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THERMOELASTIC RAYLEIGH WAVE IN A ROTATING AND TRANSVERSELY ISOTROPIC SOLID HALF-SPACE WITH MAGNETIC FIELD AND INITIAL STRESSES

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ABSTRACT

The governing equations of an initially stressed, rotating and transversely isotropic thermoelastic solid permeated with magnetic field are solved for surface wave solutions. The appropriate particular solutions in the half-space satisfy the required boundary conditions at a thermally insulated stress free surface. A velocity equation is obtained for wave speed of thermo-elastic Rayleigh wave. A special case is derived for small values of reduced frequency. Some particular cases are also derived in absence of rotation, thermal, initial stress and magnetic parameters.

Keywords Anisotropy, Magneto-elasticity, Thermoelasticity, Initial stress, Rotation, Rayleigh wave.

I. INTRODUCTION

Biot [1] proposed a coupled theory of thermoelasticity, where the heat equation is of diffusion type. In this theory, the speeds of propagation for thermal signals are observed infinite. Lord and Shulman [2] extended the Biot's coupled theory to a generalized theory, where the heat equation is of hyperbolic type with a relaxation time. A generalized theory of thermoelasticity with two relaxation times was developed by Green and Lindsay [3]. Ignaczak and Ostoja-Starzewski [4] reviewed these generalized thermoelasticity in their book. Dhaliwal and Sherief [5] developed generalized theory of anisotropic thermoelasticity. Chandrasekhariah [6] formulated the governing equations for heat-flux generalized thermoelasticity. Thermoelasticity has numerous applications in various engineering fields. For example, thermoelasticity has applications in polymer coating and evaluating the stress redistribution in ceramic matrix composites as shown by Mackin and Purcell [7] and Barone and Patterson [8].

Wave propagation phenomenon in solids is important due to its relevance in composite engineering, geology, seismology, seismic exploration, control system and acoustics. The amplitudes of seismic signals are applicable not only in investigating the internal structure of the earth, but also in exploration of valuable materials, oils, water, chemicals etc. In view of the fact that most large bodies, like the earth, the moon, and other planets, have angular velocity and their own magnetic field. Surface waves in elastic solids were first studied by Lord Rayleigh [9] for an isotropic elastic solid. The extension of surface wave analysis and other wave propagation problems to thermoelastic solids with various parameters has been the subject of many studies; see, for example, [10-32].

The study on wave propagation in a generalized thermoelastic media becomes more relevant when we include additional parameters (e.g. rotation, initial stresses, electric field, magnetic field, anisotropy, porosity, viscosity, microstructure and micro-temperature). In the present paper, the governing equations are formulated for an initially stressed, rotating and transversely isotropic thermoelastic solid permeated with magnetic field. These equations are solved for surface wave solutions and a velocity equation of thermoelastic Rayleigh wave is obtained. Some special and particular cases are also discussed.

II. FORMULATION OF THE PROBLEM AND SOLUTION

We consider an infinite homogeneous thermoelastic medium with reference temperature T_0. We restrict our analysis to plane strain parallel to x-z plane where displacement vector $\vec{u}=(u, 0, w)$ and $\frac{\partial}{\partial y}\equiv 0$. The origin is taken on the plane surface and the positive z-axis pointing into the medium ($z \ge 0$). The medium is assumed transversely isotropic, where the planes of isotropy are taken perpendicular to z-axis. Further, the medium is assumed rotating with angular velocity $\vec{\Omega}=(0, \Omega, 0)$, subjected to hydrostatic state of initial stress and permeated by a constant magnetic field $\vec{H}=(0, H_0, 0)$. Following Agarwal [33], Yu and Tang [34], De and Sengupta [35] and Schoenberg and Censor [36], the governing equations in x-z plane are written as

