

A REVIEW OF GRAVITY WAVES IN MESOSPHERIC REGION

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ABSTRACT

In this paper role of gravity waves, wave packets and relation between observed periods and unobserved wave numbers have been studied. The effect of gravity wave breaking on eddy diffusion has been discussed. Parameterization of turbulence and stress as well as identify the magnitude of known mechanism of generating gravity waves are attempted to study.

Keywords: Wave Packet, Group Velocity, Flow Acceleration,, Eddy Diffusivity, Wave Number.

I. INTRODUCTION

The gravity waves play a major role in mesospheric dynamics. Many Researchers (Balgiano, Consensus et.al) are review that the wind irregularities were due to turbulence [1,3]. The irregularities were due to internal gravity waves argued by Hines [13, 25]. The most perfunctory review of the parameterization will be given including a description of some improvement found in Holton and of some recent thoughts on turbulence due to sub-breaking waves [19-20]. In this paper role of gravity waves, wave packets and relation between observed periods and unobserved wave numbers are studied. The horizontal as well as vertical propagation of gravity wave packet in order to develop some intuition about the origin of Mesospheric gravity waves are also considered [4-8].

II. PARAMETERIZATION OF TURBULENCE AND STRESS

Lindzen introduced the simplest model capable of describing the effect of breaking gravity waves [9-12, 19]. He considered zonally travelling gravity waves which were standing waves in the meridional direction, i.e., waves in the following form

$$e^{ik(x-ct)-\delta x^2} [\text{Cos}(ly + \varphi) + \text{Sin}(ly + \varphi)] , 0 \leq \delta \leq 1 \quad (1)$$

Where

x = eastward distance

y = northward distance

t = time

k = eastward wavenumber

c = eastward phase speed

l= northward wavenumber

φ = arbitrary phase constant

In slowly varying medium, using the WKB approximation,

$$\delta T = A \left[\Gamma^{1/2} T^{-1/2} \lambda^{1/2} e^{i \int \lambda dz} e^{z/2H} e^{ik(x-ct) - \delta x^2} [\cos(\ly + \varphi) + \sin(\ly + \varphi)] \right] \quad (2)$$

A = Amplitude factor

$$\Gamma = \text{Static Stability} = \frac{dT}{dz} + \frac{g}{c_p}$$

T = Basic temperature

δT = Perturbation temperature

$$\lambda = |N/(\bar{u} - c)| \left(1 + \frac{c^2}{k^2}\right)^{1/2} \quad (3)$$

\bar{u} = Mean zonal flow

$$N^2 = \frac{g}{T} \left(\frac{dT}{dz} + \frac{g}{c_p} \right)$$

From eq (2)

$$\frac{d\delta T}{dz} = \frac{i}{2H} A \left[\Gamma^{1/2} T^{-1/2} \lambda^{3/2} e^{i \int \lambda dz} e^{z/2H} e^{ik(x-ct) - \delta x^2} [\cos(\ly + \varphi) + \sin(\ly + \varphi)] \right] \quad (4)$$

Breaking occurs when,

$$\left| \frac{d\delta T}{dz} \right| = \Gamma$$

From eq (4)

$$\frac{A}{2H} \left[\Gamma^{1/2} T^{-1/2} \lambda^{3/2} e^{z/2H} \right] = \Gamma \quad (5)$$

If the breaking height, z_{break} , is observed then from eq (5) determines A.

CASE-I

$$z > z_{break}$$

It is assumed that sufficient turbulence is generated to prevent $\frac{d\delta T}{dz}$ from growing further. According to Lindzen,

in the absence of damping, $\frac{d\delta T}{dz}$ would grow exponentially with a local exponent given by

$$\frac{1}{2H} \left[\frac{3}{2} \frac{1}{(\bar{u} - c)} \frac{d\bar{u}}{dz} \right] \quad (6)$$

Damping rise to imaginary part of c, which produces an imaginary contribution to [14-17]. The damping so that λ_i is exactly equal to the growth exponent given by eq (6), so that growth is exactly cancelled. This degree of damping, sufficiently small to permit Lindzen to relate c_i to eddy diffusivity, given by

$$kc_i = \lambda^2 D_{eddy} \quad (7)$$

and requirement that growth be cancelled then leads to

$$D_{eddy} = k \frac{|u-c|^4}{N^2 \left(1 + \frac{c^2}{k^2}\right)^{3/2}} \left| \frac{1}{2H} - \frac{3}{2} \frac{1}{(\bar{u} - c)} \frac{d\bar{u}}{dz} \right| \quad (8)$$

CASE-II

$z = z_{break}$, we have

$$\overline{\dot{w}u} = \frac{kN^2}{2\lambda^3} \quad (9)$$

Where

\dot{w} = perturbation vertical velocity

u = Perturbation zonal velocity

For plane wave of the form eq (1) in the absence of damping, the Eliassen-palm theorem requires

$$\frac{d}{dz}((\rho_0 \overline{\dot{w}u})) = 0 \quad (10)$$

Implying no acceleration of the mean flow. In the presence of damping due to wave breaking eq (10) is replaced by

$$\rho_0 \dot{w}u = \rho_0(z_{break}) \frac{kN^2}{2\lambda^3} \Big|_{z_{break}} = e^{-2} \int_{z_{break}}^z \lambda dz \quad (11)$$

From eq (6) the flow acceleration F_x is given by

$$F_x = -\frac{1}{\rho_0} \frac{d}{dz}((\rho_0 \overline{\dot{w}u})) = \overline{\dot{w}u} \left(\frac{1}{H} - \frac{3}{(\bar{u}-c)} \frac{d\bar{u}}{dz} \right) \quad (12)$$

Taking the account of filtering properties of troposphere and stratospheric winds concluded that for winter $c=0$ while for summer $c=20$ m/s. k was chosen to be the smallest value consistent with

$$k(\bar{u} - c) \geq f^1 \quad (13)$$

Where

$$F = 2\Omega \sin\varphi$$

φ = latitude

$$\Omega = 2\pi/day$$

Eq (13) being necessary for vertical propagation in the presence of rotation.

