ERROR COVARIANCE ESTIMATION IN OBJECT TRACKING SCENARIOS USING KALMAN FILTER

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ABSTRACT

Detection and Tracking is considered as the primary step in the computer vision applications such as video surveillance, target tracking applications in defense, optimization of traffic control and in human interaction. Especially in object tracking scenarios there will be an uncertainty in finding the exact location of the target object or objects. In order to measure the uncertainty, error covariance estimation is considered. However a best estimate is made by combining the knowledge of prediction and correction mechanism which were incorporated as part of Kalman filter design. The tracking results obtained are presented for discussion.

Keywords: Computer Vision, Error Covariance, Frame Differencing, Gaussian Noise, Kalman Gain, Kalman Filter.

I INTRODUCTION

Computer vision applications are primarily concerned with scene analysis. The scene analysis is about recognizing the activities in a scene and assimilating the information from the sequences of images as a whole. There are two types of scene analysis \cite{1} one is static and the other is dynamic. Static scene analysis is about recognizing and understanding the activities of static objects from a stationary camera. However there is a little scope of its application in current day applications. Whenever we observe a scene from a video, activities in scene despite being static keep on changing or moving rapidly within a snapshot of time. The change in the scene may be due to motion of camera or object. However system should be able to detect the changes that are going on rapidly over stationary or non stationary background. The dynamic scene analysis can be done in three ways i.e. stationary camera among moving objects (SCMO), moving camera among stationary objects (MCSO) and moving camera among moving objects (MCMO). As the researcher goal is to identify the objects of interest, track the motion and compute the characteristics of the motion, SCMO gained much attention in this regard. However an active research has been going progressively in this field over a few decades and as a result, several algorithms have been evolved. Nevertheless a robust, accurate and high performance approach is still a great challenge today in this regard. Most challenges arise from the image variability of video because video objects generally are moving objects. As an object moves through the field of view of a camera, the images of the object may change dramatically. This variability comes from three principle sources: variation in target pose or target
deformations, variation in illumination, and partial or full occlusion of the target. In our paper the discussion is about determining the motion characteristics of moving object when encountered with occlusions. For critical evaluation frame differencing algorithm are chosen for object detection and kalman filter is chosen for object tracking. Kalman filter algorithm is used for tracking single object from a stationary camera under two cases. One is the target object moving with constant velocity without occlusions and the other is with occlusions. In each case tracking efficiency is determined with error covariance estimation.

II PRESENT WORK

To start with, the experiment is conducted on single object moving with a variable and constant speed without any occlusions. In this perspective a video is captured by Canon 550D camera with lens of 18.55mm and is used for motion analysis. For target identification frame differencing technique is applied. In this frame differencing technique [2] a current frame is always subtracted from its previous frame by using image subtraction operator. The resulting differenced image is transformed to binary image by applying grey thresholding. For the removal of blur [3] (considered as gaussian noise) in differenced image due to linear motion or unfocussed optics, filtering operation is applied. As it is known that Wiener filter is suitable for reconstruction of signal from the noisy image, it is chosen for image filtering operation. Finally morphological operations are applied for removal of image imperfections. For each of the moving object that is identified in the preprocessed image, the centroid is computed. This centroid represents the moving object in each of the differenced images [4]. The whole process is summarized as follows:

2.1 Algorithm

1. Read the video file
2. Starting from the 2nd frame find the difference between two successive frames using image subtraction operator.
3. Calculate the threshold (T) value for the differenced image by applying grey thresholding technique.
4. Apply the threshold to each of the differenced frame and convert to binary image.
5. For each of the differenced frame containing binary image apply, image filtering techniques for noise removal and morphological operations for image perfection
6. Store the centroid values for all the differenced frames
7. Generate the trajectory for the detected locations (centroids).

It is found that previous observations that tracking results are not precise, as it is encountered with false detections and also estimated path of the moving object is distracted. In order to make good estimate, prediction and correction mechanisms are implemented as part of Kalman filter Design. In this regard frame differencing algorithm is chosen for object detection but the motion of each track is estimated by Kalman filter. As point representations are more suitable [5] for representing objects occupying small regions in a space, an object is represented as centroid for motion analysis and is used as image measurement for tracking. In the first step [6] state of an object is predicted with a dynamic model and error covariance is estimated and is corrected with the
observation model so that error covariance is minimized. The procedure is repeated for successive iterations till the end of the frame

2.2 Mathematical Model

Dynamic model describes [7] the transformation of state vector over time.

\[ \dot{x}(t) = \frac{dx}{dt} = f(x(t), m(t)) \ldots \ldots (1) \]

Where \( \dot{x}(t) = \frac{dx}{dt} \);

\( m(t) \) represents white noise

\( x(t) \) represents the state vector of an object.

