VARIANCE OF TIME TO RECRUITMENT FOR A TWO GRADED MANPOWER SYSTEM WITH CORRELATED INTER-DECISION TIMES AND INDEPENDENT INTER-EXIT TIMES

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ABSTRACT

In this paper, the problem of time to recruitment for a two graded manpower system subject to attrition which takes place due to policy decisions is studied using a univariate policy of recruitment. Assuming that the policy decisions and exits occur at different epochs, the variance of time to recruitment is obtained using a univariate policy of recruitment when the inter-policy decision times are exchangeable and constantly correlated exponential random variables and inter- exit times form an ordinary renewal process. The analytical results are obtained by using Laplace transform technique in the analysis and the results are numerically illustrated. The effect of the nodal parameters on the performance measure is studied.

Keywords: Two Grade Manpower System; Decision and Exit Epochs; Correlated Inter-Decision Times; Univariate CUM Policy of Recruitment; Ordinary Renewal Process; Variance of Time to Recruitment AMS MSC 2010: 90B70, 91B70

I. INTRODUCTION

Exodus of personnel is a common phenomenon in any marketing organization whenever the organization announces revised policies regarding sale target, revision of wages, incentives and perquisites. This in turn produces loss in man-hours, which adversely affects the sales turnover of the organization. Frequent recruitment is not advisable as it will be expensive due to the cost of recruitment. As the loss of manhours is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. One univariate recruitment policy which is often used in the literature is based on shock model approach for replacement of system in reliability theory. In this policy, known as univariate CUM policy of recruitment, the cumulative loss of manpower is permitted till it reaches a level, called the breakdown threshold and when this cumulative loss exceeds the threshold, recruitment is carried out. In [1, 2] the authors have discussed several manpower planning models using Markovian and renewal theoretic approach. In [3] this problem is studied for the first time using this policy. In [4] the authors have considered a two grade manpower system and obtained the mean time to recruitment using CUM policy of recruitment when (i) the loss of manpower and the inter-decision times form separately a sequence of independent and identically distributed exponential random variables with different means (ii) the threshold for the cumulative loss of manpower in the organization is the maximum of the threshold for the cumulative loss of manpower in the two grades. In [5, 7] the authors have obtained the mean time to recruitment for a two grade manpower system having SCBZ thresholds. In [6] the author has obtained the mean and variance of time to recruitment for a two grade manpower system involving combined thresholds. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. This aspect is taken into account for the first time in [13] and the variance of the time to recruitment for a single grade manpower system is obtained when the inter-decision times and exit times are independent and identically distributed exponential random variables using a univariate policy for recruitment by Laplace transform technique. In [14], the authors have studied the research in [13] using a different probabilistic analysis. Again in [15, 16], the authors have obtained the mean time to recruitment for a single grade manpower system by considering different epochs for decisions and exits with correlated inter-decision times. In [17], the authors have extended the work in [14] for a two grade manpower system. The present paper extends the research work of [15] & [17] for a two grade manpower system by considering exchangeable and constantly correlated exponential inter-decision times using Laplace transforms techniques.

II. MODEL DESCRIPTION AND ANALYSIS

Consider an organization with two grades (grade-1and grade-2) taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) in the organization due to ith exit point with probability distribution $M(.)$, density function $m(.)$, laplace transform $\bar{m}(.)$ and mean $\frac{1}{\lambda}$ (λ >0). Let S_i be the cumulative loss of manpower up to i-th decision. Let U_i, the time between (i-1)th and ith decisions, be exchangeable and constantly correlated exponential random variable with probability distribution function $F(.)$, density function $f(.)$ and mean u. Let R be the correlation between U_i and U_j, $i \neq j$ and $b = u(1-R)$, $A(s) = \frac{1}{1+bs}$. Let R_i be the time between (i-1)th and ith exits with distribution function $G(.)$ and density function $g(.)$. Let $F^*(.)$ and $G^*(.)$ be the Laplace Stieltjes transform of $F(.)$ and $G(.)$ respectively. Let Y_1, Y_2 be continuous random variables representing the thresholds for the cumulative loss of manhours in grade – 1 and grade – 2 respectively. Let Y be the breakdown threshold for the cumulative loss of manhours in the organization with distribution function $H(.)$ and density function $h(.)$. Let q (q≠0) be the probability that every policy decision produces an attrition. Let T be a continuous random variable denoting the time for recruitment with mean $E(T)$ and variance $V(T)$.

