\right)\left(1+0.1\cos\left(\alpha_z z\right)\right)$$

where $\left|A_{z0}\right|$= 0.5 is the amplitude of the homogenous pump 3D-IAW. A finite difference with predictor-corrector method was utilized for the evolution in time with step size of $dt=5\times10^{-5}$. To solve system of dimensionless equation (24), we have studied the algorithm for the well-known modified nonlinear Schrödinger (NLS) equation. The accuracy was determined by consistency of the number $N=\sum_k\left|A_{zk}\right|^2$ in the case of NLS equation. The quantity N was conserved up to the order of 10^{-5} during computation. After testing algorithm of NLS, it has been modified for solving dimensionless equations (24) which is used to study nonlinear coupling of 3D-IAW with PMSW and resulting turbulent spectrum and formation of localized structures.

The values of ξ_1 and ξ_2 can be estimated from the low-β plasma parameters. For application purpose in low-β plasma, the typical parameters for auroral altitude of 1700 km (*Wu et al.*, 1996) are as follows: $B_0 \approx 0.3G,\ \ n_0 \approx 5\times10^3 cm^{-3},\ T_e \approx 1.16\times10^4 K$. Using these parameters, one can find: $V_A \approx 9.25\times10^8 cm/s$,

$V_{te} \approx 3.23\times10^7 cm/s,\ \lambda_e \approx 7.42\times10^3 cm,\ \omega_{ci} \approx 2.87\times10^3 rad/\sec,\ \rho_S \approx 481.80\, cm,$ $c_s = 1.38\times10^6 cm/s$. For $k_{0r}\lambda_e \approx 0.11$ and $\omega_0/\omega_{ci} \approx 0.49$, one can calculate $k_{0r} \approx 1.46\times10^{-5} cm^{-1}$, $k_{0z} \approx 1.80\times10^{-6} cm^{-1}$ and $\omega_0 \approx 1.43\times10^3 rad/\sec$. The normalising parameter values are $x_n \approx 7.34\times10^3\ cm,\ z_n \approx 2.35\times10^7\ cm,\ t_n \approx 0.03\sec,\ n_n \approx 256\, cm^{-3}$ and $A_{zn} \approx 1.45\times10^4\ G-cm$.

V. RESULTS AND DISCUSSION

In Sec. 3, numerical simulation has been carried out for dimensionless equations (24) to study formation of localized structures due to the nonlinear interaction 3D-IAW with PMSW, applicable to low beta plasmas like auroral region.

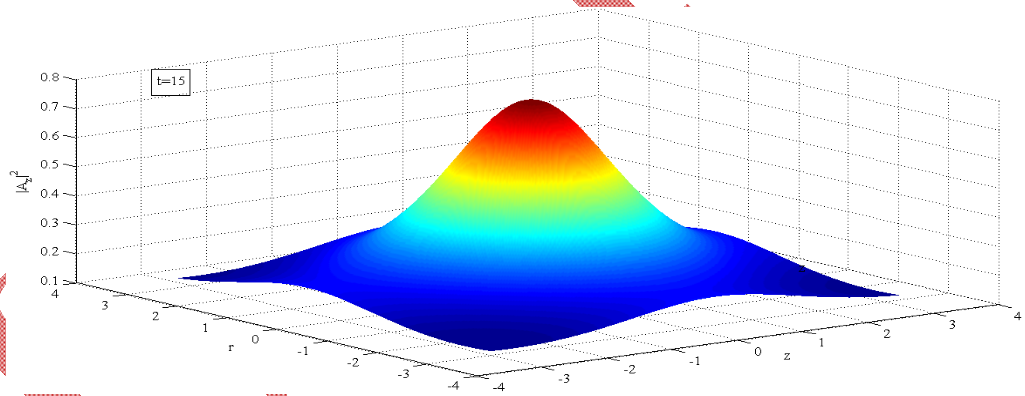

Fig. 1: The propagation dynamics of 3D-IAW (auroral region) at time t=15

Here, we have selected two instants of time namely at t = 33s where system reaches to quasi steady state and t=15s, which is a few steps before the system reaches in the quasi-steady state. Figure (1) and figure (2) reveals the formation of nonlinear localized structure at t=15s and t=33s respectively. On account of the nonlinear ponderomotive force (which changes with time), density of PMSW starts getting altered and hence affects the dynamics of 3D-IAW which leads to the localization of the 3D-IAWs. From the figure (1) and figure (2), it is observed that the nonlinear localized structures are less complex in behavior at early stage (t=15s) and becomes more complex at t=33s (quasi steady state). From the figures (2), one can observe that localized structures are form at location r = 2.53 and z = 0.31 (figure (2)), but as time increases, localized structures become intense and more complex in behavior.

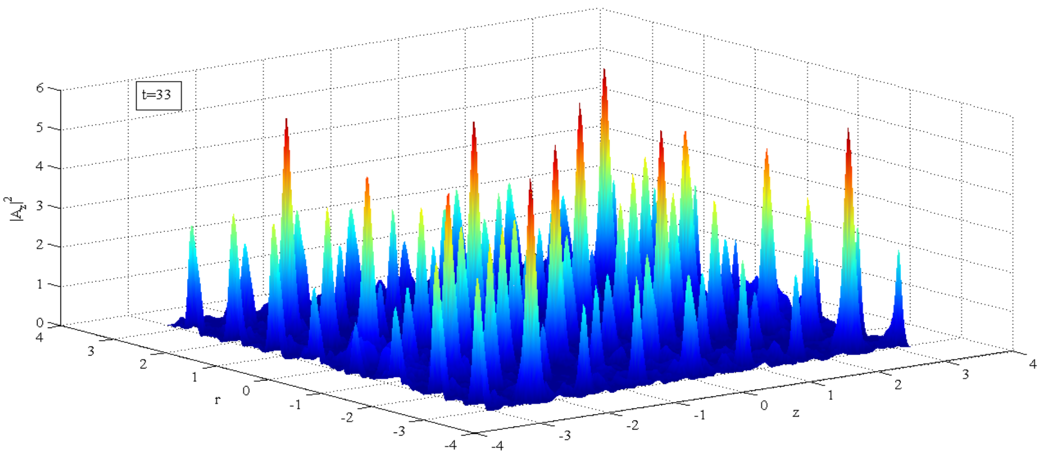

Fig. 2: The propagation dynamics of 3D-IAW (auroral region) at t=33

The formation of localized structures at higher wave numbers are one of the possible reasons for heating and particles acceleration.

VI. CONCLUSION

To summarize, we have investigated nonlinear interaction of 3D-IAW and PMSW for low β - plasma. Numerical simulation of equations (24), governing the dynamics of 3D-IAW and PMSW, has been carried out applicable to low beta plasmas like auroral region. The background density of magnetosonice wave gets altered due to the ponderomotive force of the pump 3D-IAW, which 3D-IAW gets localized/filamentation. Therefore, 3D-IAW breaks up into coherent structures as different origin of free energy accessible due to the inhomoginity of density. The result reveals that localized structures becomes progressively more complex and grows towards smaller length scales. The formation of localized structure depends on the magnitude of the ponderomotive force and the behaviour of the interacting waves. The localized structures of 3D-IAW, in the direction perpendicular to background magnetic field, could play an crucial role in dissipation of inertial Alfvén wave and can lead to the plasma heating and particles acceleration.

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