$$c_{11}\frac{\partial^2 u}{\partial x^2}+(c_{13}+c_{44})\frac{\partial^2 w}{\partial x\partial z}+c_{44}\frac{\partial^2 u}{\partial z^2}-\beta_{11}\frac{\partial T}{\partial x} +\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\partial z}\right)-p_0\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)=\rho\left(\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t}\right), \tag{1}$$

$$c_{44}\frac{\partial^2 w}{\partial x^2}+(c_{13}+c_{44})\frac{\partial^2 u}{\partial x\partial z}+c_{33}\frac{\partial^2 w}{\partial z^2}-\beta_{33}\frac{\partial T}{\partial z} +\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial z^2}\right)-p_0\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}\right), \tag{2}$$

$$K_{11}\frac{\partial^2 T}{\partial x^2}+K_{33}\frac{\partial^2 T}{\partial z^2}=\rho C_E\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right) +\beta_{11}T_0\left(\frac{\partial^2 u}{\partial t\partial x}+\tau_0\frac{\partial^3 u}{\partial t^2\partial x}\right)+\beta_{33}T_0\left(\frac{\partial^2 w}{\partial t\partial z}+\tau_0\frac{\partial^3 w}{\partial t^2\partial z}\right), \tag{3}$$

where

$$\beta_{11}=(c_{11}+c_{12})\lambda_1+c_{13}\lambda_2,\quad \beta_{33}=2c_{13}\lambda_1+c_{33}\lambda_2,$$

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and *u*, *w* are components of displacement vector, ρ is density of the medium μ_e is magnetic permeability of the medium, p_0 is hydrostatic state of stress, c_{ij} are material constants, C_E is the specific heat at constant strain, K_{11}, K_{33} are the components of thermal conductivity tensor, β_{ij} are the thermal coefficients, λ_1 is the coefficient of linear expansion in direction perpendicular to z-axis and λ_2 is the coefficient of linear expansion in the z-direction

Equations (1) and (2) are written as

$$a\frac{\partial^2 u}{\partial x^2}+e\frac{\partial^2 u}{\partial z^2}+b\frac{\partial^2 w}{\partial x\partial z}-\beta_{11}\frac{\partial T}{\partial x} = \rho[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t}], \tag{4}$$

$$e\frac{\partial^2 w}{\partial x^2}+d\frac{\partial^2 w}{\partial z^2}+b\frac{\partial^2 u}{\partial x\partial z}-\beta_{33}\frac{\partial T}{\partial x} = \rho[\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}], \tag{5}$$

where,

$$a=c_{11}+\mu_e H_0^2-p_0,\ b=c_{13}+c_{44}+\mu_e H_0^2,\ d=c_{33}+\mu_e H_0^2-p_0,\ e=c_{44}-p_0.$$

We consider the following surface wave solutions of equations (3) to (5)

$$\{u,w,T\}=\{A,B,C\}\exp\{-\eta z+i(sx-pt)\},, \tag{6}$$

where *s* is wave number, *p* is angular velocity and η is constant to be determined.

Using (6) in equations (3) to (5), we obtain

$$(e\eta^2+\rho p^2-as^2+\rho\Omega^2)A+(2ip\rho\Omega-is\eta b)B-is\beta_{11}C = 0, \tag{7}$$

$$(-2ip\rho\Omega-is\eta b)A+(d\eta^2+\rho p^2-es^2+\rho\Omega^2)B+\eta\beta_{33}C = 0, \tag{8}$$

$$spT_0\beta_{11}^*A+ip\eta T_0\beta_{33}^*B-(K_{33}\eta^2+i\rho pC_E^*-K_{11}s^2)C = 0, \tag{9}$$

where

$$\beta_{11}^*=\beta_{11}(1-i\tau_0 p),\quad \beta_{33}^*=\beta_{33}(1-i\tau_0 p),\ C_E^*=C_E(1-i\tau_0 p).$$

Equations (7) to (9) are homogenous equations in A, B, C . For existence of non-trivial solution of these equations, the determinant of coefficients of A, B, C must vanish i.e.,

$$\eta^6+X_1\eta^4+X_2\eta^2+X_3=0, \tag{10}$$

where X_1,X_2 and X_3 are given in *Appendix I*.