The univariate CUM policy of recruitment employed in this paper is stated as follows: **Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold Y**.

The survival function of T is written as

$P(T > t) = \sum_{k=0}^{\infty}$ {Probability that exactly k-exit points in (0, t] $\times$ (probability that the total loss of manhours in these k-exit points does not cross the threshold Y)}

From renewal theory [12],

$$P(T > t) = \sum_{k=0}^{\infty} [G_k(t) - G_{k+1}(t)] P(S_k < Y) \quad \text{……….(1)}$$

By the law of total probability [11],

$$P(S_k < Y) = \int_0^{\infty} P(S_k \le y) dP(Y \le y)$$

$$i.e,\ P(S_k < Y) = \int_0^{\infty} M_k(y) h(y) dy \quad \text{……….(2)}$$

We now consider different forms for Y and obtain the variance of the time to recruitment by assuming specific distributions to Y_1 and Y_2.

Model – I: $Y = Max(Y_1, Y_2)$

Case(i): Y_1 **and** Y_2 **~exponential distribution with parameters** α_1 **and** α_2 **respectively.**

In this case

$$h(y) = \alpha_1 e^{-\alpha_1 y} + \alpha_2 e^{-\alpha_2 y} - (\alpha_1 + \alpha_2) e^{-(\alpha_1 + \alpha_2) y} \quad \text{…..……..(3)}$$

Substituting (3) in (2) and on simplification, we get

$$P(S_k < Y) = A_1^k + A_2^k - A_3^k \quad \text{………….(4)}$$

where $A_1 = \overline{m}(\alpha_1)$, $A_2 = \overline{m}(\alpha_2)$ and $A_3 = \overline{m}(\alpha_1 + \alpha_2)$ ……………(5)

From (1) & (4),

$$P(T > t) = 1 + (A_1 - 1)\sum_{k=1}^{\infty} G_k(t) A_1^{k-1} + (A_2 - 1)\sum_{k=1}^{\infty} G_k(t) A_2^{k-1} - (A_3 - 1)\sum_{k=1}^{\infty} G_k(t) A_3^{k-1} \quad \text{……….(6)}$$

Since $L(t) = 1 - P(T > t)$, from (6)

$$L(t) = (1 - A_1)\sum_{k=1}^{\infty} G_k(t) A_1^{k-1} + (1 - A_2)\sum_{k=1}^{\infty} G_k(t) A_2^{k-1} - (1 - A_3)\sum_{k=1}^{\infty} G_k(t) A_3^{k-1} \quad \text{……….(7)}$$

Taking Laplace Stieltjes Transform on both sides of (7) & using convolution theorem for Laplace transform,

$$L^*(s) = \frac{(1-A_1)G^*(s)}{\left[1-A_1G^*(s)\right]^2} + \frac{(1-A_2)G^*(s)}{\left[1-A_2G^*(s)\right]^2} - \frac{(1-A_3)G^*(s)}{\left[1-A_3G^*(s)\right]^2} \qquad \ldots\ldots\ldots(8)$$

