ARTERIAL FLOW THROUGH AN INDENTED TUBE IN THE PRESENCE OF ERYTHROCYTES

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ABSTRACT

The aim of present investigation is to study the effect of an arterial blood flow through an indented tube in the presence of mild atherosclerosis. The flowing blood is treated to be Newtonian in character. A computational analysis of the fluid mechanics of blood flow is also performed for the assumed situation. Here we analyzed, the blood flow characteristic-the volumetric flow rate, pressure gradient of the fluid in stenotic region, wall shear stress at the surface of the stenosis, and the effect of hematocrit of red blood cell and frequency parameter. The blood is indicate to usefulness of its rheological character in the functioning of the diseased arterial circulation. Out of this, theoretical the numerical solution of wall shear stress and pressure gradient are shown graphically for better understanding of the problem.

Keywords: Atherosclerosis, Poiseuille Flow, Erythrocytes, Wall Shear Stress

I. INTRODUCTION

In the present century arterial stenosis is one of the major causes behind death in all parts of the globe. Arterial stenosis is abnormal narrowing or restriction present in the inner wall of blood vessels due to the deposition of the cholesterol, fatty materials cellular waste etc. It may happen in all large or small arteries, commonly in coronary artery, cardioid artery and peripheral artery. Arteries are narrowed by the development of atherosclerotic plaques that protrude into the lumen, resulting arterial stenosis. When an obstruction developed in an artery, one of the most serious consequences is the increased resistance and the associated reduction of the blood flow to the particular vascular bed supplied by the artery. Thus, the presence of a stenosis leads to the serious circulatory disorder. Several theoretical and experimental attempts were made to study the blood flow characteristics in the presence of stenosis. The assumption of Newtonian behavior of blood is acceptable for high shear rate flow through large arteries. Blood being a suspension of cells in plasma, exhibits non-Newtonian behavior at low rate. The study of flow characteristic in a constricted tube has received much attention as it represents a mathematical model for the blood flow in an artery with stenosis (abnormal and unnatural growth in the lumen of the artery). Several researchers, Young 1968, Lee and Fung 1970, Padmanabhan 1980 have studied the flow of blood in stenosesd artery by considering it as a Newtonian fluid. It is well known that blood at low shear rates and during its flow through narrow blood vessels, behaves like a non- Newtonian. Chaturaniet. al. 1985 gave an idea of the study of non-Newtonian aspects of blood flow through stenosed arteries and its applications in arterial diseases and observed the effect of wall shear stress on arterial wall. In the same year *How* and Black worked on pressure losses in non- Newtonian flow through rigid wall tapered tubes and found pressure drop and pressure gradient on tapered tubes. Haldar 1987 worked on oscillatory flow of blood in a stenosed artery. In this analysis analytical results obtained for the oscillatory flow of blood which behaves a Newtonian fluid. Numerical solution was presented for the instantaneous flow rate, resistive impedance, wall shear stress and phase lag. K. Venkateswarlu and J. Anand Roa 2004 worked on the unsteady blood flow through an indented tube with atherosclerosis and obtained the effect of hematocrit and frequency parameter. G.C Shit and M. Roy 2012 studied the hydro magnetic pulsating flow of blood in a constricted porous channel and obtained the graphical results for different values of the physical parameters. Blood is made up of a suspension of particles in a solution of proteins and electrolytes called plasma. Erythrocytes, leukocytes and platelets are the main constituents of blood .The erythrocytes or Red Blood Cells (RBCs) are more than thousand times more numerous than the leukocytes or White Blood Cells(WBCs) and much larger than platelets. For this reason the flow properties of blood mainly involve the RBCs. The Hematocrit (percentage of the blood volume that is made up of red blood cells) is the major determinant of blood viscosity. R.N. Pralhad and D.H. Schultz in 2004 discussed the Modeling of arterial stenosis and its applications to blood diseases, and flow is assumed to be represented by a couple stress fluid. Flow parameters such as velocity, resistance to flow, and shear stress distribution have been computed for different suspension concentrations (hematocrit), and for the blood diseases; polycythemia, plasma cell dyscrasias, and for Hb SS (sickle cell). The results have been compared with the case of normal blood and for other theoretical models. B. BasuMallik in 2013 studying blood flow through an atherosclerotic artery with slip velocity at wall. A power law fluid model of the blood has been utilized in this study to account for the presence of red cells (erythrocytes) in plasma. It is interesting to note that the rheological parameters, the radius, height and length of the stenosis influence the flow characteristics qualitatively and quantitatively. The investigation provides a scope for ascertaining the dominating role of slip velocity in different conditions of arteriosclerosis. It is to conclude that the results will help the physicians in predicting the stenotic range, the critical location and severity of the disease, so that they may take crucial decision for treatment through medicine or through surgery. In the same year S. Kumar analyzed the Hematocrit effects of the axisymmetric blood flow through an artery with stenosis. In this investigation, we observe the effects of red cell concentration (hematocrit) on blood flow characteristics in the presence of stenosis, and it is found that the flow resistance and the wall shear stress increases with hematocrits. Sanjeev and Chandrashekhar 2013 worked on the Hematocrit effects of the axisymmetric blood flow through an artery with stenosis arteries In this investigation, they observed the effects of red cell concentration (hematocrit) onblood flow characteristics in the presence of stenosis, and it is found that the flow resistance and the wall shear stress increases with hematocrits. In this paper we analyzed the effect of hematocrit, frequency parameter, height of the stenosis, parameter determining the shape of the constriction on velocity field, volumetric flow rate, and pressure gradient and wall shear stress.