Equation (10) is cubic in η^2. Let η_1^2 , η_2^2 ,and η_3^2 be three roots of equation (10). The appropriate particular solutions in the half-space z > 0 are

$$u=A_1\exp\{-\eta_1 z+i(sx-pt)\}+A_2\exp\{-\eta_2 z+i(sx-pt)\} +A_3\exp\{-\eta_3 z+i(sx-pt)\} \tag{11}$$

$$\begin{aligned} w &= B_1 \exp\{-\eta_1 z + i(sx-pt)\} + B_2 \exp\{-\eta_2 z + i(sx-pt)\} \\ &+ B_3 \exp\{-\eta_3 z + i(sx-pt)\} \end{aligned} \tag{12}$$

$$\begin{aligned} T &= C_1 \exp\{-\eta_1 z + i(sx-pt)\} + C_2 \exp\{-\eta_2 z + i(sx-pt)\} \\ &+ C_3 \exp\{-\eta_3 z + i(sx-pt)\} \end{aligned} \tag{13}$$

where B_j and C_j are written in terms of A_j as

$$B_j = i\left[\frac{\beta_{33}\eta_j(e\eta_j^2+\rho p^2-as^2+\rho\Omega^2)+s\beta_{11}(2p\rho\Omega+s\eta_j b)}{\beta_{33}\eta_j(2p\rho\Omega-s\eta_j b)+s\beta_{11}(d\eta_j^2+\rho p^2-es^2+\rho\Omega^2)}\right]A_j,\ (j=1,2) \tag{14}$$

$$C_j = (-i)\frac{(d\eta_j^2+\rho p^2-es^2+\rho\Omega^2)(e\eta_j^2+\rho p^2-as^2+\rho\Omega^2)-4\rho^2p^2\Omega^2+s^2\eta_j^2b^2}{s\beta_{11}(d\eta_j^2+\rho p^2-es^2+\rho\Omega^2)+\beta_{33}(2p\rho\Omega\eta_j-s\eta_j^2 b)}A_j,\ \ (j=1,2) \tag{15}$$

$$B_3 = i\frac{(e\eta_3^2+\rho p^2-as^2+\rho\Omega^2)(K_{33}\eta_3^2+i\rho p C_E^*-K_{11}s^2)-is^2pT_0\beta_{11}^*\beta_{11}}{(2p\rho\Omega-s\eta_3 b)(K_{33}\eta_3^2+i\rho p C_E^*-K_{11}s^2)-isp\eta_3T_0\beta_{11}\beta_{33}^*}A_3, \tag{16}$$

$$C_3 = \frac{pT_0[\beta_{33}^*\eta_3(e\eta_3^2+\rho p^2-as^2+\rho\Omega^2)-s\beta_{11}^*(2p\rho\Omega-s\eta_3 b)]}{(s\eta_3 b-2p\rho\Omega)(K_{33}\eta_3^2+i\rho p C_E^*-K_{11}s^2)+isp\eta_3T_0\beta_{11}\beta_{33}^*}A_3, \tag{17}$$

The boundary conditions at thermally insulated stress-free surface z = 0 are vanishing of

$$\tau_{zz}=0,\quad \tau_{zx}=0,\quad \frac{\partial T}{\partial z}=0, \tag{18}$$

where

$$\tau_{zz}=(b-e)\frac{\partial u}{\partial x}+d\frac{\partial w}{\partial z}-\beta_{33}T\ ,\quad \tau_{zx}=e\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right).$$

