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PETRI NET BASED ALGORITHMIC APPROACH FOR VECHICAL ROUTING PROBLEM

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ABSTRACT

In this paper, we consider the Vehical Routing Problem (VRP). Where VRP is modelled as a VRP-graph, in which each edge is treated as a parallel combination of oppositely directed edges. We model VRP-graph as a Petri Net-graph, where Petri Net- graph is an underlying graph of VRP-graph. Then solve VRP by defining suitable binary operation on elements of columns in sign incidence matrix representation of Petri Net-graph. In Petri Net-graph, we find a set of places which is both Siphon and Trap with minimum sum of capacities, whose set of input transitions equals to the set of output transitions, and both of them are equal to the set of all transitions in Petri Net. Then edges in VRP-graph corresponding to these places in Petri Net-graph will form a shortest route for the seller to return the point of origin, after traversing all the cities exactly ones. For the solution of VRP, we describe a new algorithm, based on siphon-trap and bounded-ness property of the Petri Nets. ***2000 Mathematics Subject Classification:*** *68R10, 90C35, 94C15.*

Keywords: Travelling Seller's Problem, Weighted Directed Graph, Spanning Cycle, Petri Net, Siphon and Trap.

I. INTRODUCTION

The Vehical Routing Problem (VRP) is one of the most intensely studied problems in computational mathematics [5]. Mathematical problems related to the VRP were treated in the early nineteenth century by W.R Hamilton and British mathematician T. P. Kirkman. Although there are many algorithms given for the solution of VRP [6, 7, 8, 13, 14], yet no effective solution is known for the general case for the VRP. In this paper, we address the same problem with a different approach, using Petri Net model. Here we present a new algorithm to solving a VRP using the siphon-trap and bounded-ness property of the places in the One-one Petri Net model of given VRP-graph. For the VRP we find a set of places in Petri Net, which is both Siphon and Trap [1, 3], with minimum sum of capacities, having the property that set of input transitions equals to the set of output transitions, and both of them are equal to the set of all transitions given in the Net. Then edges in VRP-graph corresponding to these places form a shortest route for the seller.

In Petri Net theory, Petri Net is a formal tool which is particularly well suited for discrete event systems. Its application has emerged from the initial seminal PhD thesis of C. A. Petri, so C.A. Petri is considered as the originator of Petri Net applications [10, 11]. The computational algorithmic aspects of graph theory are emphasized in the study of the Petri Nets. The most interesting connections between graph theory and Petri Nets have been brought out by T.Murata [9].

This paper is organized as follows: Section 2 provides the necessary preliminaries, Section 3 formulates the problem, Section 4 describes the algorithm for VRP with illustrative example and Section 5 briefly concludes this paper.

II. AN OVERVIEW OF PETRI NET APPROACH

This section of the paper provides the necessary preliminaries for the readers who are not familiar with Petri nets.

As Petri Nets are also called place-transition net (PT-Net), it is a particular kind of directed graphs together with an initial state called the *initial marking*. In general a Petri Net is an underlying graph of any directed graph, which is in essence a directed bipartite graph with two types of nodes called *places* and *transitions.* The arcs are either from places to transitions (output of places) or from transitions to places (input of places). In the Petri Net graph a place is denoted by a circle, a transition by a box or a bar and an arc by a directed line. A Petri Net is a PT-Net with tokens assigned to its places denoted by black dots, and the token distribution over its places is done initially by a marking function denoted by M_0. A *token* is interpreted as a command given to a condition (place) for the firing of an event (transition). An event can happen, when its all input conditions are fulfilled, See Fig.1.

