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FRAMING CONTROLLERS APPLYING PADÉ APPROXIMATION TECHNIQUE

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ABSTRACT-*This brief presents a method to commence a reduced order system for a given SISO (stable) linear continuous time system. This approach is about to retain the stability while converting the higher order system into its lower order approximant using Padé approximant and then a controller is put together in the reduced order system attaining stability and necessary parameters of the system. A numerical example is also presented to illustrate the behavior of the original system with its reduced order approximation and then attaching a controller and check the stability of the reduced order system with controller.*

***Keywords*: *Controller Designing, Model Truncation, Padé Approximation Technique, SISO Linear Continuous Time System*.**

I INTRODUCTION

In many cases, it is quite essential to illustrate a high order system by a lower order system. System reductions of continuous and discrete systems have been broadly examined. There are several techniques which are Aggregation method [14], Moment matching technique [15], Padé approximation [16], Routh approximation [17], L^{∞} optimization technique [18]. Padé approximation provides computational modesty and fitting of time moments.

But in many cases it provides instability in the reduced order model while the early system is stable. To get a truncated order system; Shamash [19] has already provided a technique of merging the Routh approximation and time moment matching. Just have a glimpse [20] of the technique in which denominator of the system is taken by keeping the dominant poles of the system and the numerator is achieved by comparing the time moments. These techniques provide certainty in obtaining a truncated order system and these are often called Partial Padé approximation in the frequency domain. A time domain version of these Padé approximation techniques have been described by Bandyopadhyay and Lamba [21].A consolidation of frequency and time domain Padé approximation is also illustrated in [22].

Reduced order control modeling techniques, Anderson and Liu [23] are characterized in two types, direct and indirect techniques. In Direct techniques controller order is confined firstly and then find gain by extension, while indirect technique truncate the size of high order controller. Optimal projection theory, Gangsaas ***et al.***[24]and Bernstein and Hyland are the direct techniques and the parameter optimization approach.

II. PADÉ APPROXIMATION TECHNIQUE

Let us assume the transfer function illustrate a stable single input single output (SISO) system which is given below:

$$G(s) = \frac{\Omega_1 s^{n-1} + \Omega_2 s^{n-2} + \ldots \Omega_n}{S^n + \Gamma_1 S^{n-1} + \ldots \Gamma_n} \qquad (1)$$

$$= t_1 + t_2 S + \_\_\_\_ + t_n S^{n-1} + \_\_\_\_\_ \qquad (2)$$

(Expansion around s=0)

$$= M_1 S^{-1} + M_2 S^{-2} + \_\_\_ + M_n S^{-n} + \_\_\_\_ \qquad (3)$$

(Expansion around s=∞)

To drive its poise compressed order (rth. order) approximant, the transfer function will be:

$$G_{comp}(S) = \frac{\nu_1 S^{r-1} + \nu_2 S^{r-2} + \ldots + \nu_r}{S^r + X_1 S^{r-1} + \ldots + X_r} \qquad (4)$$

$$= \Delta_1 + \Delta_2 S + \ldots + \Delta_n S^{n-1} + \ldots \qquad (5)$$

$$= N_1 S^{-1} + N_2 S^{-2} + \ldots + N_n S^{-n} + \ldots \qquad (6)$$

To acquire the Pade approximant, comparing the first 2r items of equation (2) to the 2r items of equation (5) respectively. Sometimes Padé approximant gives ambiguity in the response of the system. By introducing many stable reduction methods planted on the retention of r items this issue can be overthrown; have a look, for illustration [1]-[12].Many a times, it is not plenty enough to compare r items for a satisfactory total time response approximation[5];both time moments and Markov parameters must be taken. To maintain stability, a number of methods are in existence that helps in fully retaining r-items. Here according to the previous results Vimal Singh[13] by using Routh-Padé approximants it is viewed that, the denominator of the system must be taken, So as to reduce error between the (r+1)th and the consecutive time period. To retain stabilized system; Markov parameters are introduced where the numerator can be obtained in such a way, by fully restraining the first r-time moments /markov parameters of the system