II. MATHEMATICAL MODEL

We consider an axially symmetric laminar unsteady poiseuille and fully developed flow of blood in the $\bar{z}$ direction through an artery with mild stenosis in fig. 1. The geometry of stenosis is considered to be symmetrical and cosine shaped. Let the length of the tube be Z and z be the axis along which the blood flows. For mathematically convenience we have taken artery is to cylindrical form(r, θ, z). We also assumed blood flow in the tube is suspension of red cells in plasma and the fluid is incompressible. The equation of motion governing the flow field in the tube are:

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial p}{\partial r} = 0 \tag{2}$$

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{\rho}{\mu}\frac{\partial w}{\partial t} = \frac{1}{\mu}\frac{\partial p}{\partial z} \tag{3}$$

where w is the velocity components in the, z direction, ρ is the density of the fluid and ν is the coefficient of kinematics viscosity of blood. $\mu = \mu_0\{1 + \beta h(r)\}$ the viscosity of the blood given by Einstein.μ_0 is the coefficient of viscosity of plasma, β is the constant, $h(r) = h_m\{1 - (\frac{r}{R_0})^n\}$ is the hematocrit, $h_m$is the max hematocrit at the center of the tube R_0 is the radius of normal artery. The rate of growth into the lumen is expected to be a function of time t, specifically it is assumed that the rate of change of the radius R is given in the form;

$$\frac{\partial R}{\partial t} = -\alpha\left(1 + \cos\frac{\pi z}{z_0}\right) e^{-\frac{t}{T}} \tag{4}$$

For $-z_0 \leq z \leq z_0$ and $\frac{\partial R}{\partial t} = 0$. The parameter T is the time constant for the stenotic growth and α is a constant. Integrated equation (4) then we gets;

$$R = R_0 - T_\alpha\{1 - e^{-t/T}\}\{1 + \cos\frac{\pi z}{z_0}\} \tag{5}$$

Where $R = R_0$ for $t = 0$. It is noticed that as $t \to \infty$

$$R = R_0 - \frac{\delta_m}{2}(1 - e^{-t/T})(1 + \cos\frac{\pi z}{z_0}) \tag{6}$$

Where R_0 the radius of the artery without stenotic region is $\delta_h(T)$represents the maximum protuberance of the stenotic into lumen of the artery, at the time T. However, since the rate of the stenosis is very small and δ can be treated as a constant in the present analysis. Here equation (6) can be written as;

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right) \tag{7}$$

which defines the geometry of the stenosis where $\delta = \delta_m\left(1 - e^{-t/T}\right)$ is represents the maximum height of the growth.