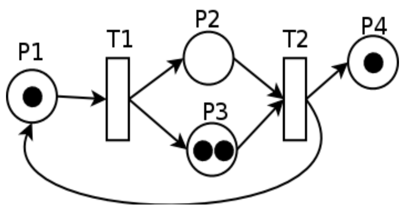

Fig.1

There are many subclasses of Petri Nets such as One-one Petri Nets, free choice Petri Nets, colored and stochastic Petri Nets etc. Here we introduce only One-one Petri Net, as it is an ordinary Petri Net such that each place P has exactly one input transition and one output transition having weight one on each edge, but we ignore these weights generally in the model representation of the Net PN. One-one Petri Net is also called as Marked Graph [4]. In our paper standard notation PN is treated as One-one Petri Net. More detailed and formal description of Petri Nets is given in [2, 9, 10, 15]. We include here some basic definitions, which are relevant to this paper.

***Definition 2.1*:** A place-transition net (PT-Net) is a quadruplet PN = $\langle P, T, F, W\rangle$, where P is the set of places, T is the set of transitions, such that $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$, $F \subseteq (P \times T) \cup (T \times P)$ is the set of arcs and W: F $\rightarrow$ {1, 2 ...} is the weight function. PN is said to be an ordinary PT-Net if and only if W: F$\rightarrow$ {1}.

A *marking* is a function M_0: P$\rightarrow$ {0, 1, 2, ...},which distributes the tokens to the places initially. Here M_0 (p) is the number of tokens in the place p at initial marking M_0, it is a non- negative integer less then or equal to the capacity of the place. Capacity of the place is defined as the capability of holding the maximum no. of tokens at any reachable marking M from M_0. A marking M is said to reachable to M_0 if there exist a firing sequence σ = $\{t_1, t_2, \ldots, t_n\}$ such that M can be obtained from M_0 as firing of transitions $t_1, t_2, \ldots, t_n$. A Petri Net structure PN =

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$\langle P, T, F, W\rangle$ without any specific initial marking is denoted by PN and Petri Net with the given initial marking is denoted by $\langle$ PN, $M_0\rangle$. *x and x* are the set of input transitions (or places) and the set of output transitions (or places) respectively, as $x\in$ P (or T). Here | *x | and | x* | stands for number of the input transitions (or places) and the output transitions (or places) respectively. Thus a One-one Petri Net is an ordinary PT-Net such that $\forall$ p $\in$ P: |*p|=|p*| =1, i.e., the number of the input transitions for p $\in$ P equals to the number of the output transitions and both of them are equal to one.

For a PT-Net, a path is a sequence of nodes $\rho = \langle x_1, x_2 \dots x_n \rangle$ where $(x_i, x_{i+1}) \in F$ for i = 1, 2… n-1. ρ is said to be elementary if and only if it does not contain the same node more than once and

a cycle is a sequence of places $\langle p_1, p_2, \dots p_n \rangle$ such that there exist $t_1, t_2, \dots, t_n \in T : \langle p_1, t_1, p_2, t_2, \dots, p_n, t_n \rangle$ forms an elementary path and $(t_n, p_1) \in F$.

Definition 2.2: For a PT-Net $\langle$PN, $M_0\rangle$, a place p is said to be *k*-bounded (or bounded by *k*) where $k\in R^+$, if and only if M_0 (p) < *k*., denotes the capacity of the place in the Net. (PN, $M_0\rangle$ is said to be k-bounded if and only if every place is *k*-bounded.

Definition 2.3: A non-empty subset of places S is called a *Siphon* if *p$\subseteq$ p* $\forall$ p$\in$S denoted also *S$\subseteq$ S*; i.e., every transition having an output place in S has an input place in S. Likewise a non-empty subset of places Q is called a *Trap* if p*$\subseteq$ *p $\forall$ p$\in$Q denoted also Q* $\subseteq$ *Q; i.e., every transition having an input place in Q has an output place in Q.

III. PROBLEM FORMULATION: VEHICAL ROUTING PROBLEM (VRP)

For a given network of cities and the cost of travel between each pair of them, the Vehical Routing Problem or VRP for short, is to find the shortest route for the seller, visiting to all of the cities exactly ones and returning to the starting point. In the standard version of VRP, the travel costs or distances are symmetric in the sense that travelling from city X to city Y costs just as much as travelling from Y to X. Any round-trip tour that goes through every city exactly once is a feasible tour with a given cost, if it is smaller than the other minimum cost tour.