III. PRIMARY RESULTS

Considering the outcomes [13], we can have an idea that with the help of Padé approximation technique a higher order system can be reformed into lower order system retaining its stability. Here the closed loop transfer function of a system is given below:

$$G(s) = \frac{8s^2 + 6s + 2}{s^3 + 4s^2 + 5s + 2} \qquad (7)$$

Equation (8) is attained by Routh-Padé approximation techniques; planted on the contemplation of time moments and it nearly preserves r+2 time moments. So equation (8) is the reduced order approximant which is given in equation (7).

$$G_{comp(s)} = \frac{8.0s + 8.129004}{s^2 + 4.307413s + 8.129004} \qquad (8)$$

IV. CONTROLLER ARCHITECTURE

For a given control system; fig (1) Gpri(s) and H(s) are already provided. Our main goal is to acquire the transfer function of the controller Cp(s) and with the help of Cp(s) desired response of the closed loop system is obtained. For modeling of controller Cp(s) an indirect technique is used here. To model and acquire the closed loop transfer function of the controller; assumptions for model specification of the compressed order model has been taken.

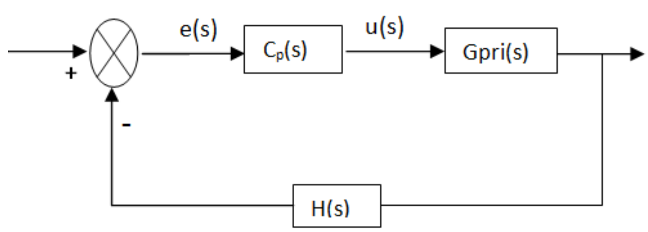

Fig. (1) Control Structure

$$G_{fpp} = \frac{Cp_{(s)}Gpri_{(s)}}{1 + Cp_{(s)}Gpri_{(s)}H_{(s)}} \qquad (9)$$

So, $$Gcomp_{(s)} = \frac{Cp_{(s)}Gpri_{(s)}}{1 + Cp_{(s)}Gpri_{(s)}H_{(s)}} \qquad (10)$$

After modification for controller, the transfer function will be:

$$Cp_{(s)} = \frac{Gcomp_{(s)}}{Gcomp_{(s)}\left[1 - Greq_{(s)}\right]} \qquad (11)$$

With the help of Padé approximation technique $G_{pri}(s)$ can be easily estimated by a reduced order transfer function $G_{comp}(s)$ shown in fig. (2)

$$Gfcc_{(s)} = \frac{Cpcomp_{(s)}Gcomp_{(s)}}{1 + Cpcomp_{(s)}Gcomp_{(s)}H_{(s)}} \quad (12)$$

$$Gfcp_{(s)} = \frac{Cpcomp_{(s)}Gpri_{(s)}}{1 + Cpcomp_{(s)}Gpri_{(s)}H_{(s)}} \quad (13)$$

To get a reduced order controller, $C_{pcomp}(s)$; the method has been explained.

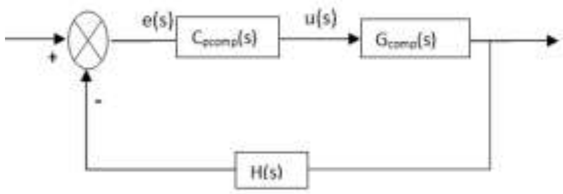

Fig.(2) Closed Loop control with $C_{pcomp}(s)$ and $G_{comp}(s)$

V. NUMERICAL EXAMPLE

The transfer function of the primary system is [13]

$$Gpri_{(s)} = \frac{8s^2 + 6s + 2}{s^3 + 4s^2 + 5s + 2} \quad (14)$$

Let us consider a reference model. In this example, a standard second-order transfer function is taken with damping ratio ϵ =0.7 and natural frequency w_n=1.5 rad/sec. Therefore

$$Gref_{(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n \varepsilon s + \omega_n^2} \quad (15)$$

$$Gref_{(s)} = \frac{2.25}{s^2 + 2.1s + 2.25} \quad (16)$$

A second order model given in the equation (17) is obtained by Padé approximation technique.