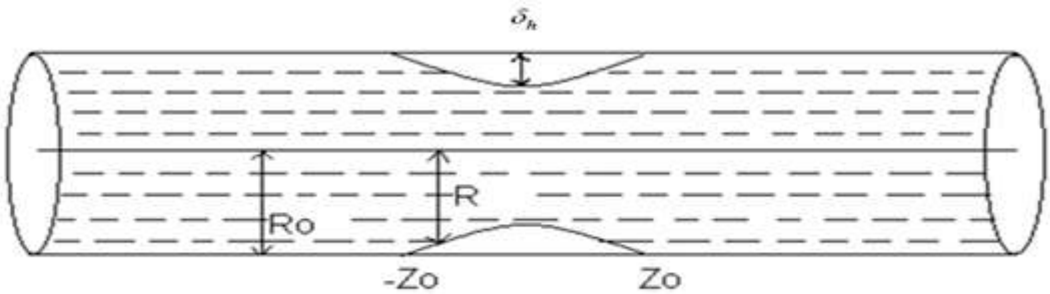

Fig: 1 Geometry of the Stenosesd Artery

The boundary conditions are provided by no-slip velocity at the wall and axially symmetry of the flow;

$$w=0 \quad at \quad r=R$$
$$\frac{\partial w}{\partial r}=0 \quad at \quad r=0 \tag{8}$$

to solve the governing equations of the problem, It is convenient to introduce a transform as $y=R/R_0$. On substituting this transform into equation (3) then we get;

$$\frac{\partial^2 w}{\partial y^2}+\frac{1}{y}\frac{\partial w}{\partial y}-\frac{\rho R_0^2}{\mu}\frac{\partial w}{\partial t}=\frac{R_0^2}{\mu}\frac{\partial p}{\partial z} \tag{9}$$

$$w=0 \quad at \quad y={R}/{R_0}$$
$$\frac{\partial w}{\partial y}=0 \quad at \quad y=0 \tag{10}$$

The pressure gradient as a periodic function of time with frequency $f={n}/{2\pi}$ to represents the arterial pulse. Using the Fourier series to represent the pressure gradient, since the change in pressure gradient is periodic, it can be expressed as;

$$\frac{\partial P}{\partial z}=Ae^{i\,fnt} \tag{11}$$

Where $A=a_n+ib_n$,

pressure gradient can be expressed as a Fourier series with Fourier coefficients;

$$\frac{\partial p}{\partial z}=a_0+\sum_{n=1}^{N}a_n\cos(fnt)+\sum_{n=1}^{N}b_n\sin(fnt) \tag{12}$$

these coefficient can be easily calculate using Mat lab and Mathematica. Since we are assuming the flow is Poiseuille so that;

$$\frac{\partial^2 w}{\partial y^2}+\frac{1}{y}\frac{\partial w}{\partial y}-\frac{1}{\upsilon}\frac{\partial w}{\partial t}=-\frac{Ae^{ifnt}R_0^2}{\mu} \tag{13}$$

If we consider a separation of variables for the unknown velocity w of the form;

$$w(r,t)=u(r)e^{i\,fnt} \tag{14}$$

we substitute the value of w in equation (13) and dividing both side $e^{i\,fnt}$ then we get the following equation;

$$\frac{\partial^2 u}{\partial y^2}+\frac{1}{y}\frac{\partial u}{\partial y}+\frac{i^3\,fn}{\upsilon}u=-\frac{AR_0^2}{\mu} \tag{15}$$

Solving equation (15) by using the boundary conditions which is in equation (10) then we get;

$$u(r)=\frac{AR_0^{\,2}}{\mu\,\alpha^2}i\left[1-\frac{J_0\left(\Lambda\frac{R}{R}\right)}{J_0(\Lambda)}\right] \tag{16}$$

Here J_0 is the Bassel Function of zero order with complex argument. Then the resulting expression for the axial velocity in the tube is given by;

$$w(r,t) = \frac{AR^2}{\mu\ \alpha^2} i \left[1 - \frac{J_0\left(\Lambda \frac{r}{R_0}\right)}{J_0\left(\Lambda \frac{R}{R_0}\right)} \right] e^{i\,fnt} \tag{17}$$