III. CALCULATION OF GRAVITY WAVE

All proposed mechanism for gravity wave generation fall into some given broad categories [2, 24]:

3.1 Mountain Waves

Mountain waves are forced by flow over topography. The forcing appears in the lower boundary condition where

$$\dot{w}(0) = U_0 \frac{\partial}{\partial x} h(x) \text{ at } z = 0 \quad (14)$$

Where

h = surface elevation

$U_0 =$ Surface mean wind

We assume surface elevation of the form

$$h(x) = h e^{(ikx - \delta x^2)} [\cos(ly + \varphi) + \sin(ly + \varphi)]$$

Eq (14) becomes

$$\dot{w}(0) = (ik - 2\delta x) h U_0 e^{(ikx - \delta x^2)} [\cos(ly + \varphi) + \sin(ly + \varphi)] \quad (15)$$

From Lindzen, we have

$$\dot{w}(z) = A \lambda^{-1/2} e^{i \int_0^z \lambda dz} e^{z/2H} e^{(ikx - \delta x^2)} [\cos(ly + \varphi) + \sin(ly + \varphi)] \quad (16)$$

Where, for $c = 0$

$$\lambda^2 = \frac{N^2}{U_0^2} \left(1 + \frac{l^2}{k^2}\right)$$

From eq (15), we have

$$A = ik U_0 h \lambda_0^{1/2}$$

We also have from Lindzen (1981)

$$\dot{u} = \frac{\lambda}{k(1 + \frac{l^2}{k^2})} \dot{w} \quad (18)$$

From eq (16) and eq (18) we get by averaging over x ,

$$\overline{\dot{w}\dot{u}} = \frac{k U_0 h^2 N_0 e^{z/H}}{2(1 + \frac{l^2}{k^2})^{1/2}} \quad (19)$$

From eq (9), we have

$$\overline{\dot{w}\dot{u}} \Big|_{z_{break}} = \frac{k}{2} \frac{U^3(z_{break})}{N(z_{break})(1 + \frac{l^2}{k^2})^{3/2}} \quad (20)$$

From eqs (19) and (20), we have

$$e^{z_{break}/H} = \frac{U^3(z_{break})}{N(z_{break}) U_0 h^2 N_0 (1 + \frac{l^2}{k^2})} \quad (21)$$

For simplicity we will take

$$N = N_0 = 2 \frac{\pi}{300s}$$

We will also take

$U_0 = 10\text{m/s}$, which probably excessive

3.2 Shear Collapse

This is probably the most efficient mechanism for generating gravity waves with phase speeds greater than zero [18]. Thus, invoking phase speeds greater than average troposphere wind speeds; we must consider what could possibly be trying to maintain an unstable shear layer at that speed.

IV. WAVE NUMBER AND OBSERVED PERIOD

To extend the periods, τ , are observed, they are related to phase speed by the simple relation

$$\tau = \frac{2\pi}{kc} \quad (22)$$

A clue to the privacy of periods or phase speed is given by eq (3) where we see tendency of all wave with the same phase speed to have the same vertical wavelength independent of k and hence period [21-23]. This is in line with currently available data.

V. WAVE PACKETS AND GROUP VELOCITY

Let us for a moment consider a gravity wave packet travelling in the x - z plane. Locally we have

$$\sigma = N \frac{k}{m} \quad (23)$$

Where

σ = Frequency observed in moving frame following mean flow = $k(c-u)$

k = wave number in x direction

m = wave number in z direction

N = Brunt-Vaisala frequency

The group velocity in x - direction is given by

$$C_{gx} = U + \frac{\partial \sigma}{\partial k} = U + \frac{N}{m} = U + \frac{\sigma}{k} = U + (c-U) = c \quad (24)$$

In the Z - direction given by

$$C_{gz} = \frac{\partial \sigma}{\partial m} = -N \frac{k}{m^2} = -\frac{\sigma}{m} = -k \frac{(c-U)}{m} = -(c-U) \frac{k}{N}$$

From eqs (23) and (24), we have

$$\frac{C_{gx}}{C_{gz}} = \frac{m}{k} \frac{c}{(c-U)} \quad (25)$$

VI. CONCLUSIONS

In this paper we have conclude that the parameterization of Lindzen wherein the effect of gravity wave breaking on the generation of eddy diffusion and on the deposition of wave momentum flux can be seen in the simplest. The origin of gravity wave observed in the mesosphere is likely to be far removed horizontally. We also investigated that gravity wave propagate through significant planetary scale stationary waves. It is very important variable in determining D_{eddy} and F_x due to breaking waves.

ACKNOWLEDGMENT

Authors are grateful to the referee for his constructive criticism and valuable suggestions for the improvement of this paper.

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