3.1 Modelling VRP as a Graph (VRP-graph):

A pair G = {V, E}, where V= $\{v_1,v_2,v_3,\dots,v_n\}$ is the set of vertices and E = $\{e_1,e_2,\dots,e_m\}$ is the set of edges such that each edge e_i having some weights $w_i\in W$, where $W:E\rightarrow R^+$ is the weight function and $w_i = w(e_i)$ is the weight associated with edge e_i . Further when the edges $v_i\, v_j$ and $v_j\, v_i$ are considered different then G={V,E} is called a *weighted directed graph*. In weighted directed graph, a *directed cycle* is a closed sequence of directed edges without repetition of vertices except terminals and it said to be *Spanning* if it contains all the vertices of the graph. If any edge from the sequence is deleted then cycle becomes open. Thus solving a VRP amounts to finding a minimum weight spanning cycle.

As an illustration we consider a weighted graph on four vertices denoting the four cities A, B, C, D, where each city has a direct link with the other three. The numbers (weights) associated with the edges denotes the physical distance between the cities, See Fig. 2. We construct a directed weighted graph (VRP-graph) of the same graph in Fig. 3.

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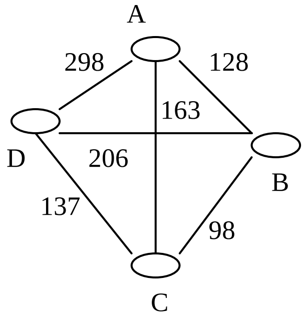

Fig. 2 (Network)

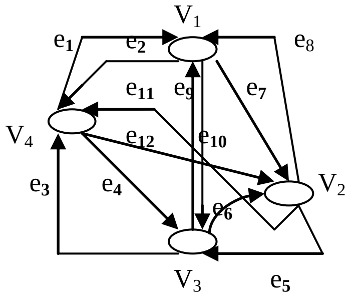

Fig. 3 (TSP- Graph)

	V_1	V_2	V_3	V_4
V_1	-----	128	163	298
V_2	128	-----	98	206
V_3	163	98	-----	137
V_4	298	206	137	-----

Distance between two cities in Fig.3 can be represented in terms of adjacency matrix as depicted below:

3.2 Modelling VRP-graph by Petri Nets:

A Petri Net PN is modelled from VRP-graph given in Fig. 3 as follows: edges e_s are transformed into places p_s and vertices v_k are transformed into transitions t_k so that the place p_s has an input from a transition t_i, and an output to a transition t_j, if e_s is an directed edge v_i v_j in graph, then weights (distance between two cities) of e_s's are replaced by capacities k_s of corresponding places p_s and number of tokens for places p_s's is the value of $\lfloor k_s \rfloor$.

Fig. 4 shows the One-one Petri Net model say PN, of the VRP-graph, having the set of transitions T= $\{t_1\ t_2,\ t_3,\ t_4\}$ and the set of places is P = $\{p_1, p_2, \ldots, p_{12}\}$ corresponding to the set of vertices$\{v_1,v_2,v_3,v_4\}$ and the set of edges $\{e_1,e_2,\ldots,e_{12}\}$ respectively in the given VRP-graph. For the sake of clarity, we observe that t_1 is the input transitions for the places $p_2,$ $p_7,$ p_{10} and the output transition for the places $p_1,$ $p_8,$ and $p_9.$ Similarly p_1 is the input place for the transition t_1 and output place for the transition $t_4.$ Using this same procedure, we find a set of the places in Net which is both siphon and trap with minimum sum of capacities, whose set of the input transitions equals to the set of output transitions and both of them are equal to the set of all transitions in the Net PN.

