# MODEL FOR PREDICTING THE STRENGTH OF MASONRY CONSTRUCTION

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**ABSTRACT**

*The design of masonry structures is currently done by utilizing the characteristic strength of masonry construction which is normally obtained from tables in BS 5628. Such characteristic strength is determined by relating the compressive strength of units (bricks, blocks) and the class of mortar. Use of tables in the design is cumbersome as the designer needs a number of hard copy tables because there is no any mathematical model that could be used for the same purpose. In addition, the current practice poses difficulty in writing masonry design software. To solve these problems, a study has been carried out with the objective of developing a mathematical model that can be used to determine the characteristic strength of masonry construction for utilizing in the design. Hence, laboratory tests were carried out to determine the compressive strengths of bricks and mortars. Using the same samples of bricks and mortars, sample walls were constructed then tested for the compressive strength at the age of 28 days. The obtained results from tests were used to develop a mathematical model, which after being tested has been found to be suitable for determining the characteristic strength of masonry construction.*

***Keywords: Masonry, Mortar, Bricks, Blocks, Design, Model, Equation, Masonry Structures***

## I. INTRODUCTION

In order to achieve durable, robust and strong masonry constructions, there must be good design, good materials and good construction practices. The first step in realizing such a project is the production of good designs which comply to the established standards, after which the construction has to be carried out adhering to the specifications given in the design documents which include structural drawings. Currently, the design of masonry structures here in Tanzania is based on the code of BS 5628 Parts 1 and 2[**1**]. In the said code, the characteristic compressive strength of masonry construction is determined using the compressive strengths of masonry elements such as bricks and blocks and the mortar class that reflects a certain compressive strength. Masonry construction strength is obtained by using tables which relate the compressive strengths of units and mortar class. During design, always there must be tables in hard copy because there is no any mathematical model which could be used instead of the said tables. Therefore, there is a need to develop a mathematical model that can be utilized to predict the characteristic strength of masonry construction and use it in the design of masonry structural elements. In addition, the mathematical model can easily be used in writing a masonry design software.

## II. OBJECTIVES

The main objective of this paper is to develop a mathematical model that can be utilized in determining the characteristic strength of masonry construction so that it can be used in the design of masonry structural elements. With respect to the main objective, the specific objectives embrace the following:

i) To establish the compressive strength of masonry units

ii) To establish the compressive strength of mortar

iii) To establish the strength of masonry walls built from masonry units and mortars in (i) and (ii) above

iv) And finally, to develop a mathematical model and test for its suitability.

## III. OVERVIEW ON MASONRY DESIGN

Masonry construction is defined as an assembly of bricks, stones or block units bonded together by mortar in predetermined pattern known as bond. The strength of masonry structural elements such as walls and columns depends on several parameters, the chief of which being the following:-

- the strength of individual units
- the strength of mortar

Other minor parameters include:

- the thickness of mortar, and
- the type of bond pattern

Masonry units are required to have sufficient engineering properties such as strength, form of brick or block, and an appropriate size. The compressive strength of a brick is the maximum stress to which the unit can be subjected by gradually increasing load applied perpendicular to the bedding plane or normal position. The ultimate compressive strength of masonry wall is less than the strength of blocks or bricks. Table 1 shows the common strength values for different masonry units that are frequently used in masonry construction works. Also, in the same Table 1, shown are the mortar classes and the expected characteristic strengths of masonry constructions. The sub-tables (a) and (b) in Table 1 are among the tables found in BS 5628 [**1**] that are used to determine the characteristic strengths; $f_{k,}$ of masonry construction used for the design of structural elements

**Table 1: Characteristic Compressive Strength of Masonry, $F_{k,}$ In N/Mm$^2$**

(a) Constructed with standard format bricks

| Mortar Designation | Compressive strength of unit (N/mm$^2$) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **5** | **10** | **15** | **20** | **27.5** | **35** | **50** | **70** | **100** |
| **(i)** | 2.5 | 4.4 | 6.0 | 7.4 | 9.2 | 11.4 | 15.0 | 19.2 | 24.0 |
| **(ii)** | 2.5 | 4.2 | 5.3 | 6.4 | 7.9 | 9.4 | 12.2 | 15.1 | 18.2 |
| **(iii)** | 2.5 | 4.1 | 5.0 | 5.8 | 7.1 | 8.5 | 10.6 | 13.1 | 15.5 |
| **(iv)** | 2.2 | 3.5 | 4.4 | 5.2 | 6.2 | 7.3 | 9.0 | 10.8 | 12.7 |