It can be proved that, $G(x)$ satisfies the relation

$$G(x) = \sum_{n=1}^{\infty} q(1-q)^{n-1} F_n(x) \qquad \ldots\ldots\ldots(9)$$

and from [10],

$$F_k^{\,*}(s) = \frac{(A(s))^k}{1+\left(\frac{kR(1-A(s))}{(1-R)}\right)} \qquad \ldots\ldots\ldots(10)$$

$$\left.\begin{aligned} &\text{Since } E(T) = -\left[\frac{d}{ds}L^*(s)\right]_{s=0} \\ &\text{and } E(T^2) = \left[\frac{d^2}{ds^2}L^*(s)\right]_{s=0} \end{aligned}\right\} \qquad \ldots\ldots\ldots(11)$$

from (8), (9) (10) & (11), we get

$$E(T) = \frac{b}{(1-R)q}\left[\frac{1}{(1-A_1)} + \frac{1}{(1-A_2)} - \frac{1}{(1-A_3)}\right] \qquad \ldots\ldots\ldots(12)$$

$$E(T^2) = \frac{2b^2}{q^2(1-R)^2}\left[\frac{1}{(1-A_1)^2} + \frac{1}{(1-A_2)^2} - \frac{1}{(1-A_3)^2}\right] + \frac{2b^2R^2(1-q)}{q^2(1-R)^2}\left[\frac{1}{(1-A_1)} + \frac{1}{(1-A_2)} - \frac{1}{(1-A_3)}\right] \qquad \ldots\ldots\ldots(13)$$

V(T) can be calculated from

$$V(T) = E(T^2) - [E(T)]^2 \qquad \ldots\ldots\ldots(14)$$

(12) together with (13) & (14) give $V(T)$ for the present case.

Case (ii): Y_1 and Y_2 ~ extended exponential distribution [8] with shape parameter 2 and scale parameters α_1 and α_2 respectively.

In this case

$$\begin{aligned} h(y) = {} & 2\alpha_1 e^{-\alpha_1 y} - 2\alpha_1 e^{-2\alpha_1 y} + 2\alpha_2 e^{-\alpha_2 y} - 4(\alpha_1+\alpha_2)e^{-(\alpha_1+\alpha_2)y} + 2(2\alpha_1+\alpha_2)e^{-(2\alpha_1+\alpha_2)y} \\ & - 2\alpha_2 e^{-2\alpha_2 y} + 2(\alpha_1+2\alpha_2)e^{-(\alpha_1+2\alpha_2)y} - (2\alpha_1+2\alpha_2)e^{-(2\alpha_1+2\alpha_2)y} \end{aligned} \qquad \ldots(15)$$

$$P(S_k < Y) = 2A_1^k + 2A_2^k - 4A_3^k - A_4^k - A_5^k + 2A_6^k + 2A_7^k - A_8^k \qquad \ldots\ldots\ldots(16)$$

$$E(T) = \frac{b}{(1-R)q}\begin{bmatrix} \frac{2}{(1-A_1)} + \frac{2}{(1-A_2)} - \frac{4}{(1-A_3)} - \frac{1}{(1-A_4)} \\ -\frac{1}{(1-A_5)} + \frac{2}{(1-A_6)} + \frac{2}{(1-A_7)} - \frac{1}{(1-A_8)} \end{bmatrix} \quad ....(17)$$

and

$$E(T^2) = \frac{2b^2}{(1-R)^2 q^2}\begin{bmatrix} \frac{2}{(1-A_1)^2} + \frac{2}{(1-A_2)^2} - \frac{4}{(1-A_3)^2} - \frac{1}{(1-A_4)^2} \\ -\frac{1}{(1-A_5)^2} + \frac{2}{(1-A_6)^2} + \frac{2}{(1-A_7)^2} - \frac{1}{(1-A_8)^2} \end{bmatrix} + \frac{2b^2R^2(1-q)}{(1-R)^2 q^2}\begin{bmatrix} \frac{2}{(1-A_1)} + \frac{2}{(1-A_2)} - \frac{4}{(1-A_3)} - \frac{1}{(1-A_4)} \\ -\frac{1}{(1-A_5)} + \frac{2}{(1-A_6)} + \frac{2}{(1-A_7)} - \frac{1}{(1-A_8)} \end{bmatrix} \quad ....(18)$$

where A_1, A_2 & A_3 are given in (5) and

$$A_4 = \bar{m}(2\alpha_1); A_5 = \bar{m}(2\alpha_2); A_6 = \bar{m}(2\alpha_1 + \alpha_2); A_7 = \bar{m}(\alpha_1 + 2\alpha_2); A_8 = \bar{m}(2\alpha_1 + 2\alpha_2) \quad ....(19)$$

(17) together with (18) & (14) give $V(T)$ for the present case.