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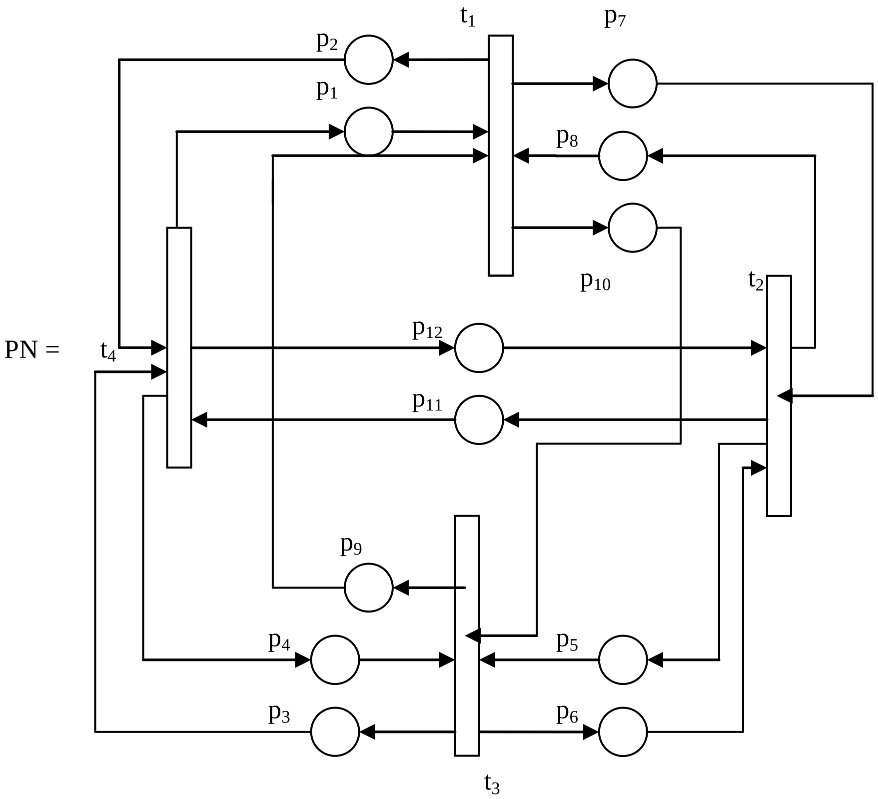

Fig. 4

IV. DESCRIPTION OF ALGORITHM FOR TRAVELLING SELLER'S PROBLEM

As in above we model the VRP-graph as a Petri Net-graph. Now here we present an algorithm for solving the Travelling Seller's Problem. For the description of the algorithm for VRP; we introduce some new notations as follows [16]:

In a Petri Net PN with n-transitions and m-places, the sign incidence matrix $I = [a_{ij}]$ is the n x m matrix, whose entries are defined as,

$a_{ij} = +$ if place j is an output place of transition i.

$a_{ij} = -$ if place j is an input place of transition i.

$a_{ij} = \pm$ if place j is both input and output places of transition i

(i.e., transition i and place j form a self loop)

$a_{ij} = 0$ otherwise.

As an illustration, the Sign incidence matrix I of PN is:

$$\begin{array}{c} \\ t1 \\ t2 \\ t3 \end{array} \begin{array}{cccccccccccc} p1 & p2 & p3 & p4 & p5 & p6 & p7 & p8 & p9 & p10 & p11 & p12 \\ \hline - & + & 0 & 0 & 0 & 0 & + & - & - & + & 0 & 0 \\ 0 & 0 & 0 & 0 & + & - & - & + & 0 & 0 & + & - \\ 0 & 0 & + & - & - & + & 0 & 0 & + & - & 0 & 0 \end{array}$$

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t4	+	-	-	+	0	0	0	0	0	0	-	+

Hare we introduce a commutative binary operation, denoted by $\oplus$ on the set U= (0, +, - ,±).

$\oplus$ defined as following:

$+ \oplus - = \pm$ '+' entry is said to be neutralized by adding a '–'entry to get a '±' entry.