(b) Constructed with blocks having a ratio of height to least horizontal dimension of 0.6

| Mortar Designation | Compressive strength of unit (N/mm$^2$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **2.8** | **3.5** | **5.0** | **7.0** | **10** | **15** | **20** | **35 ≤** |
| **(i)** | 1.4 | 1.7 | 2.5 | 3.4 | 4.4 | 6.0 | 7.4 | 11.4 |
| **(ii)** | 1.4 | 1.7 | 2.5 | 3.2 | 4.2 | 5.3 | 6.4 | 9.4 |
| **(iii)** | 1.4 | 1.7 | 2.5 | 3.2 | 4.1 | 5.0 | 5.8 | 8.5 |
| **(iv)** | 1.4 | 1.7 | 2.2 | 2.8 | 3.5 | 4.4 | 5.2 | 7.3 |

### 3.1 Brick Sizes and Forms

According to **EN 771-1: 2003[2], b**ricks are defined as walling units designed to be laid in mortar with dimensions not exceeding 337.5 x 225.0 x 112.5 mm (length x thickness x height). The standardized format module introduces a modular size of length x width x height of 215 x 102.5 x 65 [mm] and mortar joints of 10 mm thick. The working size of the bricks therefore includes the mortar thickness and comes to be 225 x 122.5 x 75 mm.

There are three different forms of bricks; solid, cellular ad hollow, and within each form a variety of products are available thus providing versatility to brickwork construction both in style and function. The British Standard; BS 6093: Part 1[**4**] defines a block as a masonry unit of large size in all dimension than specified for bricks but no dimension should exceed 650mm nor should the height (in its normal aspect) exceed either its length or six times its thickness.

To ascertain the quality of bricks/blocks, they are usually checked for good shape, sharp edges, flat surface (no arching), clean surfaces and hard. Also the water absorption should be within acceptable values and some simplified soundness and strength tests are employed. The advantages of masonry constructions include the following:

- does not require the use of formworks and therefore saves cost,
- forms lighter structures if compared to reinforced concrete and steel structures, they provide greater strength to weight ratio, a suitable condition for seismic areas
- have high fire resistant due to the nature of the masonry materials,
- provides good aesthetics,
- they are relative strong.

### 3.2 Structural Design

In the theory of masonry design, it is assumed that masonry can take large compressive forces and very small tensile stresses [**3**]. Axial forces which are eccentric may be split up into two components; an axial force ***P*** and a bending moment ***M* = *P.e*** , where ***e*** is the eccentricity at the section being considered as shown in Figure 1.

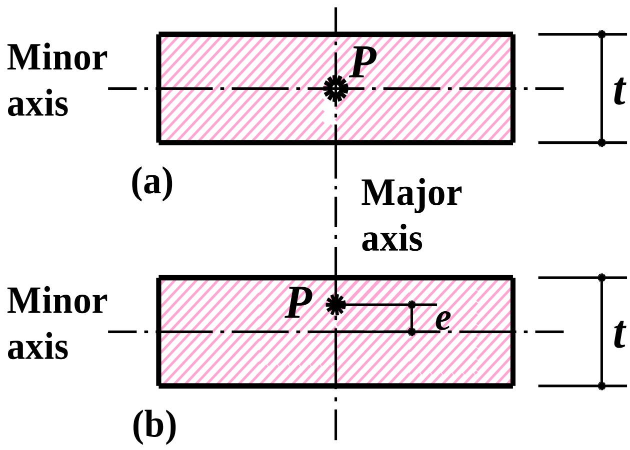


**Figure 1: Masonry Wall Plan With: (A) Axial Vertical Load, (B) Eccentric Vertical Load**

When a masonry structural element is acted upon by an axial compressive force, the developed stresses in the bed joints act as shown in Figure 2.

The failure of masonry walls or columns under vertical compression action can be either buckling or crushing.

