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ANALYSIS OF FUNCTIONALLY GRADED CYLINDER SUBJECTED TO VARYING THERMAL LOAD IN AXIAL DIRECTION

Yogesh Kumar Bhardwaj[1], Vikas Bansal[2], Bhanu Prakash Mahur[3]

Department of Mechanical Engineering, Rajasthan Technical University, Kota, (India)

ABSTRACT

In this paper functionally graded cylinder of finite length is analyzed which is subjected to the varying thermal load in axial direction with a sinusoidal function. In this work study made on the effect of temperature changes in axial direction and see how it respond. Using the software COMSOL 4.2 Multiphysics we have found the temperature and various stress distribution. The result of FEM simulation in COMSOL 4.2 Multiphysics are compared with analytical result in which gradation index is taken as 5 by power law and we use the sigmoid law. The result shows a good agreement in between analytical and numerical results. The results also conclude the advantage sigmoid gradation of material over power law gradation.

Keywords* : *Functionally Graded Cylinder, Sigmoid Gradation Law, FEM Simulation, Thermal Stress

I INTRODUCTION

Functionally graded materials (FGM) are nonhomogeneous materials designed to work in a high temperature environment. Many research works have been carried out for thermal problems of functionally graded structures [1–4]. Obata and Noda studied the one dimensional steady thermal stresses in a functionally graded circular hollow cylinder and hollow sphere by use analytical method [5]. By introducing the theory of classical laminate theory, Ootao and Tanigawa treated the three dimensional transient thermal stresses of functionally graded rectangular plates [6,7]. Kim and Noda researched the two-dimensional unsteady thermoelastic problems of functionally graded infinite hollow cylinders by using a Green's function approach[8,9]. Jabbari derived analytical solutions for one-dimensional steady-state thermoelastic problems of functionally graded circular hollow cylinders in the case of material models expressed as power functions of r [10].

Erashlan obtained analytical solutions for thermally induced axisymmetric deformations in nonuniform heat-generating composite tubes [12]. Liew obtained analytical solutions of a functionally graded circular hollow cylinder by employing the solutions of homogenerous circular hollow cylinders [13]. Shao and Wang obtained analytical solutions of mechanical stresses of a functionally graded hollow cylinder with finite length [14]. Shoa obtained the thermal and mechanical stresses in finite length cylinder usig power law for gradation [15].

In this paper, we consider a steady-state thermal stress problem of a functionally graded circular hollow cylinder with finite length. The thermal load applied on the cylinder is varying in the axial direction. In order to obtain solutions of temperature stresses for the two-dimensional thermal the model consider as 2D axisymmetric. The gradation of material is done with sigmoid law of gradation in radial direction. For these purposes, first analytical solution of thick-walled FGM pressure vessels is considered. Then, numerical study is conducted using the COMSOL Multiphysics 4.2 software. The volume fraction is calculated for n=5 by sigmoid gradation law.

II BASIC EQUATION

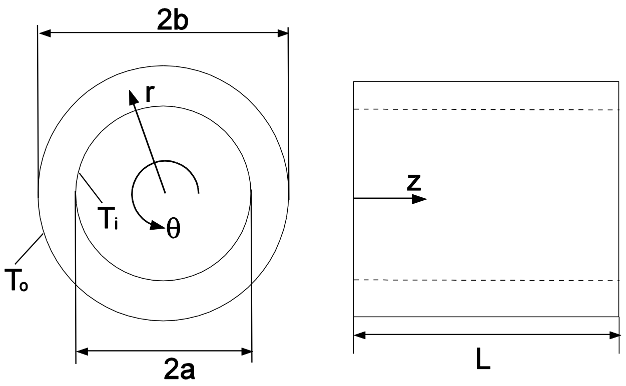


Figure 1 : Hollow Cylinder Subjected to Thermal Load

In this diagram T_i is the temperature at the inner wall above the initial temperature. T_o is the temperature at the outer wall above the initial temperature. To obtain the temperature distribution we consider a steady state heat conduction through the cylinder under the given thermal boundary condition.

$$\left[\frac{\partial^2}{\partial r^2}+\left(\frac{1}{r}\right)\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right]T=0 \tag{1}$$