Stresses and displacement tends to zero as $z\to\infty$ i.e. Re(η)>0, the temperature fluctuation T satisfy the radiation condition. Putting the value of u, w, and T from equations (11) to (13) in boundary conditions (18), we get

$$\begin{aligned} &is(b-e)A_1+is(b-e)A_2+is(b-e)A_3-d\eta_1B_1-d\eta_2B_2-d\eta_3B_3 \\ &-\beta_{33}C_1-\beta_{33}C_2-\beta_{33}C_3=0, \end{aligned} \tag{19}$$

$$\eta_1A_1+\eta_2A_2+\eta_3A_3-isB_1-isB_2-isB_3=0, \tag{20}$$

$$(-\eta_1)C_1+(-\eta_2)C_2+(-\eta_3)C_3=0, \tag{21}$$

Using relations (14) to (17) and eliminating A_1, A_2, A_3, we obtain

$$\chi_1\xi_2\gamma_3-\chi_1\xi_3\gamma_2-\chi_2\xi_1\gamma_3+\chi_2\xi_3\gamma_1+\chi_3\xi_1\gamma_2-\chi_3\xi_2\gamma_1=0\,, \tag{35}$$

where $\chi_1, \chi_2, \chi_3,\ \xi_1, \xi_2,\ \xi_3, \gamma_1, \gamma_2$ *and* γ_3 are given in *Appendix II*.

The equation (25) is the required velocity equation of Rayleigh wave in a rotating and initially stressed transversely isotropic medium. This equation involves the frequency and hence the wave is dispersive. The imaginary part of speed (being non-zero) is called attenuation.

III. SPECIAL CASE (SMALL VALUES OF THE REDUCED FREQUENCY)

Dividing equation (10) by s^6, setting $\frac{\eta}{s}=\alpha$, $\frac{p^2}{s^2}=v^2$, $\frac{aC_E^*}{K_{33}}=p^*$ and introducing dimensionless quantities

$$\frac{K_{11}}{K_{33}}=N', \frac{\beta_{11}}{\beta_{33}}=N, \frac{\rho v^2}{e}=x^*, \frac{p}{p^*}=\chi, \frac{\Omega^2}{s^2}=\Omega^{*2}, \frac{\rho\Omega^2}{e}=\Omega_0$$

$$\frac{i\rho p C_E^*}{s^2 K_{33}}=\frac{i\rho v^2}{a\chi}, \frac{T_0\beta_{33}^*\beta_{33}}{\rho C_E^* d}=\varepsilon_{33}, \frac{T_0\beta_{11}^*\beta_{11}}{\rho C_E^* d}=\varepsilon_{11}, \frac{\beta_{11}^*}{\beta_{33}^*}=N,$$

$$\frac{ipT_0\beta_{33}^*\beta_{33}}{s^2 d K_{33}}=\frac{i\rho v^2\varepsilon_{33}}{a\chi}, \frac{ipT_0\beta_{11}^*\beta_{11}}{s^2 d K_{33}}=\frac{i\rho v^2\varepsilon_{11}}{a\chi},$$

we obtain

$$\alpha^6+Y_1\alpha^4+Y_2\alpha^2+Y_3=0, \qquad (26)$$

where Y_1, Y_2 and Y_3 are given in *Appendix I*.

Since $\frac{p}{p^*}=\chi$ is very small, we neglect the higher power of χ in comparison with unity and then one root of equation (26) say α_1^2 tends to infinity, while the other two roots α_2^2 and α_3^2 are given by

$$(1+\varepsilon_{33})\alpha^4+\{(1+\frac{e}{d}+\varepsilon_{33})x^*+\frac{b^2-e^2-ad}{ed}-\varepsilon_{33}(\frac{a}{e}-\frac{2b}{e}N+\frac{d}{e}N^2)+\Omega_0((1+\frac{e}{d}+\varepsilon_{33})\}\alpha^2$$
$$+(x^*-1)\{\frac{e}{d}(x^*-\frac{a}{e})-\varepsilon_{11}\}+\Omega_0\{(\Omega_0-2x^*-\frac{a}{e}-1)\frac{e}{d}-\varepsilon_{11}\}=0,$$