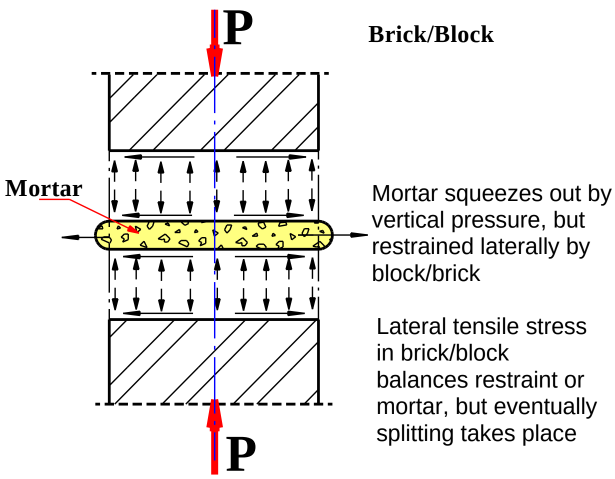


**Figure 2: Forces in a Masonry Joint**

Masonry elements in bending will usually fail when the developed tensile stress is greater than the masonry tensile strength. For solid rectangular walls which are able to accept some tension, the stresses at either side of the wall are given by equation (1) and depicted in Figure 3.

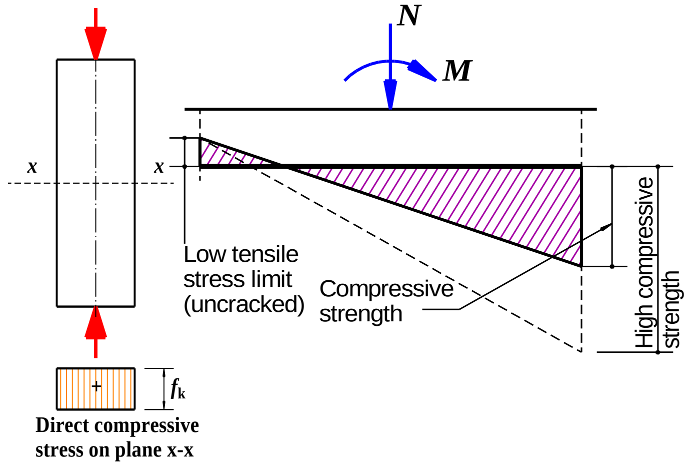


Figure 3: Stresses In Plain Masonry

$$\sigma_{1,2} = \frac{P}{A}\left(1 \pm \frac{6.e}{t}\right) \qquad \text{….(1)}$$

Where P = axial load

A = Area of wall

t = thickness of wall

e = eccentricity

BS 5628; Part 1[**1**], states that the design vertical load of resistance of a wall must be greater than the design vertical load given by the following equation:

$$P_{Rd} = \frac{\beta.t.f_k}{\gamma_m} \qquad \text{….( 2)}$$

where $\gamma_m$ = partial safety factor for material strength , ranging from 2.5 to 3.5 depending on the category of construction, manufacturing and structural control

$\beta$ = capacity reducing factor allowing for the effects of slenderness and eccentricity

$f_k$ = characteristic compressive strength of masonry as defined in Table 1

$t$ = thickness of the wall

The vertical design load of resistance of a rectangular column is defined by equation (3), thus:

$$P_{Rd} = \frac{\beta . b . t . f_k}{\gamma_m} \quad \text{....(3)}$$

where $b$ = width of the column

$t$ = thickness of the column

other parameters are as defined in equation ( 2) The process of designing masonry walls involves the determination of values of design loads of resistance in compression, flexural or shear. The values obtained are then compared with the actual design loads acting in the walls.

### 3.3 Design Loads

The vertical design load at ultimate limit state is determined by the expression:

$$P = \gamma G_k + \gamma Q_k \quad \text{….(4)}$$

Where $\gamma$ = partial safety factor for dead load $G_k$ and live load $Q_k$

The conformance criteria must always be fulfilled, thus:

$$P_{Rd,} \geq P \quad \text{....(5)}$$

The applied design moment is calculated by using equation (6), thus:

$$M = \alpha H.\ \gamma_f . l^2 \text{ or } M = \alpha H.\ \gamma_f . h^2 \quad \text{….(6)}$$

depending on the lesser between $l$ and $h$.

where $\alpha$ = bending moment coefficient per unit width or length of the wall.

The design moment of resistance of the wall is given by

$$M_{Rd} = f_{kx} . z / \gamma_m$$

Or ….(7)

$$M_{Rd} = f_{ka} . z / \gamma_m$$

Where, $f_{kx}$ = the characteristic flexural strength of the wall spanning horizontally,

$f_{ka}$ = effective flexural strength of the wall and is equal to $f_{kx} + \gamma_m g_d$ i.e including the design dead vertical load of the wall.