Case (iii): Distribution of Y_1 and Y_2 has Setting the Clock Back to Zero property [9] with parameter $(\alpha_1, \beta_1, \beta_2) and (\alpha_2, \beta_3, \beta_4)$ respectively.

In this case

$$\begin{aligned} h(y) = {} & p_1(\alpha_1 + \beta_1)e^{-(\alpha_1+\beta_1)} + p_2(\alpha_2 + \beta_3)e^{-(\alpha_2+\beta_3)} + q_1\beta_2 e^{-(\beta_2)} + q_2\beta_4 e^{-(\beta_4)} \\ & - p_1q_2(\alpha_1 + \beta_1 + \beta_4)e^{-(\alpha_1+\beta_1+\beta_4)} - q_1p_2(\alpha_2 + \beta_2 + \beta_3)e^{-(\alpha_2+\beta_2+\beta_3)} \\ & - p_1p_2(\alpha_1 + \alpha_2 + \beta_1 + \beta_3)e^{-(\alpha_1+\alpha_2+\beta_1+\beta_3)} - q_1q_2(\beta_2 + \beta_4)e^{-(\beta_2+\beta_4)} \end{aligned} \quad ....(20)$$

$$P(S_k < Y) = p_1B_1^k + p_2B_2^k + q_1B_3^k + q_2B_4^k - p_1q_2B_5^k - q_1p_2B_6^k - p_1p_2B_7^k - q_1q_2B_8^k \quad ......(21)$$

$$E(T) = \frac{b}{(1-R)q}\begin{bmatrix} \frac{p_1}{(1-B_1)} + \frac{p_2}{(1-B_2)} + \frac{q_1}{(1-B_3)} + \frac{q_2}{(1-B_4)} \\ -\frac{p_1q_2}{(1-B_5)} - \frac{q_1p_2}{(1-B_6)} - \frac{p_1p_2}{(1-B_7)} - \frac{q_1q_2}{(1-B_8)} \end{bmatrix} \quad ....(22)$$

and

$$E(T^2)=\frac{2b^2}{(1-R)^2q^2}\begin{bmatrix}\frac{p_1}{(1-B_1)^2}+\frac{p_2}{(1-B_2)^2}+\frac{q_1}{(1-B_3)^2}+\frac{q_2}{(1-B_4)^2}\\-\frac{p_1q_2}{(1-B_5)^2}-\frac{q_1p_2}{(1-B_6)^2}-\frac{p_1p_2}{(1-B_7)^2}-\frac{q_1q_2}{(1-B_8)^2}\end{bmatrix}$$
$$+\frac{2b^2R^2(1-q)}{(1-R)^2q^2}\begin{bmatrix}\frac{p_1}{(1-B_1)}+\frac{p_2}{(1-B_2)}+\frac{q_1}{(1-B_3)}+\frac{q_2}{(1-B_4)}\\-\frac{p_1q_2}{(1-B_5)}-\frac{q_1p_2}{(1-B_6)}-\frac{p_1p_2}{(1-B_7)}-\frac{q_1q_2}{(1-B_8)}\end{bmatrix} \quad \text{.........(23)}$$

where

$$B_1=\overline{m}(\alpha_1+\beta_1);B_2=\overline{m}(\alpha_2+\beta_3);B_3=\overline{m}(\beta_2);B_4=\overline{m}(\beta_4);B_5=\overline{m}(\alpha_1+\beta_1+\beta_4)$$
$$B_6=\overline{m}(\alpha_2+\beta_2+\beta_3);B_7=\overline{m}(\alpha_1+\alpha_2+\beta_1+\beta_3);B_8=\overline{m}(\beta_2+\beta_4) \quad \text{.........(24)}$$

(22) together with (23) & (14) give $V(T)$ for the present case.