In the experiment we have chosen position, distance and velocity as the parameters of state vector.

\[ x(t) = \begin{bmatrix} s(t) \\ v(t) \end{bmatrix} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2) \]

Where \( s(t) \) represents distance

\( v(t) \) represents velocity

\[ \dot{x}(t) = A \cdot x(t) + m(t) \ldots \ldots \ldots (3) \]

A is a Transformation matrix which is treated as a constant.

The observation model represents the relationship between the state and measurements

\[ l(t_i) = H \cdot x(t_i) + w(t_i) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4) \]

Where \( H \) is the observation matrix and is constant, \( w(t_i) \) is the measurement noise with covariance matrix \( R(t_i) \).

The predicted state \( \left( x^{-}(t) \right) \) is calculated by neglecting the dynamic noise and solving differential equation

\[ \dot{x}^{-}(t) = A \cdot x^{-}(t) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5) \]

The state vector at time \( t \) can be expresses by a Taylor series with respect to the approximate state \( x^{-}(t_0) \).

\[ x^{-}(t) = x^{-}(t_0) + \dot{x}^{-}(t_0)(t-t_0) + \frac{1}{2} \ddot{x}^{-}(t_0)(t-t_0)^2 + \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6) \]

By using equation (4) this can be rewritten as

\[ x^{-}(t) = x^{-}(t_0) + A \cdot x^{-}(t_0)(t-t_0) + \frac{1}{2} A^2 x^{-}(t_0)(t-t_0)^2 + \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7) \]

In other words the actual state is expressed as linear combination of the initial state \( x^{-}(t_0) \).

\[ x^{-}(t) = \phi^T o \cdot x^{-}(t_0) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (8) \]

\( \phi^T o \) is called the state transition matrix, which transforms any initial state \( x(t_0) \) to its corresponding state \( x(t) \) at time \( t \).

By Substituting eq (7) and (8) in eq (5), we get

\[ \dot{x}^{-}(t) = A \cdot x^{-}(t) = A \cdot \phi^T o \cdot x^{-}(t_0) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (9) \]

Substitute eq(9) in eq(1)

\[ \dot{x}^{-}(t) = \frac{dx}{dt} x^{-}(t) = \frac{d}{dt} \phi^T o \cdot x^{-}(t_0) = \left[ \frac{d}{dt} \phi^T o \right] \cdot x^{-}(t_0) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (10) \]
By comparing eq (9) and (10)

\[ \frac{d}{dt} \Phi^\dagger_0 = A \Phi^\dagger_0 \]  

(11)

With the initial matrix \( \Phi^0_0 = I \), because \( x^- (t_0) = I \) \( x^- (t_0) \)

The covariance matrix \( P^- (t_i) \) of the predicted state vector is obtained by the law of error propagation and is given as

\[ P^- (t_i) = \Phi^{ti}_{t-1} P (t_{i-1}) \Phi^{ti}_{t-1}^T + Q \]  

(12)

**Q-system noise**

Error propagation is the problem of finding the distribution of a function of random variables.

As covariance matrix of the noise \( Q (t) \) is a function of time eq(12) is rewritten as

\[ P^- (t_i) = \Phi^{ti}_{t-1} P (t_{i-1}) \Phi^{ti}_{t-1}^T + \int_{t_{i-1}}^{t_i} Q (t) dt \]  

(13)

**In the correction step the predicted state vector** \( x^- (t_i) \) is improved with observations made at the time \( t_i \), the corrected state has the form

\[ x^+ (t_i) = x^- (t_i) + \Delta x (t_i) \]  

(14)

With the covariance matrix

\[ P^+ (t_i) = P^- (t_i) + \Delta P (t_i) \]  

(15)

Where \( \Delta P (t_i) = E[\Delta x (t_i) \Delta x (t_i)^T] \)

This condition is compiled with

\[ \Delta x (t_i) = P^- H^T (H P^- H^T + R(t_i))^{-1} (l(t_i) - H x^- (t_i)) \]  

R(t) – measurement noise

\[ \Delta x (t_i) = K (t_i) . (l(t_i) - l^- (t_i)) \]  

(16)

With

\[ K (t_i) = P^- H^T (H P^- H^T + R(t_i))^{-1} \]

Where K is called Kalman Gain matrix used for minimizing the variance of estimator. The difference \( (l(t_i) - l^- (t_i)) \) is called the measurement residual. It reflects the discrepancy between the predicted measurement and the actual measurement \( l(t_i). \)

Finally the corrected state is obtained by

\[ x^+ (t_i) = x^- (t_i) + K (t_i) . (l(t_i) - l^- (t_i)) \]  