Where $\Lambda = i^{3/2}\alpha$ and $\alpha = R\sqrt{\frac{f}{\upsilon}}$

The volumetric flow rate Q is given by;

$$Q = 2\pi \int_0^R wrdr \tag{18}$$

Using equation (15) and integrating equation (18) then we get;

$$Q = \frac{\pi A R_0^{\ 4}}{i\mu\ \alpha^2} \frac{R}{R_0} \left[\frac{R}{R_0} - \frac{2J_1\left(\Lambda \frac{R}{R_0}\right)}{i^{3/2} J_0\left(\Lambda \frac{R}{R_0}\right)} \right] e^{i\,fnt} \tag{19}$$

now we using following relation;

$$J_0\left(zi^{3/2}\right) = M_0(z)e^{i\theta} \tag{20}$$

$$J_1\left(\alpha i^{3/2}\right) = M_1(z)e^{i\theta_1} \tag{21}$$

we also use the fact that the real part of $Ae^{i\,fnt}$ is $M_n \cos(f\,nt + \psi)$ where $M_n = \sqrt{a_n^2 + b_n^2} = A$ since $A = a_n + ib_n$

by using these relation we get the flow rate Q is;

$$Q = \frac{\pi M R^4 M_{10}^!}{\mu\ \alpha^2} \sin\left(fmt + \psi + \phi_{10} \right) \tag{22}$$

The pressure gradient is;

$$\frac{\partial p}{\partial z} = \frac{Q\mu\alpha^2}{\pi R_0^{\ 4}\left(\frac{R}{R_0}\right)} \left[\frac{R}{R_0} - \frac{2J_1\left(\Lambda \frac{r}{R_0}\right)}{i^{3/2} J_0\left(\Lambda \frac{R}{R_0}\right)} \right]^{-1} \tag{23}$$

The shear stress at the wall $r = R$ is defined by;

$$\tau = \mu \left(\frac{\partial u}{\partial r}\right)_{r=R} \tag{24}$$

$$\frac{\tau_R}{Q} = i\frac{\mu\beta^2}{\pi R_0^3} \frac{J_1\left(\Lambda \frac{R}{R_0}\right)}{\Lambda\left(\frac{R}{R_0}\right)^2 J_0\left(\Lambda \frac{R}{R_0}\right) - 2\left(\frac{R}{R_0}\right) J_1\left(\Lambda \frac{R}{R_0}\right)} \tag{25}$$

If τ is normalized with the steady flow solution given by;

$$\tau_N = \frac{4\mu Q}{\pi R^3} \tag{26}$$

The wall shear stress is define as;

$$\tau = \frac{\tau_R}{\tau_N} = 2\frac{R}{R_0}\frac{J_1\left(\Lambda\frac{R}{R_0}\right)\left\{\frac{R}{R_0} - \frac{2J_1\left(\Lambda\frac{R}{R_0}\right)}{J_0\left(\Lambda\frac{R}{R_0}\right)}\right\}}{\left[\Lambda\left(\frac{R}{R_0}\right)^2 J_0\left(\Lambda\frac{R}{R_0}\right) - 2\left(\frac{R}{R_0}\right) J_1\left(\Lambda\frac{R}{R_0}\right)\right]} \tag{27}$$

III. RESULT AND DISCUSSION

Fig: 2-3 represents the axial velocity field w versus y for different values of hematocrit h_m and frequency parameter α. In this figure we observed the axial velocity decreases when hematocrit and frequency parameter increases. Fig: 4 represents the volumetric flow rate Q verses y for different values of maximum hematocrit. It is observed that it is negligible initially and there after gradually increases. In stenotic region the volumetric flow rate decreases as hematocrit increases. Fig: 5 shows the variation of flow rate with frequency parameter α for different values of stenotic height. It is observed that for $\alpha = 0$ each curve attains a maximum flow rate and there is no appreciable change in it as αincreases up to a value less then unity. Thus it is observed that the values of the frequency parameter α is the steeply falling part of the curve. Again it is also seen that a particular values of the frequency parameter α, the flow rate decreases with increases stenotic height δ/R_o . Fig: 6 shows the variation of the wall shear stress with frequency parameter $\alpha$for different values of stenotic height. This graph represents the wall shear stress increases with increases stenosis heightδ/R_0. Fig:7 represents the shear stress at the surface of the stenosis. Shear stress plotted versus z for different values of maximum hematocrit at$\delta/R_0 = .01$. We observed that the shear stress is maximum at z= 0 and minimum at $z = 0.5$. Shear stress gradually decreases with in region $0 < z < 0.5$ and increases with in region$0.5 < z < 1.0$ for different values of. It is also observed that an increase in maximum h_m and maximum height of the stenosis decreases the shear stress in stenotic region. From the above discussion, it is clear that the pressure gradient increases with the increase of hematocrit value.