Model – II: $Y = Min(Y_1, Y_2)$

Suppose Y_1 and Y_2 are as in case (i) of Model – I

In this case, $h(y)=(\alpha_1+\alpha_2)e^{-(\alpha_1+\alpha_2)y}$ ....(25)

$$P(S_k<Y)=A_3^k \quad \text{..........(26)}$$

$$E(T)=\frac{b}{(1-R)q}\left[\frac{1}{(1-A_3)}\right] \quad \text{............(27)}$$

and

$$E(T^2)=\frac{2b^2}{q^2(1-R)^2}\left[\frac{1}{(1-A_3)^2}\right]+\frac{2b^2R^2(1-q)}{q^2(1-R)^2}\left[\frac{1}{(1-A_3)}\right] \quad \text{.............(28)}$$

(27) together with (28) & (14) give V(T) for the present case.

Suppose Y_1 and Y_2 are as in case (ii) of Model – I

In this case

$$h(y)=4(\alpha_1+\alpha_2)e^{-(\alpha_1+\alpha_2)y}-2(2\alpha_1+\alpha_2)e^{-(2\alpha_1+\alpha_2)y}$$
$$-2(\alpha_1+2\alpha_2)e^{-(\alpha_1+2\alpha_2)y}+(2\alpha_1+2\alpha_2)e^{-(2\alpha_1+2\alpha_2)y} \quad \text{...........(29)}$$

$$P(S_k<Y)=4A_3^k-2A_6^k-2A_7^k+A_8^k \quad \text{..........(30)}$$

$$E(T)=\frac{b}{(1-R)q}\left[\frac{4}{(1-A_3)}-\frac{2}{(1-A_6)}-\frac{2}{(1-A_7)}+\frac{1}{(1-A_8)}\right] \quad \text{.........(31)}$$

and

$$E(T^2) = \frac{2b^2}{(1-R)^2 q^2}\left[\frac{4}{(1-A_3)^2} - \frac{2}{(1-A_6)^2} - \frac{2}{(1-A_7)^2} + \frac{1}{(1-A_8)^2}\right] + \frac{2b^2R^2(1-q)}{(1-R)^2 q^2}\left[\frac{4}{(1-A_3)} - \frac{2}{(1-A_6)} - \frac{2}{(1-A_7)} + \frac{1}{(1-A_8)}\right] \quad \ldots\ldots\ldots\ldots(32)$$

(31) together with (32) & (14) give V(T) for the present case.

Suppose Y_1 and Y_2 are as in case (iii) of Model – I

In this case

$$h(y) = p_1q_2(\alpha_1+\beta_1+\beta_4)e^{-(\alpha_1+\beta_1+\beta_4)} + q_1p_2(\alpha_2+\beta_2+\beta_3)e^{-(\alpha_2+\beta_2+\beta_3)} + p_1p_2(\alpha_1+\alpha_2+\beta_1+\beta_3)e^{-(\alpha_1+\alpha_2+\beta_1+\beta_3)} + q_1q_2(\beta_2+\beta_4)e^{-(\beta_2+\beta_4)} \quad \ldots\ldots\ldots\ldots(33)$$

$$P(S_k < Y) = p_1q_2B_5^k + q_1p_2B_6^k + p_1p_2B_7^k + q_1q_2B_8^k \quad \ldots\ldots\ldots\ldots(34)$$