$a \oplus a = a \quad \forall\, a \in U$

$\pm \oplus a = \pm \quad \forall\, a \in U$

$0 \oplus a = a \quad \forall\, a \in U$

For the VRP, we choose a subset of k places X = $\{p_1, p_2 \ldots p_k\}$ in sign incidence matrix I of PN, which is both siphon and trap and also *X= X* = T, i.e., set of the input transitions of X is equals to the set of output transitions of X, and both of them equal to the set of all transition in T. This equality holds only if the addition under the operation $\oplus$, of the k column vectors say $C_1, C_2, C_3, \ldots, C_k$ i.e., $C_1 \oplus C_2 \oplus C_3 \oplus \ldots \oplus C_k$ contains only ± entries everywhere, where C_j, j = 1, 2... k, denotes the column vector corresponding to the place p_j in I.

Now let $C_1 \oplus C_2 \oplus C_3 \oplus \ldots \oplus C_k$ will be a column vector denoted by $\boldsymbol{\gamma} = [\gamma_i]$ where γ_i denotes the i^{th} element of the column vector $\boldsymbol{\gamma}$, have as elements from set U. Then from the definition of I under the operation$\oplus$, we interprets about γ_i as.

$\gamma_i = 0$ means no place in X is an input or output place of transition $_i$.

$\gamma_i = -$ means some place in X is an input place of transition $_i$.

$\gamma_i = +$ means some place in X is an output place of transition $_i$.

$\gamma_i = \pm$ means some place in X is an input place as well as output place for transition $_i$.

From the above it can be seen that every transition having an output place in X has an input place in X only if $\gamma_i \neq +$, and likewise every transition having an input place in X has an output place in X only if $\gamma_i \neq -$. So X is both siphon and trap if and only if $\boldsymbol{\gamma}$ has either 0 or ± entries. And if $\boldsymbol{\gamma}$ has only ± entries everywhere, then the places corresponding to columns $C_1, C_2, C_3, \ldots, C_k$ in $C_1 \oplus C_2 \oplus C_3 \oplus \ldots \oplus C_k$ forms a set of places, which is both siphon and trap, whose input transitions equals to the output transitions and both of them are equal to the set of all transitions T, with having some capacities. We select only those set of places X such that *X = X* = T, having minimum sum of capacities among all. Algorithm discussed below, gives us a siphon–trap set of places X with minimum sum of capacities, having the property *X = X* = T, whose corresponding edges in VRP-graph will form a shortest route for the seller, we follow for this [16].

ALGORITHM:

Input Sign incidence matrix I of order n x m.

Step 1 Select C_j, the first column in the sign incidence matrix I having '+' entry whose corresponding place and capacity is denoted as $PLACE_j$ and $CAPACITY_j$

Set recursion level r to 1

Set $V_{jr} = C_j$

Set $PLACE_{jr} = PLACE_j$;

Set $CAPACITY_{jr} = CAPACITY_j$;

Set X =∅, W=∅, Sum =0 and

K =∅.

Step 2 If V_{jr} has a '±' entry at the i^{th} row then $PLACE_{jr}$ is a self loop with transition i, Go to Step 5.

Step 3 If V_{jr} has a '+' entry in the k^{th} row find a column C_s which contains a '–'entry at k^{th} row.

(i) If no such column C_s exists then go to Step 5.

(ii) If such C_s exists, add it to V_{jr} to obtain $V_{j(r+1)} = V_{jr} \oplus C_s$, containing a '±' entry at k^{th} row. Then

$$PLACE_{j(r+1)} = PLACE_{jr} \cup PLACE_s$$

$$CAPACITY_{j(r+1)} = CAPACITY_{jr} \cup CAPACITY_s$$

(iii) Repeat this step for all neutralizing columns C_s. This gives a new set of $V_{j(r+1)}$'s, $PLACEj_{(r+1)}$'s and $CAPACITY_{j(r+1)}$

Step 4 Increase r by 1, Repeat Step 3 until there are no '+'entries in each $V_{jr} = C_1 \oplus C_2 \oplus C_3 \oplus \ldots . \oplus C_{jr}$.

Step 5 Any V_{jr} with all entries as '±' represents both siphon and trap such that their input transitions equal to the output transitions and both of them equal to the set of all transitions T.

$$X = X \cup PLACE_{jr}$$

$$K= K \cup CAPACITY_{jr}$$

W=Sum + Sumw K, where Sumw K is the sum of the capacities in the set K

Store it any other set and compare it to minimum weight set at each iteration

Step 6 Delete C_j

j = j + 1 Go to Step 1.