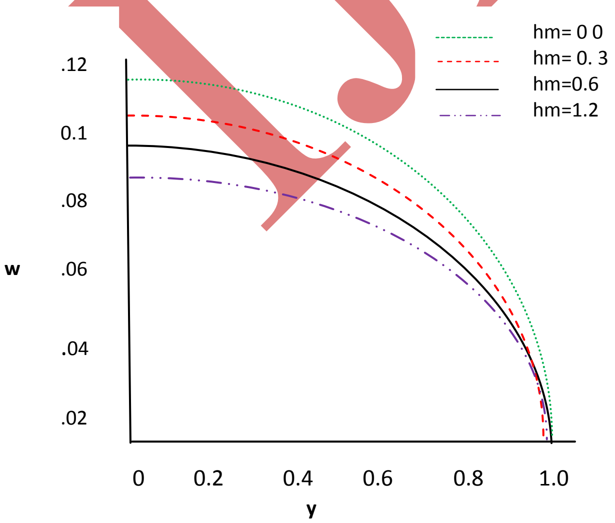

Fig: 2 Variation of axial velocity profile for different hematocrit number

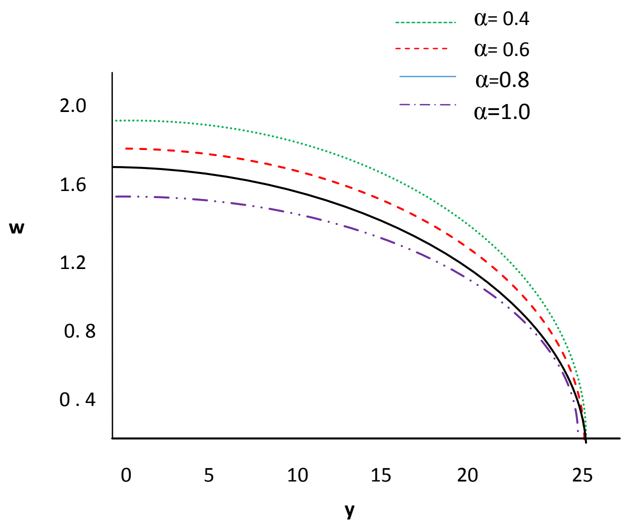

Fig: 3 Variation of axial velocity profile for different values of frequency parameter (α)

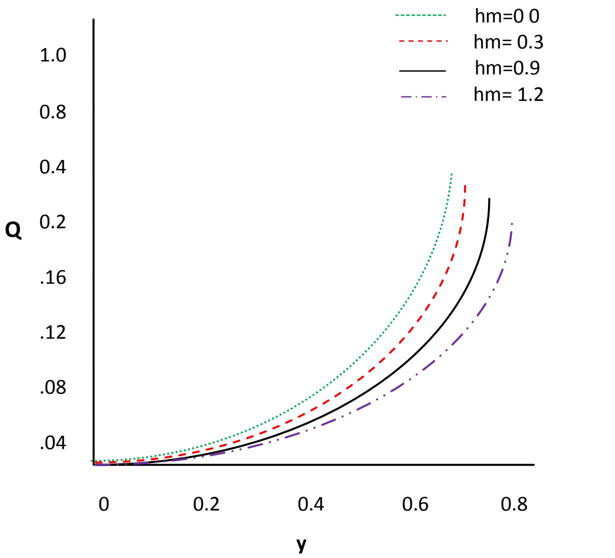

Fig: 4 Variation of volumetric flow rate in steno tic region for different values of hematocrit