$$E(T) = \frac{b}{(1-R)q}\left[\frac{p_1q_2}{(1-B_5)} + \frac{q_1p_2}{(1-B_6)} + \frac{p_1p_2}{(1-B_7)} + \frac{q_1q_2}{(1-B_8)}\right] \quad \ldots\ldots\ldots\ldots(35)$$

and

$$E(T^2) = \frac{2b^2}{(1-R)^2 q^2}\left[\frac{p_1q_2}{(1-B_5)^2} + \frac{q_1p_2}{(1-B_6)^2} + \frac{p_1p_2}{(1-B_7)^2} + \frac{q_1q_2}{(1-B_8)^2}\right] + \frac{2b^2R^2(1-q)}{(1-R)^2 q^2}\left[\frac{p_1q_2}{(1-B_5)} + \frac{q_1p_2}{(1-B_6)} + \frac{p_1p_2}{(1-B_7)} + \frac{q_1q_2}{(1-B_8)}\right] \quad \ldots\ldots\ldots(36)$$

(35) together with (36) & (14) give V(T) for the present case.

Model – III: $Y = Y_1 + Y_2$

Suppose Y_1 and Y_2 are as in case (i) of Model – I

In this case

$$h(y) = \frac{\alpha_1\alpha_2}{\alpha_1-\alpha_2}e^{-\alpha_2 y} - \frac{\alpha_1\alpha_2}{\alpha_1-\alpha_2}e^{-\alpha_1 y} \quad \ldots\ldots\ldots\ldots(37)$$

$$P(S_k < Y) = \frac{\alpha_1}{\alpha_1-\alpha_2}A_2^k - \frac{\alpha_2}{\alpha_1-\alpha_2}A_1^k \quad \ldots\ldots\ldots\ldots(38)$$

$$E(T) = \frac{b}{(1-R)q}\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)(1-A_2)} - \frac{\alpha_2}{(\alpha_1-\alpha_2)(1-A_1)}\right] \quad \ldots\ldots\ldots\ldots(39)$$

and

$$E(T^2) = \frac{2b^2}{(1-R)^2 q^2}\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)(1-A_2)^2} - \frac{\alpha_2}{(\alpha_1-\alpha_2)(1-A_1)^2}\right] + \frac{2b^2R^2(1-q)}{(1-R)^2 q^2}\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)(1-A_2)} - \frac{\alpha_2}{(\alpha_1-\alpha_2)(1-A_1)}\right] \quad \ldots\ldots\ldots(40)$$

(39) together with (40) & (14) give V(T) for the present case.

Suppose Y_1 and Y_2 are as in case(ii) of Model-I.

In this case

$$h(y) = C_1\alpha_1 e^{-\alpha_1 y} + 2C_2\alpha_1 e^{-2\alpha_1 y} + C_3\alpha_2 e^{-\alpha_2 y} + 2C_4\alpha_2 e^{-2\alpha_2 y} \quad \text{.........(41)}$$

$$\therefore P(S_k < Y) = C_1(A_1)^k + C_3(A_2)^k + C_2(A_4)^k + C_4(A_5)^k \quad \text{.........(42)}$$

$$E(T) = \frac{b}{(1-R)q}\left[\frac{C_1}{(1-A_1)} + \frac{C_3}{(1-A_2)} + \frac{C_2}{(1-A_4)} + \frac{C_4}{(1-A_5)}\right] \quad \text{.........(43)}$$

and

$$E(T^2) = \frac{2b^2}{(1-R)^2 q^2}\left[\frac{C_1}{(1-A_1)^2} + \frac{C_3}{(1-A_2)^2} + \frac{C_2}{(1-A_4)^2} + \frac{C_4}{(1-A_5)^2}\right] + \frac{2b^2R^2(1-q)}{(1-R)^2 q^2}\left[\frac{C_1}{(1-A_1)} + \frac{C_3}{(1-A_2)} + \frac{C_2}{(1-A_4)} + \frac{C_4}{(1-A_5)}\right] \quad \text{.........(44)}$$