In thermal loading, the elongations are not only in the radial and circumferential directions. Temperature loading makes a volume increase in all directions, the deflection the axis of the cylinder i.e z-direction should also need to be taken into consideration. This deflection is represented as w and its strain component shall be ε_z.

$$\varepsilon_z=\frac{\partial w}{\partial z} \tag{2}$$

For obtaining the solution, the total strain Δ, which is the sum of the strains should be calculated. After getting Δ, the radial and hoop stress equations for a thick cylinder with axisymmetric thermal loading are as follows:

$$\Delta=\varepsilon_r+\varepsilon_\theta+\varepsilon_z \tag{3}$$

$$\sigma_r = \frac{E(r)}{(1+\upsilon(r))}\left[\varepsilon_r + \frac{\upsilon(r)}{1-2\upsilon(r)}\ \Delta\right] - \frac{\alpha(r).E(r)}{1-2\upsilon(r)}\partial T \qquad (4)$$

$$\sigma_\theta = \frac{E(r)}{(1+\upsilon(r))}\left[\varepsilon_\theta + \frac{\upsilon(r)}{1-2\upsilon(r)}\ \Delta\right] - \frac{\alpha(r).E(r)}{1-2\upsilon(r)}\partial T \qquad (5)$$

$$T(r=b) = T_o \qquad (6)$$

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (7)$$

$$\sigma_r(r=a) = 0 \qquad (8)$$

$$\sigma_r(r=b) = 0 \qquad (9)$$

$$T(r=a) = T_i \qquad (10)$$

for estimation of volume fraction for property estimation the following equation of sigmod gradation law is used:-

$$V_m(r) = 1 - \frac{1}{2}\left(\frac{r-a}{r_m - a}\right)^n \qquad \text{for } a < r < r_m \qquad (11)$$

$$V_m(r) = \frac{1}{2}\left(\frac{b-r}{b-r_m}\right)^n \qquad \text{for } r_m < r < b \qquad (12)$$

Here in equtions, ‘a’ is internal radius, ‘b’ is outer radius, ‘r’ is radius at any point, ‘r_m’ is the mean radius. The distribution of volume fraction by sigmoid law for a cylinder shown in the figure 2

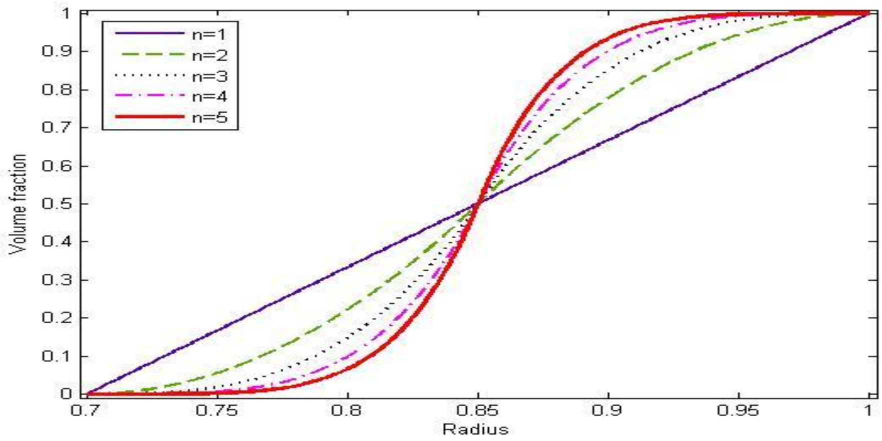


Figure 2: Volume fraction distribution along the radius for different gradation index.

As we seen in literature review many theories used to estimate the property of functionally graded material. After the calculation of volume fraction, estimation of a properties like young modulus, thermal expansion coefficient, thermal conductivity and poisson ratio by applying Rule of mixture.

$$P(r) = P_m V_m + P_c (1 - V_m) \qquad (13)$$

III RESULT AND DISCUSSION

We consider a thick functionally graded circular hollow cylinder made of mullite and molybdenum, as shown in Figure 1. The thermoelastic properties of mullite and molybdenum are given in Table 1, which were used in [15]. The geometric constants of the functionally graded circular hollow cylinder are a=0.7, b= 1.0, and L=5.

The reference values of temperature, Young's modulus, and thermal expansion coefficient are T0 =200 K,E 0=330 GPa, and α=4.9e5.