$$G_{comp(s)} = \frac{8.0s + 8.129004}{s^2 + 4.307413s + 8.129004} \quad (17)$$

Now let us calculate the transfer function of the controller with the primary system which is given in equation (19)

$$Cp_{(s)} = \frac{Gref_{(s)}}{Gpri_{(s)}\left[1 + Gref_{(s)}\right]} \qquad (18)$$

$$Cp_{(s)} = \frac{2.25s^5 + 13.725s^4 + 35.21255s^3 + 48.375s^2 + 34.7625s + 10.125}{8s^6 + 39.6s^5 + 116.48s^4 + 188.76s^3 + 188.37s^2 + 89.1s + 20.25} \qquad (19)$$

Now let us calculate the transfer function of the controller with the reduced order system which is given in equation (21)

$$Cc_{(s)} = \frac{Gref_{(s)}}{Gcomp_{(s)}\left[1 + Gref_{(s)}\right]} \qquad (20)$$

$$Cc_{(s)} = \frac{2.25s^4 + 14.416s^3 + 43.704s^2 + 60.218s + 41.153}{8s^5 + 41.729s^4 + 123.422s^3 + 204.12s^2 + 196.23s + 82.307} \qquad (21)$$

The closed loop transfer function when the controller is attached to the primary system; is given below:

$$P(s) = \frac{Cp_{(s)}Gpri_{(s)}}{\left[1 + Cp_{(s)}Gpri_{(s)}\right]} \qquad (22)$$

$$P(s) = \frac{18s^7 + 123.3s^6 + 368.544s^5 + 625.728s^4 + 638.78s^3 + 386.328s^2 + 130.276s + 20.25}{8s^9 + 71.6s^8 + 332.88s^7 + 991.98s^6 + 1973.564s^5 + 2645.068s^4 + 2334.8s^3 + 1289.568s^2 + 409.726s + 60.75} \qquad (23)$$

The closed loop transfer function when the reduced order controller is attached to the primary system; is given below:

$$Q(s) = \frac{Cc_{(s)}Gpri_{(s)}}{\left[1 + Cc_{(s)}Gpri_{(s)}\right]} \qquad (24)$$

$$Q(s) = \frac{18s^6 + 128.828s^5 + 440.628s^4 + 772.8s^3 + 774.94s^2 + 367.354s + 82.306}{8s^8 + 73.729s^7 + 348.338s^6 + 1051.281s^5 + 2153.906s^4 + 2907.471s^3 + 2496.558s^2 + 1171.349s + 246.92} \qquad (25)$$

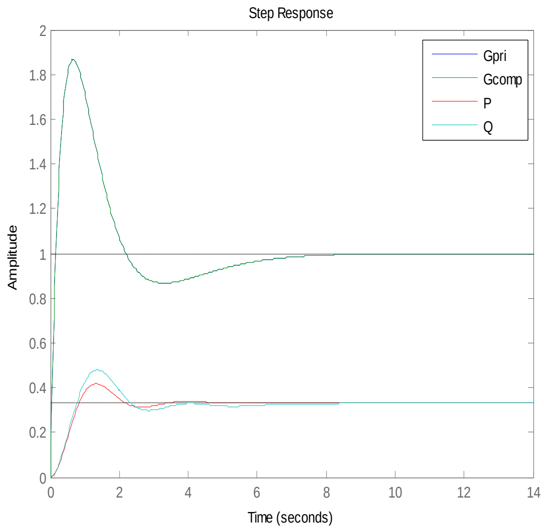

Fig.(3)-1.Step responses of Gpri(s) and Gcomp(s), 2. Step responses of P(s) and Q(s)

VI. CONCLUSION

In the present work a controller is designed using the classical approach. The system is first approximated by a low order model using Pade approximation technique and a controller is designed for this low order model. For the design of the controller, a reference model $G_{ref}(s)$ with has been chosen. Then this reduced order controller is attached to the original higher order system and it was found that the step response of the primary system with reduced order controller is a good approximant to the step response of primary system with higher order controller. The present technique has been applied to the continuous systems further it would be interesting to implement the same idea to the discrete systems as well.

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