where

$$\left.\begin{aligned} C_1 &= 2 + \frac{2\alpha_1}{\alpha_1 - 2\alpha_2} - \frac{4\alpha_1}{\alpha_1 - \alpha_2} \\ C_2 &= \frac{4\alpha_1}{2\alpha_1 - \alpha_2} - \frac{2\alpha_1}{2\alpha_1 - 2\alpha_2} - 1 \\ C_3 &= \frac{4\alpha_1}{\alpha_1 - \alpha_2} - \frac{4\alpha_1}{2\alpha_1 - \alpha_2} \\ C_4 &= \frac{2\alpha_1}{2\alpha_1 - 2\alpha_2} - \frac{2\alpha_1}{\alpha_1 - 2\alpha_2} \end{aligned}\right\} \quad \text{.........(45)}$$

(43) together with (44)&(14) give V(T) for the present case.

Suppose Y_1 and Y_2 are as in case(iii) of Model-I.

In this case

$$h(y) = D_1(\alpha_1+\beta_1)e^{-(\alpha_1+\beta_1)y} + D_2\beta_2 e^{-\beta_2 y} + D_3(\alpha_2+\beta_3)e^{-(\alpha_2+\beta_3)y} + D_4\beta_4 e^{-\beta_4 y} \quad \text{.........(46)}$$

$$\therefore P(S_k < Y) = D_1(B_1)^k + D_2(B_3)^k + D_3(B_2)^k + D_4(B_4)^k \quad \text{........(47)}$$

$$E(T) = \frac{b}{(1-R)q}\left[\frac{D_1}{(1-B_1)} + \frac{D_2}{(1-B_3)} + \frac{D_3}{(1-B_2)} + \frac{D_4}{(1-B_4)}\right] \quad \text{.........(48)}$$

$$E(T^2) = \frac{2b^2}{(1-R)^2 q^2}\left[\frac{D_1}{(1-B_1)^2} + \frac{D_2}{(1-B_3)^2} + \frac{D_3}{(1-B_2)^2} + \frac{D_4}{(1-B_4)^2}\right] + \frac{2b^2R^2(1-q)}{(1-R)^2 q^2}\left[\left[\frac{D_1}{(1-B_1)} + \frac{D_2}{(1-B_3)} + \frac{D_3}{(1-B_2)} + \frac{D_4}{(1-B_4)}\right]\right] \quad \ldots\ldots\ldots(49)$$

where

$$\left.\begin{aligned}
D_1 &= p_1 - \frac{p_1p_2(\alpha_1+\beta_1)}{(\alpha_1+\beta_1-\alpha_2-\beta_3)} - \frac{p_1q_2(\alpha_1+\beta_1)}{(\alpha_1+\beta_1-\beta_4)} \\
D_2 &= q_1 + \frac{q_1p_2\beta_2}{(\beta_2-\alpha_2-\beta_3)} - \frac{q_1q_2\beta_2}{(\beta_2-\beta_4)} \\
D_3 &= \frac{p_1p_2(\alpha_1+\beta_1)}{(\alpha_1+\beta_1-\alpha_2-\beta_3)} + \frac{q_1p_2\beta_2}{(\beta_2-\alpha_2-\beta_3)} \\
D_4 &= \frac{p_1q_2(\alpha_1+\beta_1)}{(\alpha_1+\beta_1-\beta_4)} - \frac{q_1q_2\beta_2}{(\beta_2-\beta_4)}
\end{aligned}\right\} \quad \ldots\ldots\ldots(50)$$

(48) Together with (49) & (14) give V (T) for the present case.