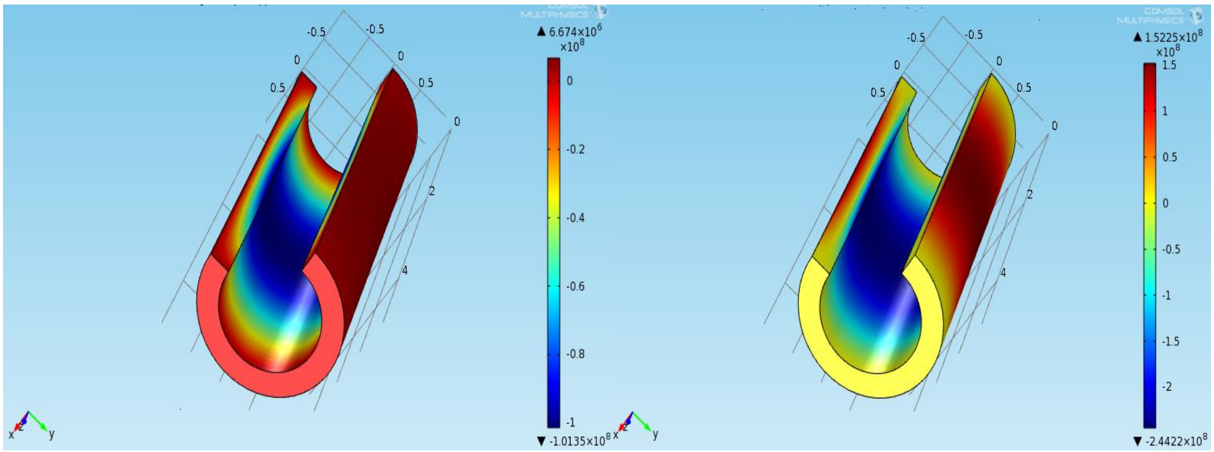
Table 1: Thermoelastic properties of Mullite and Molebdenum

Material Properties	Mullite	Molybdenum
Thermal Conductivity Coefficient, W(m K^{-1})	5.9	138
Thermal Expansion Coefficient, $x10^{-6}$ K^{-1}	4.8	4.9
Young's Modulus, GPa	225	330
Poisson's Ratio	0.27	0.30
Specific Heat i.e. Cp, J/g-^{o}C	.950	10.22
Density, g/cm^3	2.80	.217

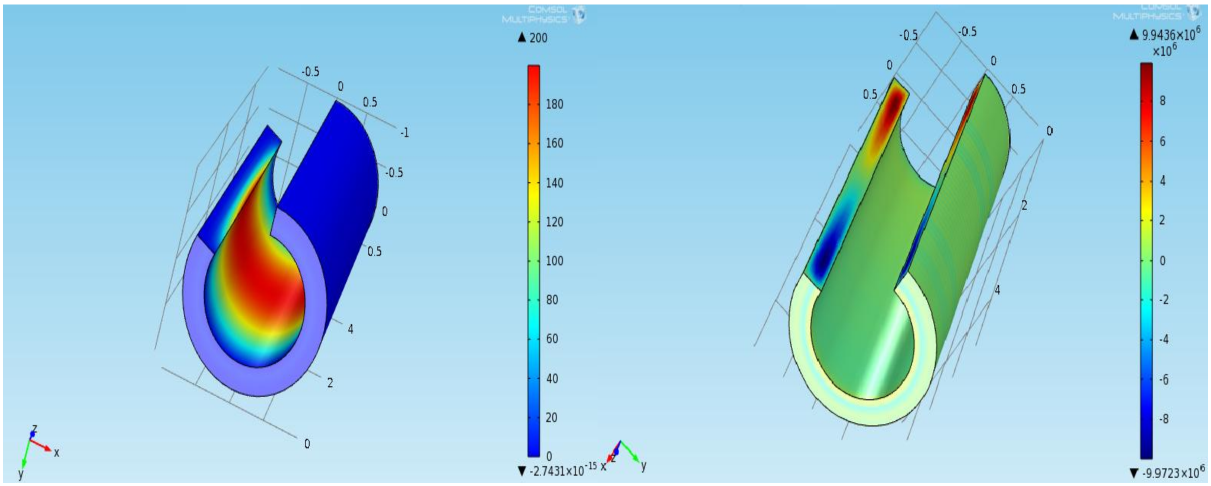
Now the boundary condition for temperature is as follows:

$$T_o = 200\ \mathrm{Sin}(\Pi Z/5),\ T_i = 0\ \mathrm{K} \qquad (14)$$

The analysis of thick-walled cylindrical vessel is performed using the COMSOL Multiphysics 4.2 software. The element to the model in simulation used is triangular with fine meshing with total degree of freedom 2064. For the simulation perform by 2D - axissymmetric physics with stationary solution is selected. This model is a precise approximation method to simulate the FGM vessel.

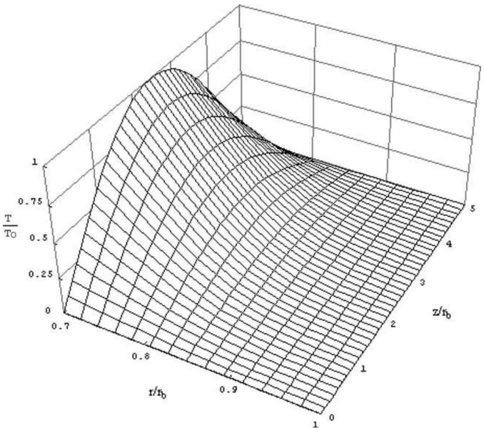


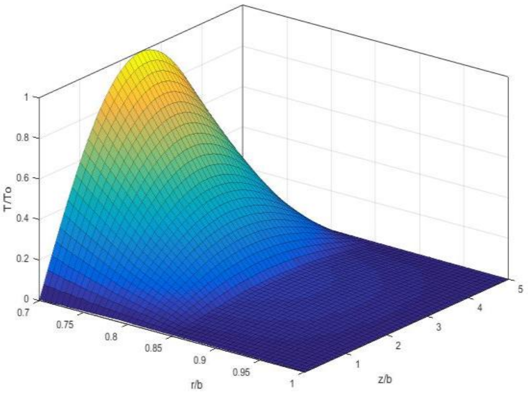
(a) (b)



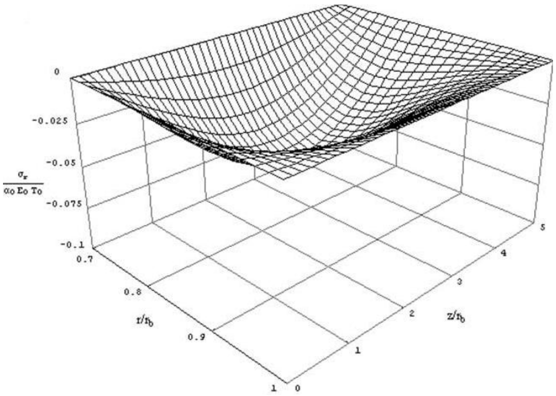
(c) (c)

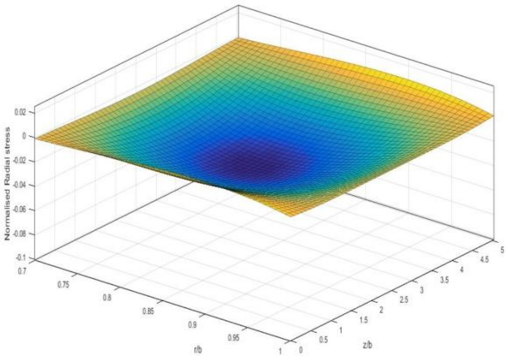
Figure 3 Different result distribution in FG cylinder under thermal load (a) Radial stress (b) Axial stress (c) Temperature (d) Shear stress .



