International Journal of Advanced Technology in Engineering and Science www.ijates.com
Volume No 03, Special Issue No. 01, April 2015 ISSN (online): 2348 – 7550

A REVIEW OF A CERTAIN PROBLEMS INOPTIMIZATION

Zaharaddeen G. Kutama

Postgraduate Student, Department of Mathematic / Statistics, Sharda University, Greater Noida, (India)

ABSTRACT

In mathematics and computer science, an ***optimization problem*** *is the problem of finding the best solution from all feasible solutions. Optimization problems can be divided into two categories depending on whether the variables are continuous or discrete. An optimization problem with discrete variables is known as a combinatorial optimization problemThe aims and objectives of this paper work, is to basically study certain problems in optimization of scalar value and vector valued function, general cases, some related theorems and their proofs.*

I. INTRODUCTION

The basic elements of optimization can be found in the calculus courses where maximum and minimum (extremum) problems are concerned with those values of the independent variables for which a given function attains its maximum or minimum value. A result of the celebrated mathematician Pierre de Fermat states that if a di erentiable real-valued function of one vari-able has a maximum or minimum at a point, then its derivative is zero at that point.Moreover, optimization deals with determining the best(optimal)solutions to mathematical problems representing an important occurring in various domains, such as engineering, eco-nomics, biotechnology, military science and medical science. The origin of this topic ca be traced to the following classical results of Weierstrass and Euler respectively:'every-real valued continuous function de ned on a closed and bounded interval of a real numbers attain its min-imum and maximum' on that interval, and 'the shortest path joining origin to a point in the plane is a straight line'. However, the importance of this topic has been realized only after 1950 and most of the results in this area has been discovered in the last four decades.

Moreover, the act of achieving the best possible result under given circumstances in design, construction, maintenance, engineers have to take decisions. The goal of all such decisions is either to minimize e ort or to maximize bene t. The e ort or the bene t can be usually ex-pressed as a function of certain design variables. Hence, optimization is the process of nding the conditions that give the maximum or the minimum value of a function.It is obvious that if a pointx_1 corresponds to the minimum value of a function f(x), the same point corresponds to the maximum value of the function f(x). Thus, optimization can be taken to be minimization. There is no single method available for solving all optimization problems e ciently. Hence, a number of methods have been developed for solving di erent types of problems. Optimum seeking methods are also known as mathematical programming techniques, which are a branch of operations research. Operations research is coarsely com-posed of the following areas.Mathematical programming methods. These are useful in nding the minimum of a function of several variables under a prescribed set of constraints. Stochastic process techniques.

These are used to analyze problems which are described by a set of random variables of known dis-tribution. Statistical methods. These are used in the analysis of experimental data and in the construction of empirical models. However, readers may nd detailed accounts of non-smooth optimization problems in [Cl 83, ClWo 98, HiLa 93, Mi 11aNe 92, OuKo 98, Ro 81]. Current developments concerning algorithmic optimization and related software may be seen in [CoGo 2000, GoTo 2000, Ke 95, MoWr 93, Po 97, Po 87].

II. MATERIAL AND METHODOLOGY

Let X be a normed space, K a nonempty subset of X and F : K ! R a function on K into R. The general optimization problem (P) is to nd an element u 2 K such that F (u) 2 F (V), 8v 2 K. IF such an elementu exist, we say that u minimizes F on K, and write

$$F(u) = \inf$$

such a solution is called global minimum.

And in this situation, we say that f has a minimum at u. IfKX, this problem is referred to as the constrained optimization problem, while the case K = X is called the unconstrained optimization problem.

De nition 3.1 Let A be a subset of a normed space X and f a real-valued function on A. f is said to have a local or relative minimum (maximum) at a point X_o 2 A if there is an open sphere $S_r(x_o)$ of S such that $f(x_o)$ f(x)(f(x) $f(x_o)$) holds for all x 2 $S_r(x_o)$ \ A. If f has either a relative minimum or relative maximum at X_o, then f is said to have a relative ex-tremum. The set A on which an extremum problem is de ned is often called the admissible set.

Theorem 3.1 Let f : X ! R be a Gteaux di erentiable functional at x_o 2 X and f have a local extremum at X_o, then Df (x_o)t = 0 for all t 2 X.

Proof For every t 2 X, the function f(x0+ t) (of the real variable) has a local extremum at = 0. Since it is di erentiable at 0, it follows from ordinary calculus that

$$dx0 \ f(x_0 + t)_{=0} = 0$$

This means that $Df(x_0)t = 0$ for all t 2 X, which proves the theorem.

Remark 3.1

(i) It follows immediately from Theorem 6.1 that if a functional f : X ! RisFrchet di eren-tiable at x_0 2 X and has a relative extremum at x_0, then dT (x_0) = 0.