In BS 5628, the elastic design moment of resistance for a freestanding wall is expresses as:

$$M_{Rd} = [\frac{f_{kx}}{\gamma_m} + g_d]z \quad \geq \quad M \quad \text{….(8)}$$

where $f_{kx}$ = characteristic flexural strength of masonry at the critical section

$z$ = section modulus

$g_d$ = design vertical dead load per unit area

In cases where the flexural strength of masonry cannot be relied upon, a freestanding wall can only be used when there is sufficient vertical load action. The design moment of resistance can be determined using the expression:

$$M_{Rd} = \frac{n_w}{2}\left(t - \frac{n_w \gamma_m}{f_k}\right) \quad \text{....(9)}$$

where $t$ = thickness of the wall

$n_w$ = design vertical load per unit length

$f_k$ = characteristic compressive strength of masonry

### 3.4 Mortar

There are 4 mortar designations, namely type (i), type (ii), type (iii), and type (iv). Type (i) is the strongest mortar, while mortar type (iv) is the weakest. Values for the four types of mortar can be viewed in Table 2 of BS 5628 Part 1.

### 3.5 Good Properties of Mortar

The desired properties of mortar for jointing are:

- In fresh state: its workability should be plastic and coherent without bleeding.
- Good bond to units: development of early strength, as well as good final strength.
- Sufficient water resistance (low permeability in the hardened state).
- Deformation behavior should be similar to that of the block/brick units. Choice for mortar designation will be influenced by durability requirements.
- The higher the ratio of height to least horizontal dimension, the fewer the number of mortar bed joints of assembling, hence the higher the strength.

## IV. LABORATORY INVESTIGATION AND TEST RESULTS

The investigation was carried out in four stages which include preparation of mortar samples and testing, manufacturing of cement-sand bricks and testing, construction of masonry wall samples and testing, and finally was analysis of results and development of model for predicting the characteristic compressive strength of masonry construction, $f_k$.

### 4.1 Mortars

A total of 6 mortar mixes of cement to sand ratios were prepared. Ordinary Portland Cement and sand which was free from organic compounds were used. The mix ratios by weight were 1:5, 1:6, 1:8, 1:9, 1:11 and 1:12. The mixing process was done using a laboratory concrete mixer machine. Then the mixed material was filled in the moulds of size 70x70x70 mm as shown in Figure 4, and compacted using a vibration table. After 1 day, the cubes were taken off the molds and then kept in water bath for 4 days and 21 days respectively. For each mix ratio, 6 mortar cubes were prepared in which 3 were tested at the age of 7 days while the other 3 were tested at the age of 28 days. A summary of selected test results from the mortar compressive strength test are as shown in Table 2.

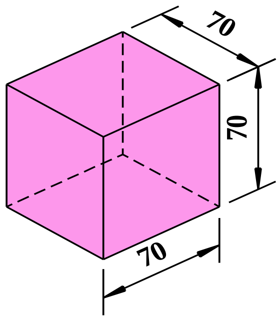

Figure 4: Mortar Cubes Of Various Ratios

**Table 2: Selected Compressive Strengths and Densities of Mortar Samples**

| No. | Ratio | Age | Sample No. | Density kg/dm$^3$ | Compressive strength N/mm$^2$ | Average Compressive strength N/mm$^2$ |
|---|---|---|---|---|---|---|
| 1 | 1:5 | 7 days | 1 | 2.025 | 5.71 | 5.70 |
| | | | 2 | 2.023 | 6.02 | |
| | | | 3 | 2.026 | 5.60 | |
| | | 28 days | 1 | 2.017 | 9.20 | 8.91 |
| | | | 2 | 1.991 | 8.67 | |
| | | | 3 | 2.008 | 8.88 | |
| 2 | 1:6 | 7 days | 1 | 1.997 | 4.081 | 3.979 |
| | | | 2 | 1.944 | 3.979 | |
| | | | 3 | 1.979 | 3.877 | |
| | | 28 days | 1 | 1.956 | 5.306 | 5.914 |
| | | | 2 | 2.128 | 6.122 | |
| | | | 3 | 2.081 | 5.714 | |