(27) Putting the values of χ_1,χ_2,χ_3, ξ_1,ξ_2,ξ_3, γ_1,γ_2 and γ_3 in equation (25), and dividing $(-\frac{i^2e^2s^4}{\beta_{33}})(\alpha_1^5)$ and taking the limit $\alpha_1\to\infty$, we obtain

$$P\alpha_2^2\alpha_3^2+K\alpha_2^3+L\alpha_3^3+Q\alpha_2^2+R\alpha_3^2+D\alpha_2+M\alpha_3+S+$$
$$\alpha_2\alpha_3[U\alpha_2^2\alpha_3^2+E\alpha_2\alpha_3^2+V\alpha_2^2+W\alpha_3^2+F\alpha_2+G\alpha_3+Z]=0, \qquad (28)$$

where

$$P = \left(\frac{e}{d}-\frac{b}{d}-\varepsilon_{33}N\right)\left(x^{*}-1+\frac{b}{e}-\frac{d}{e}N+\Omega_{0}\right),$$

$$K = \left(-\frac{2\Omega x^{*}}{p}\right)\left(1-\frac{b}{e}+\frac{d}{e}N\right)\left(\frac{b}{d}-\frac{e}{d}+N\varepsilon_{33}\right),\ L = \frac{2\Omega N x^{*}}{p}(1+\varepsilon_{33}),$$

$$Q = \left(\frac{e}{d}\left(x^{*}-\frac{a}{e}+\Omega_{0}\right)-\varepsilon_{11}\right)\left(x^{*}-1+\frac{b}{e}-\frac{d}{e}N+\Omega_{0}\right)-\left(\frac{2\Omega x^{*}}{p}\right)\left(\frac{b}{d}-\frac{e}{d}+N\varepsilon_{33}\right),$$

$$R = \left(\frac{e}{d}-\frac{b}{d}-\varepsilon_{33}N\right)\{(x^{*}-1)(x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+\Omega_{0})+\Omega_{0}(x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+\Omega_{0})-4\Omega_{0}x^{*}\},$$

$$D = \frac{2\Omega x^{*}}{p}\left(\frac{b}{d}-\frac{e}{d}+N\right)\left(x^{*}-\frac{a}{e}+\Omega_{0}-\frac{d}{e}\varepsilon_{11}\right)-\left(\frac{2\Omega x^{*}}{p}\right)\left(\frac{b}{d}-\frac{e}{d}+N\varepsilon_{33}\right)\{(1+N)x^{*}-\frac{a}{e}+$$
$$(\frac{b}{e}-1)N+(1+N)\Omega_{0}\},$$

$$M = \frac{2\Omega x^{*}e}{pd}\{(x^{*}-1)(x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+\Omega_{0})+\ \Omega_{0}(x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+\Omega_{0})-4\Omega_{0}x^{*}\}\}$$
$$+\frac{2\Omega N x^{*}}{p}\{(x^{*}-\frac{a}{e}+\frac{b^{2}}{ed}-\frac{b}{d}+\Omega_{0})+\varepsilon_{33}(x^{*}-\frac{a}{e}+(\frac{2b}{e}-1)N-\frac{d}{e}N^{2}+\Omega_{0})\},$$

$$S = \{\frac{e}{d}(x^{*}-\frac{a}{e}+\Omega_{0})-\varepsilon_{11}\}\{(x^{*}-1)(x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+\Omega_{0})+\Omega_{0}(x^{*}-\frac{a}{e}+$$
$$(\frac{b}{e}-1)N+\Omega_{0})-4\Omega_{0}x^{*}\}-\frac{2\Omega N x^{*}}{p}.(\frac{b}{d}-\frac{e}{d}+N\varepsilon_{33}),$$

$$U = (1+\varepsilon_{33})(1-\frac{b}{e}+\frac{d}{e}N),\ \ E = \left(\frac{2\Omega x^{*}}{p}\right)(1+\varepsilon_{33}),$$