(ii) Let f be a real-valued functional on a normed space X and x_0 a solution of (P) on a convex set K. If f is a Gteaux di erentiable at x_0, then

$$Df(x_0)(x \ x_0) > 0 \text{ or all} x \ 2 \ K.$$

Verication Since K is a convex set, x_0+ (x x_0) 2 K for all x 2 (01) and x 2 K. Hence

$$f(x_0)t(x \ x_0) = \frac{d}{d} f(x_0 {}^{+ (xx} 0)_{=0} > 0.$$

Theorem 3.2 Let K be a convex subset of a normed space X.

1. If J : K ! R is a convex function, then (P) has a solution u whenever J has a local minimum at u.

2. If J : O X ! R is a convex function de ned over an open subset of X containing K and J is Frchet di erentiable at a point u 2 K, then J has a minimum at u (u is a solution of (P) on K) if and only if

0(u)(v u)0 for every v 2 K

If K is open, then (3.1) is equivalent to

J(u) = 0 (often called Eulers equation)

Proof 1. Let v = u + w be any element of K. By the convexity of J J(u + w)(1)J(u) + J(v); 01

which can also be written as

J(u + w)J(u)(J(v) J(u)); 01

Since J has a local minimum at u, there exists $_0$ such that $_0$> 0 and 0 J(u + 0w) J(u), which implies that J(v) J(u).

2. By Remark 3.1(ii), the necessity of (3.1) equation holds even without convexity assumption on J. For the suciency part, we observe that

J(v)J(u) J(u)(v u)foreveryv 2 K

Since J is convex

J((1)u + v)(1)J(u) + J(v) for all 2 [0; 1]

or

$$J(v)\ J(u)\ \frac{J(u+\ (v\ u))\ J(u)}{}$$

or

$$J(v)\ J(u))\ \lim_{!0} \frac{J(u+\ (v\ u))\ J(u)}{} = J0(u)(v\ v)\ \ 0$$

This proves that if J0(u)(vu) 0, then J has a minimum at u.

A functional J de ned on a normed space is called coercive if $\lim_{jjxjj!1}$ J(x) = 1.

Theorem (3.3) (Existence of Solution in Rn) Let K be a non-empty, closed convex subset of R^n and J :Rn ! R a continuous function which is coercive if K is unbounded. Then there exists at least one solution of (P).

Proof Let u_k be a minimizing sequence of J; that is, a sequence satisfying conditions u_k 2 k for every integer K and $\lim_{k!1}$ J(u_k) = J(v). This sequence is necessarily bounded, since the functional J is coercive, so that it is possible to nd a subsequence u_k which converges to an element u 2 K (K being closed). Since J is continuous, J(u) : $\lim_{k0}$! 1J(u_{k0}) = $\inf_{v2k}$ J(v) which proves the existence of a solution of (P).

Theorem 3.4 (Existence of Solution in Innite-Dimensional Hilbert Space) Let K be a non-empty, convex, closed subset of a separable Hilbert space H and J : H ! R a convex, continuous functional which is coercive if K is unbounded. Then (P) has at least one solution.

Proof As in the previous theorem, K must be bounded under the hypotheses of the the-orem. Let u_k be a minimizing sequence in K. Then by Theorem u_k has a weakly convergent subsequence u_{u0} * u. By Corollary, J(u) liminf J(u_{k0}); u_{k0} * u which, in turn, shows that u is a solution of (P). It only remains to show that the weak limit u of the sequence u_{k0} belongs to the set K. For this, let P denote the projection operator associated with the closed, convex set K; by another Theorem 2 K implies hPu u; w·Pui 0 for every integer

The weak convergence of the sequence w· to the element u implies that

0 lim·1hPu u; w·Pui = jjPu ujj^2 0

Thus,Pu = u and u 2 K.

Remark 3.2 (i) Theorem 5.4 remains valid for re exiveBanach space and continuity re-placed by weaker condition, namely, weak lower semi-continuity. For proof, see Ekeland and Temam fEkTi^{0}feg99gorSiddiqifSi93g.

(ii) The set S of all solutions of (P) is closed and convex.

Verication Let u_1; u_2 be two solutions of (P); that is, u_1; u_2 2 S. u_1 +(1)u_2 2 K; 2 (0; 1) as K is convex. Since J is convex

J(u_1 + (1)u_2) J(u_1) + (1)J(u_2)

Let = $\inf_{v2K}$ J(v) = J(u_1), and = $\inf_{v2K}$ = J(u_2) then

J(u_1 + (1)u_2) + (1)lamda =

that is, = J(u_1 + (1)u_2) implying u_1 + (1)u_2 2 S. Therefore, S is convex.

Let u_n be a sequence in S such that u_n ! u. For proving closedness, we need to show thatu 2 S. Since J is continuous

J(v) = liminf$_n$! 1J(u_n) J(u)

This gives

J(u) = and so u 2 S

(iii) The solution of Theorem 5.4 is unique if J is strictly convex.