## 4.2 Bricks

In order to obtain varieties of bricks strengths, four different mix ratios of cement and sand were proposed. The mix ratios were **1:4, 1:6, 1:8 and 1:10 (cement: sand).** Batching was done by weight and mixing of materials was done using mixer machine at the laboratory. The amount of water varied between 0.35 to 0.50 water-cement ratio depending on the amount of cement in the mix. The size of the bricks was as depicted in Figure 5. The materials used for making bricks were Ordinary Portland cement, clean sand which was free from lumps of stones, or earth loaded dust, nor organic matter or any other deleterious substances. The water used for mixing was from tap, clean, fresh and free from harmful chemicals. Testing was done after the bricks had attained 7 and 28 days ages using a universal compression test machine in the laboratory. Some of the test results are given in Table 3.

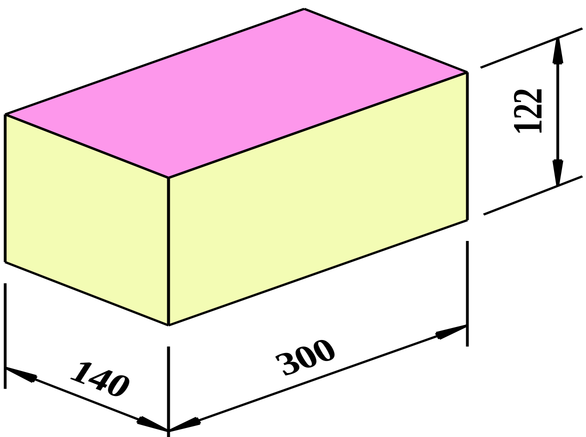


**Figure 5: Size of bricks**

**Table 3: Compressive Strengths and Densities of Sample Bricks**

| No. | Ratio | Age | Sample No. | Density kg/dm$^3$ | Compressive strength N/mm$^2$ | Average Compressive strength N/mm$^2$ |
|---|---|---|---|---|---|---|
| 1 | 1:4 | 7 days | 1 | 2.020 | 5.494 | 5.772 |
| | | | 2 | 2.035 | 5.733 | |
| | | | 3 | 2.126 | 6.091 | |
| | | 28 days | 1 | 2.064 | 9.675 | 8.839 |
| | | | 2 | 2.130 | 8.003 | |
| | | | 3 | 2.012 | 8.839 | |

### 4.3 Wall Samples

The third required parameter was the compressive strength of masonry wall elements, so that the three strength categories could be related mathematically. Hence, eight sample wall specimens of 2.5 bricks in length, 140 mm thick and 5 courses high were constructed. Bricks from one mix ratio of cement-sand were used to build 2 sample walls where by the binding mortars were different from each other. The mix ratio for bricks and that for the respective mortar that was used to bond the bricks is given in Table 4.

**Table 4: Ratios of Bricks As Well As Mortars**

| Type 1 walls | | | | Type 2 walls | | |
|---|---|---|---|---|---|---|
| Nr. | Brick ratio | Mortar ratio | | Nr. | Brick ratio | Mortar ratio |
| 1 | 1:4 | 1:5 | | 5 | 1:4 | 1:6 |
| 2 | 1:6 | 1:8 | | 6 | 1:6 | 1:9 |
| 3 | 1:8 | 1:9 | | 7 | 1:8 | 1:11 |
| 4 | 1:10 | 1:11 | | 8 | 1:10 | 1:12 |

**Figure 6: Sample walls for testing**

One photo of the constructed 4 sample walls is shown in Figure 6. All eight walls were tested after attaining an age of 28 days. In Table A1, shown are the test results for the bricks, mortars and the respective sample walls.

### 4.5 Analysis and Model Establishment

Basing on Table A1 results, the relationship between strengths of bricks and mortars against wall strengths were developed and plotted as shown in Figure 7a and 7b.