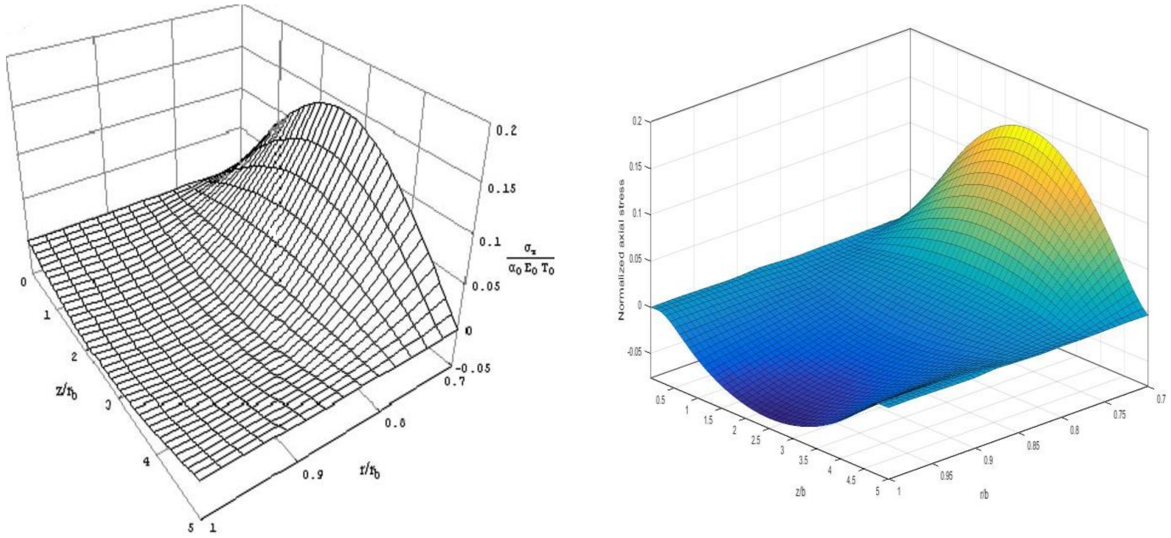


(a) Temperature Distribution **(b) Evaluated Temperature Distribution**



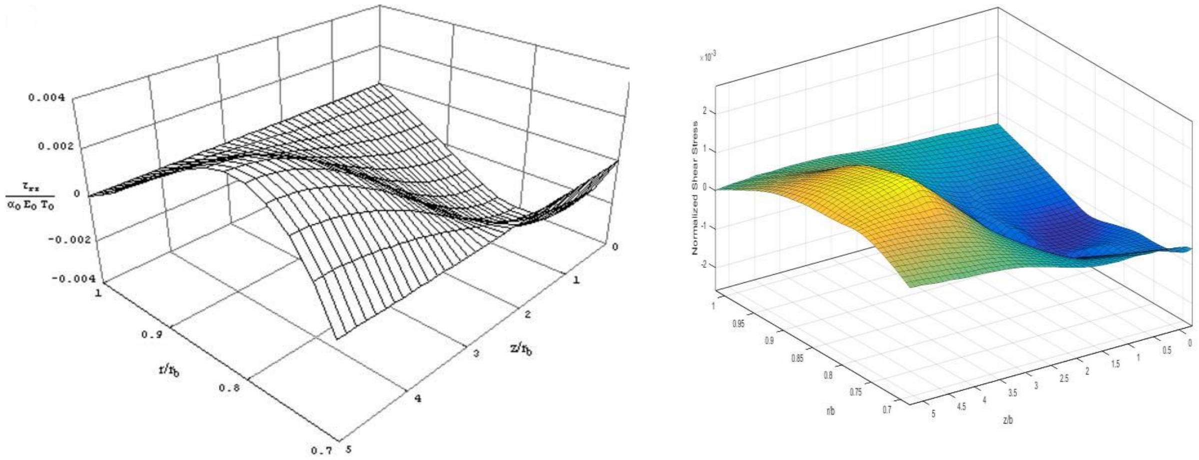


(c) Radial Stress [15] **(d) Evaluated Radial Stress**



(e) Axial Stress [15]

(f) Evaluated Axial Stress



(g) Shear stress [15]

(h) Evaluated Shear stress

Figure 4: Graphical representation of all results in a),(b),(c),(d),(e),(f),(g) and (h).

The evaluated result in the figure 4 show a good agreement with the published result with acceptable accuracy. The evaluated result shows that temperature distribution, radial stress and axial stress are symmetric about Z=2.5. Where shear stress is anti- symmetric about Z=2.5. The sharness of the evalusted surface show lesser chane in magnitude of the different result which conclude that there is less stress concentration due to the use of sigmoid law of gradation.

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