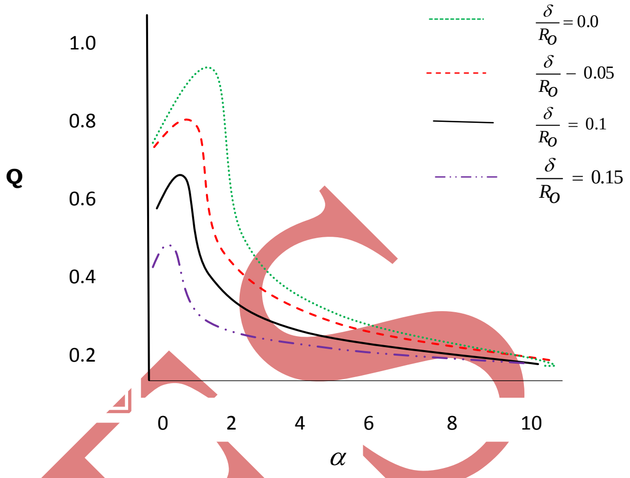

Fig: 5 Variation on instantaneous flow rate with frequency parameter for different values of stenotic height

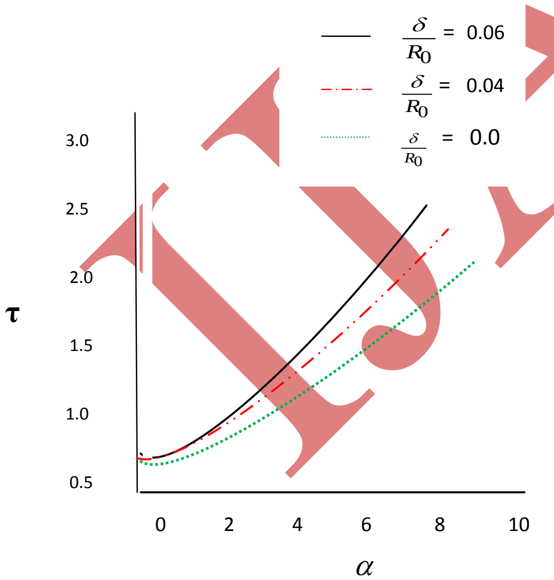

Fig: 6 Variation of wall shear stress with frequency parameter for different values of stenosis height

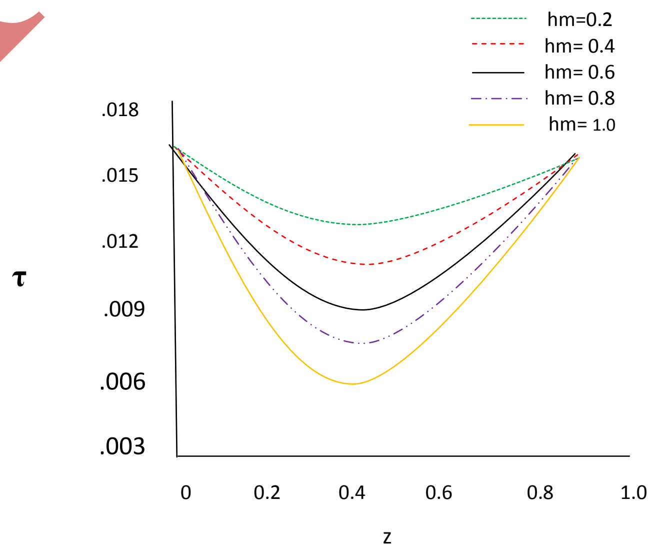

Fig: 7 Variation of axial velocity profile for different hematocrit number

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IV. CONCLUSION

From the above discussion, It is clear that narrowing of an artery is a frequent effect or a cause of vascular diseases. Such constriction disturbs normal blood flow through the vessel, and there is considerable evidence that fluid dynamic factors play a significant role in the development and progression of diseases. The results demonstrate the axial velocity and volumetric flow rate decreases with increases the hematocrit. It is also obtained if the height of the stenosis is increases the wall shear stress are also increases for the particular values of frequency parameter α. The flow decreases with increases stenosis height.

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