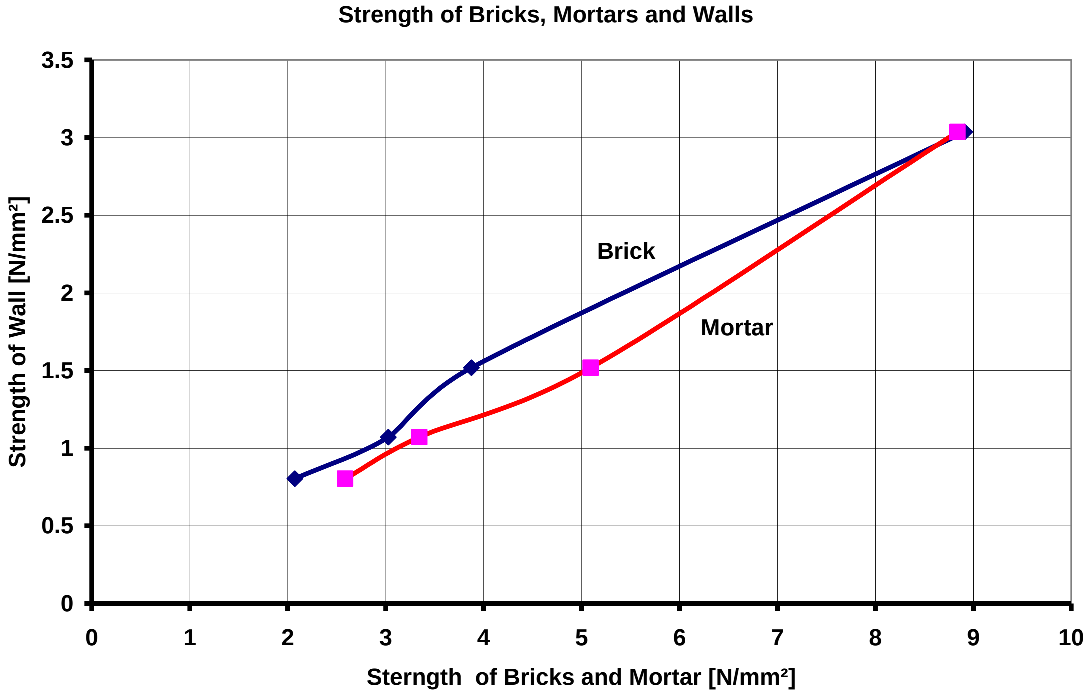


**Figure 7a: Relationship between Strengths of Bricks and Mortar to Masonry Wall Type 1**

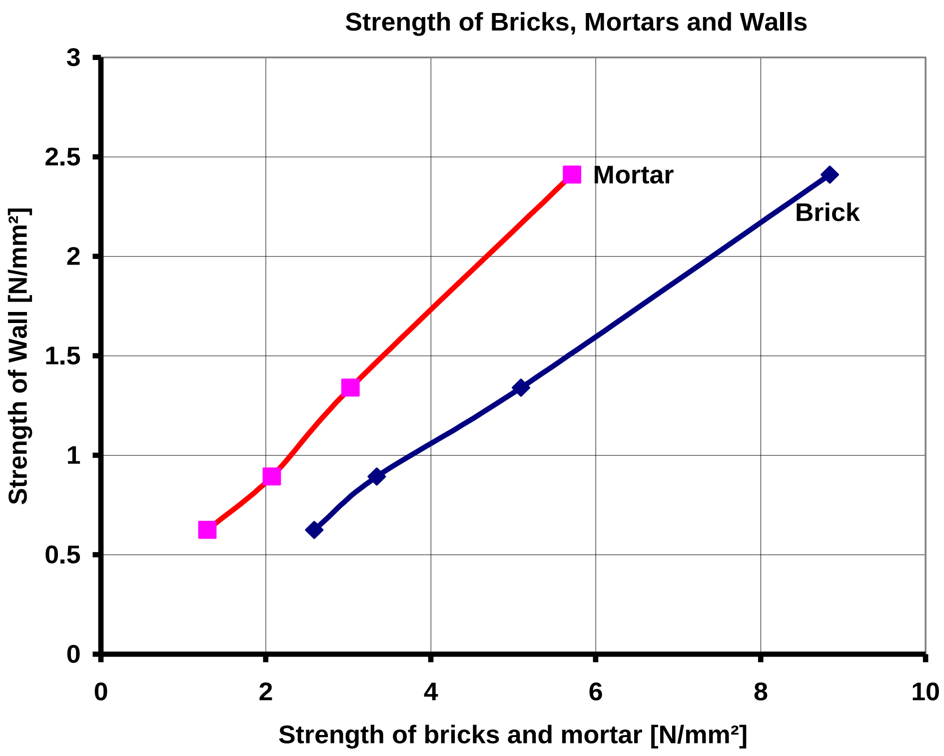


**Figure 7b: Relationship between Strengths of Bricks and Mortar to Masonry Wall Type 2**

The form of the equation from the above graphs can be in the form of

$$f_k = \beta f_b + \alpha f_m \qquad \text{....(10)}$$

Where $f_b$ = compressive strength of bricks

$f_m$ = compressive strength of mortar

$f_k$ = characteristic compressive strength of masonry construction (wall)