Output: Set X has places both siphon and trap such that their input transitions equal to the output transitions and both of them equal to the set of all transitions T with minimum sum of capacities. Whose corresponding edges set in VRP-graph, forms a shortest route for the seller.

As an illustration consider the graph in Fig 2.

Step 1 Select first column having '+' entry. Here is C_1, then

$$V_{11} = \begin{bmatrix} - \\ 0 \\ 0 \\ + \end{bmatrix} \;;\; PLACE_{11} = \{p_l\};\; CAPACITY_{11} = \{137\}$$

Steps 2, 3 and 4, V_{11} has a '+' entry at 4^{th} row. The neutralizing columns are C_2, C_3 and C_{11}.

$$V_{12}^{(1)} = V_{11} \oplus C_2 = \begin{bmatrix} - \\ 0 \\ 0 \\ + \end{bmatrix} \oplus \begin{bmatrix} + \\ 0 \\ 0 \\ - \end{bmatrix} = \begin{bmatrix} \pm \\ 0 \\ 0 \\ \pm \end{bmatrix} ;\; PLACE_{12}^{(1)} = \{p_l, p_2\};\; CAPACITY_{12}^{(1)} = \{137,137\}$$

$$V_{12}^{(2)} = V_{11} \oplus C_3 = \begin{bmatrix} - \\ 0 \\ 0 \\ + \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ + \\ - \end{bmatrix} = \begin{bmatrix} - \\ 0 \\ + \\ \pm \end{bmatrix} ;\; PLACE_{12}^{(2)} = \{p_l, p_3\};\; CAPACITY_{12}^{(2)} = \{137,206\}$$

$$V_{12}^{(3)} = V_{11} \oplus C_{11} = \begin{bmatrix} - \\ 0 \\ 0 \\ + \end{bmatrix} \oplus \begin{bmatrix} 0 \\ + \\ 0 \\ - \end{bmatrix} = \begin{bmatrix} - \\ + \\ 0 \\ \pm \end{bmatrix}; \quad \text{PLACE}_{12}^{(3)} = \{p_l, p_{11}\}; \text{CAPACITY}_{12}^{(3)} = \{137{,}298\}$$

$V_{12}^{(2)}$ has '+' entry at 3rd row. The neutralizing columns are C_4, C_5 and C_{10}.

$$V_{13}^{(1)} = V_{12}^{(2)} \oplus C_4 = \begin{bmatrix} - \\ 0 \\ + \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ - \\ + \end{bmatrix} = \begin{bmatrix} - \\ 0 \\ \pm \\ \pm \end{bmatrix}; \quad \text{PLACE}_{13}^{(1)} = \{p_l, p_3, p_4\}; \; \text{CAPACITY}_{13}^{(1)} = \{137{,}206{,}206\}$$

$$V_{13}^{(2)} = V_{12}^{(2)} \oplus C_5 = \begin{bmatrix} - \\ 0 \\ + \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ + \\ - \\ 0 \end{bmatrix} = \begin{bmatrix} - \\ + \\ \pm \\ \pm \end{bmatrix}; \quad \text{PLACE}_{13}^{(2)} = \{p_l, p_3, p_5\}; \text{CAPACITY}_{13}^{(2)} = \{137, 206{,}128\}$$

$$V_{13}^{(3)} = V_{12}^{(2)} \oplus C_{10} = \begin{bmatrix} - \\ 0 \\ + \\ \pm \end{bmatrix} \oplus \begin{bmatrix} + \\ 0 \\ - \\ 0 \end{bmatrix} = \begin{bmatrix} \pm \\ 0 \\ \pm \\ \pm \end{bmatrix}; \quad \text{PLACE}_{13}^{(3)} = \{p_l, p_3, p_{10}\}; \text{CAPACITY}_{13}^{(3)} = \{137, 206{,}98\}$$