Verication Let u1; u2S and u1u$_2$. Then $^{u1+}{}_2{}^{u2}$ 2 S as S is convex. Therefore, J($^{u1+}{}_2{}^{u2}$) = . Since J is strictly convex

J($^{u1+}{}_2{}^{u2}$) <$^1_2$ J(u_1) + $^1_2$ J(u_2) = $^1_2$ + $^1_2$ =

This is a contradiction. Hence, u_1 6= u_2 is false and u_1 = u_2.

Quadraric and Convex Programming:

For K = v 2 X= $_i$(v) 6 0; 1 6 i 6 m0; $_i$(v) = 0; m0 + 1 6 i 6 m; (P) is called a nonlinear pro-gramming problem. If $_i$ and J are convex functionals, then (P) is called a convex programming problem.

n n m; J v ^{1}< Av; v > < b; v >; A a

For X = R ; K = v 2 R $^=$ i$^{(v)6d}$i$^{;16i6}$$_{j=1}$ ( ) = 2 == ($_{ij}$),

an n n positive de nite matrix, and $_i$(v) n $a_{ij}v_j$; (p) = is called a quadratic programming

n n K v R^n= j ^{n}a v d ; i m; A

problem. If J(v) = $_{i=1}$i^{v}i$^{; X=R ;}$ P = 2 = 1 ij $_j$ 6 i 1 6 6 =

a ; n n nite matrix, then P is called a linear programming problem.

ij positive de ^{P} P

Calculus of Variations and Euler-Lagrange Equation :

The classical calculus of variation is a special case of (P) where we look for the extremum of functionals of the type

^{R} b du

J(v) = $_a$ F (x; u; u0)dx(u0(x) = $_{dx}$:::::::::::::::::::::::::::::(i) u is a twice continuously di erentiable function on [a; b], F is continuous in x, u and u0, and has continuous partial derivatives with respect to u and u0.

Theorem 4.1 A necessary condition for the functional J(u) to have an extremum at u is that u must satisfy the Euler-Lagrange equation

$$^{@F}{}_{@udx}{}^{d}(_{@x}{}^{@F}{}_{0}) = 0::::::::::::::::::::::::(ii)$$

in [ab] with boundary conditions u(a) and u(b) = .

Proof Let u(a) = u(b) = 0, then

$$J(u + v) \; J(u) = {}^{R}{}_{a}{}^{b}[F(x; u + v; u0 + v0) \; F(x; u; u0)dx]::::::::::::::::::::::::::::::(iii)$$

Using the Taylor series expansion

$$F(x; u + v; u0 + v0) = F(x; u; u0) + (v \frac{@F}{@u} + v0 \frac{@F}{u0}) + {}^{2} 2! \; (v \frac{@F}{@u} + v0 \frac{@F}{u0})^2 + :::$$

it follows from (iii) that

$$J(u + v) = J(v) + dJ(u)(v) + \frac{{}^{2}}{2!} d^2J(u)(v) + ::::: \quad ::::::::::(iv)$$

where the rst and the second Frchet di erentials are given by

$$dJ(u)(v) = {}^{R}{}_{a}{}^{b}(v\frac{@F}{@u} + v0 \frac{@F}{u0})dx::::::::::::::::::::::::(v)$$

$$d^2J(u)(v) = {}^{R}{}_{a}{}^{b}(v \frac{@F}{@u} + v0 \frac{@F}{u0})^2dx::::::::::::::::::::::::::::::::::(vi)$$

The necessary condition for the functional J to have an extremum at u is that dJ(u)v = 0 for all v 2 C2[a; b] such that v(a) = v(b) = 0; that is

$$0 = dJ(u)v = {}^{R}{}_{a}{}^{b}(v \frac{@F}{@u} + v0 \frac{@F}{u0}):::::::::::::::::::::::::::(vii)$$

Integrating the second term in the integrand in (vii) by parts, we get

$$^{R}{}_{a}{}^{b}[\frac{@F}{@u} \frac{d}{dx}(\frac{@F}{@u0})]vdx + [v \frac{@F}{@u0}]_a^b = 0::::::::::::::::::::::::(viii)$$

Since v(a) = v(b) = 0, the boundary terms vanish and the necessary condition becomes

$$^{R}{}_{a}{}^{b}[\frac{@F}{@u} \frac{d}{dx}(\frac{@F}{@x_0})]vdx = 0 \text{ for all } v \; c^2[a; b]::::::::::::::::(ix)$$

for all function v 2 c^2 [a; b] vanishing at a and b. This is possible only if

$$\frac{@F}{@u} \quad \frac{d}{dx}\left(\frac{@F}{@x0}\right) = 0$$

III. CONCLUSION

In this paper report, we have seen de nitions of: optimization and related terms, general cases of optimization, some theorems, remarks and their prof was discussed.Optimization plays an important role not only in Mathematics. However in this research work, optimization gives easy way of modelling and solving real live problem thematically. Moreover, chapter one deals with general introduction of optimization, de nition of optimization and related terms related term.

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