III. NUMERICAL ILLUSTRATION

The mean and variance of time to recruitment for all the three models are numerically illustrated by varying one parameter and all the other parameters fixed. The effect of nodal parameters on the performance measures namely mean and variance of time to recruitment is shown in the following tables. In all the computation we have taken $\alpha_1 = 0.2$, $\alpha_2 = 0.4$ & $b = 0.1$

Table- 1: Effect of λ, R and q on performance measures E(T) & V(T)

$Y = Max(Y_1, Y_2)$								
λ	q	R	case (i)		case (ii)		Case (iii)	
			E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	0.1	0.5	3.1667	12.4333	3.6500	15.0031	4.0548	17.2789
0.2	0.1	0.5	4.3333	20.9000	5.3000	26.4422	6.1095	31.5790
0.3	0.1	0.5	5.5000	31.2000	6.9500	40.1175	8.1643	48.7004
0.3	0.2	0.5	2.7500	7.6625	3.4750	9.8556	4.0821	11.9710
0.3	0.4	0.5	1.3750	1.8469	1.7375	2.3770	2.0411	2.8907
0.3	0.6	0.5	0.9167	0.7903	1.1583	1.0178	1.3607	1.2394
0.3	0.1	0.2	3.4375	10.5633	4.3438	13.6185	5.1027	16.6128
0.3	0.1	0.4	4.5833	20.4292	5.7917	26.2956	6.8036	31.9827
0.3	0.1	0.6	6.8750	52.1531	8.6875	66.9839	10.2054	81.1460

Table- 2: Effect of λ, R and q on performance measures E(T) & V(T)

$Y = Min(Y_1, Y_2)$								
λ	q	R	case (i)		case (ii)		Case (iii)	
			E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	0.1	0.5	2.3333	7.5444	2.6000	8.9144	2.8786	10.4916
0.2	0.1	0.5	2.6667	9.5111	3.2000	12.3778	3.7571	15.9563
0.3	0.1	0.5	3.0000	11.7000	3.8000	16.1900	4.6357	22.1942
0.3	0.2	0.5	1.5000	2.8500	1.9000	3.9525	2.3179	5.4327
0.3	0.4	0.5	0.7500	0.6750	0.9500	0.9406	1.1589	1.3002
0.3	0.6	0.5	0.5000	0.2833	0.6333	0.3969	0.7726	0.5521
0.3	0.1	0.2	1.8750	3.6844	2.3750	5.2020	2.8973	7.3006
0.3	0.1	0.4	2.5000	7.4500	3.1667	10.3881	3.8631	14.3696
0.3	0.1	0.6	3.7500	20.1375	4.7500	27.6481	5.7946	37.5468

Table- 3: Effect of λ, R and q on performance measures E(T) & V(T)

$Y = Y_1 + Y_2$								
λ	q	R	case (i)		case (ii)		Case (iii)	
			E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	0.1	0.5	3.5000	14.4000	6.0000	30.9556	3.4064	11.0360
0.2	0.1	0.5	5.000	25.5000	10.0000	67.2222	4.5235	16.5591
0.3	0.1	0.5	6.5000	39.1000	14.0000	114.6000	5.6406	22.5457
0.3	0.2	0.5	3.2500	9.6125	7.0000	28.3000	2.8203	5.4954
0.3	0.4	0.5	1.6250	2.3219	3.5000	6.9000	1.4101	1.3033
0.3	0.6	0.5	1.0833	0.9958	2.3333	2.9889	0.9401	0.5479
0.3	0.1	0.2	4.0625	13.3539	8.7500	40.6312	3.5254	7.1412
0.3	0.1	0.4	5.4167	25.6903	11.6667	76.4333	4.7005	14.3876
0.3	0.1	0.6	8.1250	65.1156	17.5000	187.7250	7.0507	38.7178

From the above table, the following observations are presented which agrees with reality**.**

1. When **λ** increases and keeping all the other parameter fixed, the average loss of manhours decreases and the mean time to recruitment increases for all the three cases of models I, II and III .

2. As **R** increases and keeping other parameters fixed, the mean and variance of the time to recruitment increases for all the three models.

3. As **q** decreases and keeping all the other parameter fixed, the mean and variance of the time to recruitment increases in all the models.

IV. CONCLUSION

The model discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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