$V_{12}^{(3)}$ has '+' entry at 2nd row. The neutralizing column is C_6, C_7 and C_{12}.

$$V_{13}^{(4)} = V_{12}^{(3)} \oplus C_6 = \begin{bmatrix} - \\ + \\ 0 \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ - \\ + \\ 0 \end{bmatrix} = \begin{bmatrix} - \\ \pm \\ + \\ \pm \end{bmatrix}; \quad \text{PLACE}_{13}^{(4)} = \{p_l, p_{11}, p_6\}; \text{CAPACITY}_{13}^{(4)} = \{137{,}298{,}128\}$$

$$V_{13}^{(5)} = V_{12}^{(3)} \oplus C_7 = \begin{bmatrix} - \\ + \\ 0 \\ \pm \end{bmatrix} \oplus \begin{bmatrix} + \\ - \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pm \\ \pm \\ 0 \\ \pm \end{bmatrix}; \quad \text{PLACE}_{13}^{(5)} = \{p_l, p_{11}, p_7\}; \text{CAPACITY}_{13}^{(5)} = \{137{,}298{,}163\}$$

$$V_{13}^{(6)} = V_{12}^{(3)} \oplus C_{12} = \begin{bmatrix} - \\ + \\ 0 \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ - \\ 0 \\ + \end{bmatrix} = \begin{bmatrix} - \\ \pm \\ 0 \\ \pm \end{bmatrix}; \quad \text{PLACE}_{13}^{(6)} = \{p_l, p_{11}, p_{12}\}; \text{CAPACITY}_{13}^{(6)} = \{137{,}298{,}298\}$$

$V_{13}^{(1)}$ and $V_{13}^{(6)}$ has no '+' entry , also all the entries are not '±' only, but those sets which have all entries '±' or '0' are both siphon and trap (as $V_{12}^{(1)}$, $V_{13}^{(3)}$ and $V_{13}^{(5)}$)

$V_{13}^{(2)}$ has '+' entry at 2rd row. The neutralizing columns are C_6, C_7, and C_{12}.

$$V_{14}^{(1)} = V_{13}^{(2)} \oplus C_6 = \begin{bmatrix} - \\ + \\ \pm \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ - \\ + \\ 0 \end{bmatrix} = \begin{bmatrix} - \\ \pm \\ \pm \\ \pm \end{bmatrix}; \ \text{PLACE}_{14}^{(1)} = \{p_1, p_3, p_5, p_6\}; \text{CAPACITY}_{14}^{(1)} = \{137,206,128,128\}$$

$$V_{14}^{(2)} = V_{13}^{(2)} \oplus C_7 = \begin{bmatrix} - \\ + \\ \pm \\ \pm \end{bmatrix} \oplus \begin{bmatrix} + \\ - \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pm \\ \pm \\ \pm \\ \pm \end{bmatrix}; \ \text{PLACE}_{14}^{(2)} = \{p_1, p_3, p_5, p_7\}; \text{CAPACITY}_{14}^{(2)} = \{137,206,128,163\}$$

$$V_{14}^{(3)} = V_{13}^{(2)} \oplus C_{12} = \begin{bmatrix} - \\ + \\ \pm \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ - \\ 0 \\ + \end{bmatrix} = \begin{bmatrix} - \\ \pm \\ \pm \\ \pm \end{bmatrix}; \ \text{PLACE}_{14}^{(3)} = \{p_1, p_3, p_5, p_{12}\}; \text{CAPACITY}_{13}^{(3)} = \{137,206,128,298\}$$

$V_{13}^{(4)}$ has '+' entry at 3rd row. The neutralizing columns are C_4, C_5, and C_{10}.