Letting the coefficient of mortar strength be $\alpha$ and the coefficient for brick strength be **$\beta$.** Then, solving simultaneous equation by alternating the values of compressive strength in rows due to different mix ratios from Table A1, values of $\alpha$ and $\beta$ can be obtained. Using permutation principle, a number of equations equal to 4!/2 (4 factorial)/2 was established for each wall type indicated in Table A1; for example taking walls type 1:

By using values in a permutation form of the rows, a number of equations of the form:

$\alpha f_m + \beta f_b = f_k$ were established.

The obtained equations were then solved in pairs as simultaneous equations to get the values of :

$\alpha$ and $\beta$ respectively.

The process was repeated for all equations, and a summary of the obtained results for both wall types were obtained, thus:

**Walls Type 1**

The average value of $\alpha$ was found to be 0.188 while the value of $\beta$ was 0.155

**Walls Type 2**

The average value of $\alpha$ was equals to 0.203 and that of $\beta$ was equal to 0.142

Combining the coefficients obtained from walls type 1 and type 2, the final values were established as

**$\alpha$ =0.1955 and $\beta$ =0.1485.**

Thus, from these results, the values of **$\alpha$ and $\beta$** were approximated as **0.196 and 0.149** respectively. Therefore, substituting the coefficients **$\alpha$ and $\beta$** in equation (10), the resulting model expression for estimating the characteristic compressive strength of masonry construction $f_k$ becomes:

$$f_k = 0.196\, f_m + 0.146\, f_b \qquad \text{......(11)}$$

The mathematical model (**eqn. 11**) was then tested for its suitability in determining the theoretical strength of masonry walls based on the strength of bricks as well as the respective mortars, and it was found to be suitable as illustrated in Figure 8. In the said figure, the model prediction and the actual test results are agreeing to each other since their respective graphs are over each other.

The ratio between test results and model results was found to be varying between **97.64%** and **100.96%,** the average being **99.87%** as depicted in Figure 8.