(17)

\( K (t_i) \)-kalman gain

\[ x^+ (t_i) = x^- (t_i) + \Delta x (t_i) \]

Where \( \Delta x (t_i) = K (t_i) . (l(t_i) - l^- (t_i)) \)
III RESULTS AND OBSERVATIONS

![Graph]

Fig. 1(a): The above graph is plotted with correction factor $\Delta x$ to the predicted state vector $X(t)$ on Y axis and frame number on X axis

It is observed that up to frame number <20, the correction factor drops to -22.5 from zero and from greater than 20 onwards, the correction factor rises to above 5 and immediately approximate to zero point till 82nd frame. After 82nd frame correction factor drops to -20 and within an ample amount of time, it rise to -5 . It is also observed that least value of correction factor is between -20 and -25 and The maximum value of correction factor lies between 5 and 6. Hence from the above discussion it is inferred that correction factor is sustained at a value approximately to zero for more number of frames.

Here $\Delta x$ is approximating to zero up to frame number 15, and also in the range 30 to 75, indicating that the predicted state is approximately equal to corrected state. In the other frames a correction factor of -20 /+5 is to be added to the predicted state to attain the corrected state.

In the eq (17) estimated state and the measurements are weighted and are combined to calculate the corrected state. If the estimated error covariance ($\Delta P(t_i)$) is much smaller than that of the predicted state, the measurements weight will be high and predicted state will be low. Then the uncertainty will be reduced.

The covariance matrix of the corrected state $P^+(t_i)$ is given by the law of error propagation[8] by

$$P^+(t_i)= P^-(t_i)-K(t_i)H\Delta P(t_i)= P^-(t_i)+\Delta P(t_i)\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

Where $\Delta P(t_i)= - K(t_i)H P^-(t_i)$
Fig. 1(b): variation of error covariance $\Delta P$ with respect to frame number

The above graph is plotted with error covariance $\Delta P(t_i)$ on Y axis and frame number on X axis. The results show that $\Delta P$ is reduced to zero and going to negative. As the $\Delta P$ approaches to negative, covariance of predicted state will be low and so the uncertainty is reduced.

Fig. 1(c): variation of error covariance $\Delta P$ with respect to correction factor for predicted state vector $\Delta x$

It is observed that as $\Delta x$ approaches to zero, $\Delta P=1$, at $\Delta x \geq 0$, $\Delta P$ falls to zero showing that the error covariance is minimized and so uncertainty will be reduced. At $\Delta x =0$, $x^*(t_i)=x^-(t_i)+0=x^-(t_i)$, which means that the predicted state vector will be equal to corrected state vector. Thus it can be established that Kalman filter is the best estimator.
Fig. 1(d): shows the tracking results of single object tracking without occlusions.

The algorithm is tested on a single object moving with constant velocity with occlusions and tracking results are presented for discussion.

Fig. 2(a) single object tracking with occlusion

Figure 2(b) the above graph is plotted with correction factor $\Delta x$ to the predicted state vector $\hat{x}(t_i)$ on Y axis and frame number on X axis
It is observed that up to frame number <20, the $\Delta X$ will be approximated to zero. At frame number =20, correction factor will be greater than 20. At frame number >20 the $\Delta X$ lies between -5 and +5. Hence it is inferred that the least value of $\Delta X$ is -25, maximum value $\Delta X$ is 5. The correction factor $\Delta X$ will more or less to zero value in more number of frames. $-5 \leq \Delta X \leq +5$. It is also inferred that the correction factor $\Delta X$ of -5 to +5 is to be added to the predicted state vector to get the corrected state vector.

![error covariance graph](image)

**Frame number**

**Figure 2(c): variation of error covariance $\Delta P$ with respect to frame number**

The above graph is plotted with error covariance $\Delta P (t_i)$ on Y axis and frame number on X axis. The results show that $\Delta P$ is reduced to zero and going to negative. As the $\Delta P$ approaches to negative, covariance of predicted state will be low and so the uncertainty will be reduced. Thus in this it is proved in this case also that Kalman filter is best estimator.

**IV CONCLUSION**

In order to study in depth the essence of these algorithms developed based on their mathematical and environment of their applications, complex situations like partial and full occlusion are chosen and are successfully implemented. A critical evaluation has been made in measuring uncertainties in the tracking scenarios with error covariance parameter and it is proved from the experimental observations that Kalman filter is the best estimator in the above two cases.

**V FUTURE WORK**

There is a possibility of extending the work in identifying and tracking the location of stationary and moving objects in 2D/3D space based on acoustic waves and visual ability i.e. with complex interaction of light, eyes and brain.
REFERENCES