$$V_{14}^{(4)} = V_{13}^{(4)} \oplus C_4 = \begin{bmatrix} - \\ \pm \\ + \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ - \\ + \end{bmatrix} = \begin{bmatrix} - \\ \pm \\ \pm \\ \pm \end{bmatrix}; \ \text{PLACE}_{14}^{(4)} = \{p_1, p_6, p_{11}, p_4\}; \text{CAPACITY}_{14}^{(4)} = \{137,298,128,206\}$$

$$V_{14}^{(5)} = V_{13}^{(4)} \oplus C_5 = \begin{bmatrix} - \\ \pm \\ + \\ \pm \end{bmatrix} \oplus \begin{bmatrix} 0 \\ + \\ - \\ 0 \end{bmatrix} = \begin{bmatrix} - \\ \pm \\ \pm \\ \pm \end{bmatrix}; \ \text{PLACE}_{14}^{(5)} = \{p_1, p_6, p_{11}, p_5\}; \text{CAPACITY}_{14}^{(5)} = \{137,298,128,128\}$$

$$V_{14}^{(6)} = V_{13}^{(4)} \oplus C_{10} = \begin{bmatrix} - \\ \pm \\ + \\ \pm \end{bmatrix} \oplus \begin{bmatrix} + \\ 0 \\ - \\ 0 \end{bmatrix} = \begin{bmatrix} \pm \\ \pm \\ \pm \\ \pm \end{bmatrix}; \ \text{PLACE}_{14}^{(6)} = \{p_1, p_6, p_{11}, p_{10}\}; \text{CAPACITY}_{14}^{(6)} = \{137, 298, 128, 98\}$$

$V_{14}^{(1)}, V_{14}^{(3)}, V_{14}^{(4)}$ and $V_{14}^{(5)}$ has no + entry. $V_{14}^{(2)}, V_{14}^{(6)}$ have all the entries as '±', Hence $\text{PLACE}_{14}^{(2)}$ and $\text{PLACE}_{14}^{(6)}$ form both siphon and trap whose input transitions equal to the output transitions and both of equal to the set of all transitions.

Step 5 the subsets of places, which are both siphon and trap, whose input transitions equal the output transitions and both of them equal to the set of all transitions are $\{p_1, p_3, p_5, p_7\}$ and $\{p_1, p_6, p_{11}, p_{10}\}$ with capacities sum 634 and 661 as choosing column first C_1.

Step 6 now delete C_1 from sign incidence matrix, choose next column and repeat all the steps again in similar way, we get another different sets of places $\{p_2, p_4, p_6, p_8\}$, $\{p_2, p_9, p_5, p_{12}\}$, $\{p_3, p_8, p_{10}, p_{12}\}$ and $\{p_4, p_{11}, p_7, p_9\}$ as choosing columns C_2, C_3 and C_4 respectively having sum of the capacities 634, 661, 765 and 765. The set of places which have minimum sum of capacities 634 is either $\{p_1, p_3, p_5, p_7\}$ or $\{p_2, p_4, p_6, p_8\}$. The edges set $\{e_1, e_3, e_5, e_7\}$ or $\{e_2, e_4, e_6, e_8\}$ are corresponding to places set $\{p_1, p_3, p_5, p_7\}$ and $\{p_2, p_4, p_6, p_8\}$ respectively, will form a shortest spanning cycle in underlying graph of Petri Net-graph, which give an optimal route for the seller in VRP-graph.

The finding of this can be applied to a company's logistic problem of delivering petrol to different petrol sub-stations. This delivery system is formulated as the Petri Net model, which involves finding an optimal route for visiting stations and returning to point of origin, where the inter-station distance is symmetric and known. As a standard problem, we defined it simply as the time spent or distance traveled by seller visiting n cities (or nodes) cyclically, where vehicle visits each station just once and returns the starting station. This real world application is a deceptive simple combinatorial problem and our approach is to develop solutions of such type of distribution problems, based on concept of the Petri nets.

V. CONCLUSION

In this paper, while solving the travelling seller problem, we have exploited the potentials of siphons and traps. Our analysis is based on the notion of sign incidence matrix; this helps us to relate Petri Net theory to graph theory. The complexity of the VRP is part of a deep question in mathematics, as VRP is a NP- complete problem. Here we have developed an algorithm using Petri net model, which can be executed by computer for any finite number of nodes.

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