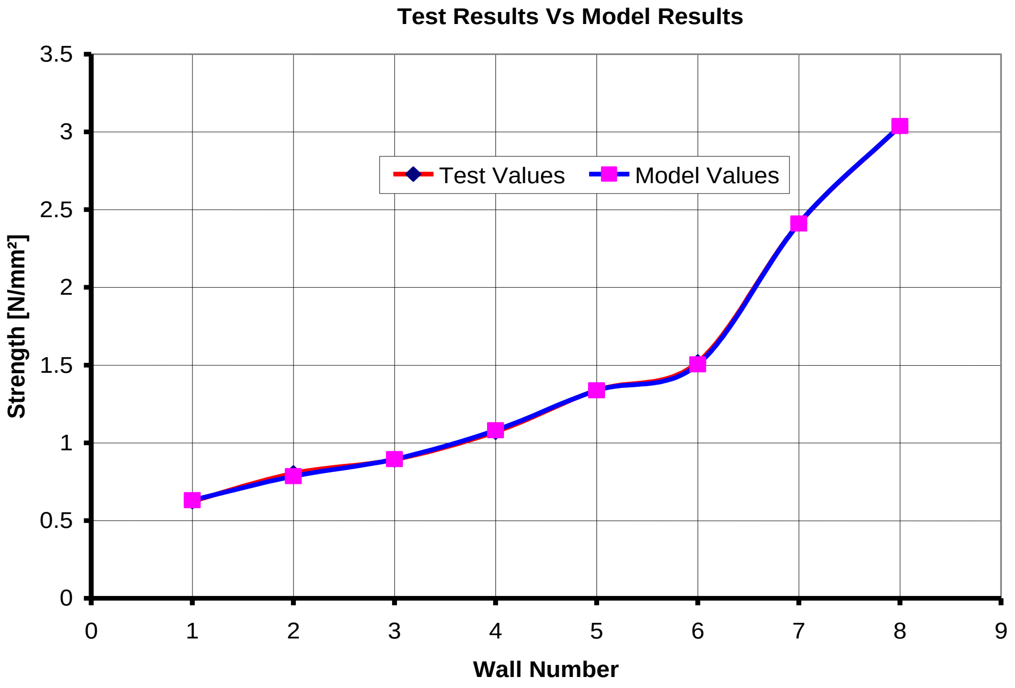


**Figure 8: Test Results Compared To Model Results**

## 4.6 Design Example

A wall that supports a floor slab is 225 mm thick and its height is 3175 mm. The end conditions are such that at bottom it offers enhanced resistance and at top it offers simple resistance. The characteristics of materials are: compressive strength of bricks **7.65 N/mm$^2$** and that of mortar is **5.75 N/mm$^2$**. Assuming a partial factor of safety of 3.1, check whether the wall is sufficiently strong to safely carry the applied loads or not. The characteristic dead load from the floor is **24.20 kN/m** and the imposed load is **16.60 kN/m.**

## 4.7 Solution

Slenderness ratio,

$$SR = \frac{h_{ef}}{t_{ef}} = \frac{0.85x3.175}{0.225} = 12.00 < 27 \text{ OK}$$

Hence the capacity reduction factor, β from BS 5628 Part 1 = 0.93

Design vertical load: Select Case (a) of Clause 22 of the Code: $1.4G_k + 1.6Q_k$.

**At top of wall:** Design vertical load,

**P = 1.4(24.2) + 1.6(16.6) = 60.44 kN/m**

Characteristic compressive strength of masonry construction is now determined using equation (**11**);

$$f_k = 0.146\, f_b + 0.196\, f_m,$$

given that

$f_b$ = 7.65 N/mm$^2$ and $f_m$ =5.75 N/mm$^2$,

then the resulting characteristic compressive strength of masonry is:

$f_k$ = 2.244 N/mm$^2$

Design vertical load resistance of the wall

$$P_u = \frac{\beta . t . f_k}{\gamma_m} = \frac{0.93x225x1000x2.244}{3.1x1000}$$

$$P_u = 151.47 \; kN / m \quad > \quad P \quad OK$$

### 4.6 Discussion of the Results

Referring to Table 1 in page 4 of this paper, or Table 2b in BS 5658: Part 1, if you take the strength of units, and a certain compressive strength value of mortar (depending on the designation), the characteristic compressive strength of masonry, $f_k$ in N/mm$^2$ seems to agree to the results of the developed Mathematical model given by equation (**11**).

These results are also in agreement with the results given by the equation of the form:

$$f_k = 0.35 f_b^{0.65} + f_m^{0.25} \qquad \text{....(12)}$$

where $f_k$ = characteristic compressive strength of masonry construction,
$f_b$ = compressive strength of bricks,
$f_m$ = compressive strength of mortar,

that was developed by Freeda et al [**5**], giving $f_k$ = **2.034 N/mm$^2$**. which is slightly lower than **2.244 N/mm$^2$** obtained by equation (**11**) developed in this study.

## V. CONCLUDING REMARKS

The developed Mathematical model depends on both values of compressive strength of masonry units (bricks, blocks) and compressive strength of mortar. The equation is useful for mortars which have definite compressive strength values as well as the strength of the respective masonry units.

Based on the test results, discussions and conclusion made above, it is recommended that the developed Mathematical Model can be used to determine the characteristic compressive strength of masonry construction for utilizing in the design of masonry elements. Also the model can be useful in developing a masonry design software.

However, it is required that a further study be done to establish the compressive strengths of a varieties of cement-sand mortars which are dominant in use in Sub-Sahara Africa; such as 1: 3, 1:4, 1:6, 1: 7, 1:8, 1:10, 1:12 (cement: sand) ratios by volume. This volume batching is much more popular in most of our construction sites.

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## VI. APPENDIX

**Table A1: Strength Test Results for Bricks, Mortars and Walls**

| Type | Strength of Bricks N/mm$^2$ | Strength of Mortar N/mm$^2$ | Strength of Wall N/mm$^2$ |
|---|---|---|---|
| Type 1 | 8.839 | 8.911 | 3.036 |
| | 5.095 | 3.877 | 1.518 |
| | 3.344 | 3.027 | 1.071 |
| | 2.587 | 2.076 | 0.804 |
| Type 2 | 8.839 | 5.714 | 2.411 |
| | 5.095 | 3.027 | 1.339 |
| | 3.344 | 2.076 | 0.893 |
| | 2.587 | 1.292 | 0.625 |