$$V = (1-\frac{b}{e}+\frac{d}{e}N)\{\ (x^{*}-\frac{a}{e}+\frac{b^{2}}{ed}-\frac{b}{d}+\Omega_{0})+\varepsilon_{33}(x^{*}-\frac{a}{e}+(\frac{2b}{e}-1)N-\frac{d}{e}N^{2}+\Omega_{0})\},$$

$$W = (1+\varepsilon_{33})\{(1+N)x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+(1+N)\Omega_{0}\},$$

$$F = \frac{e}{d}\left(\frac{2\Omega x^{*}}{p}\right)\left(x^{*}-1+\frac{b}{e}-\frac{d}{e}N+\Omega_{0}\right)+\left(\frac{2\Omega x^{*}}{p}\right)\{(x^{*}-\frac{a}{e}+\frac{b^{2}}{ed}-\frac{b}{d}+\Omega_{0})+$$
$$\varepsilon_{33}(x^{*}-\frac{a}{e}+(\frac{2b}{e}-1)N-\frac{d}{e}N^{2}+\Omega_{0})\},$$

$$G = \left(\frac{2\Omega x^{*}}{p}\right)\left(\frac{b}{d}-\frac{e}{d}-N\right)\left(1-\frac{b}{e}-\frac{d}{e}\varepsilon_{33}N\right),$$

$$Z = \left(\frac{2\Omega x^{*}}{p}\right)\left(\frac{b}{d}-\frac{e}{d}-N\right)+\{(1+N)x^{*}-\frac{a}{e}+(\frac{b}{e}-1)N+(1+N)\Omega_{0}\}.$$
$$\{(x^{*}-\frac{a}{e}+\frac{b^{2}}{ed}-\frac{b}{d}+\Omega_{0})+\varepsilon_{33}(x^{*}-\frac{a}{e}+(\frac{2b}{e}-1)N-\frac{d}{e}N^{2}+\Omega_{0})\},$$

Equation (28) does not involve the frequency and so the wave is not dispersive. The velocity equation (28) shows the dependence on elastic, rotational, magnetic, initial stress and thermal parameters.

IV. PARTICULAR CASES

(a) In absence of rotational, magnetic, transverse isotropy and initial stress parameter (isotropic thermoelastic case)

If we neglect rotation, magnetic field, anisotropy and initial stress parameters, i.e.

$$\Omega^*=0,\ \Omega_0=0, H=0,\ \ p_o=0\ ,\ \beta_{11}=\beta_{33}=\beta,\ \ ,\beta^*_{11}=\beta^*_{33}=\beta(1-i\tau_0 p)=\beta^*,$$

$$K_{11}=K_{33}=\beta,\frac{\beta_{11}}{\beta_{33}}=N=1,\frac{K_{11}}{K_{33}}=N'=1,\varepsilon_{11}=\varepsilon_{33}=\varepsilon,$$

then the equation (27) reduces to

$$(1+\varepsilon)\alpha^4+\{(1+\frac{e_2}{d_2}+\varepsilon)x_2^*+\frac{b_2^2-e_2^2-a_2^2d_2^2}{e_2d_2}-\varepsilon(\frac{a_2}{e_2}-\frac{2b_2}{e_2}+\frac{d_2}{e_2})\}\alpha^2$$
$$+(x_2^*-1)\{\frac{e_2}{d_2}(x_2^*-\frac{a_2}{e_2})-\varepsilon\}=0 \tag{29}$$

where

$$a_2=d_2=\lambda+2\mu,\ b_2=\lambda+\mu,\ \ e_2=\mu,\ x_2^*=\frac{\rho v^2}{e_2}.$$

The equation (28) also reduces to

$$P_4\alpha_2^2\alpha_3^2+Q_4\alpha_2^2+R_4\alpha_3^2+S_4+\alpha_2\alpha_3[U_4\alpha_2^2\alpha_3^2+V_4\alpha_2^2+W_4\alpha_3^2+Z_4]=0 \tag{30}$$

where

$$P_4=\ (\frac{e_2}{d_2}-\frac{b_2}{d_2}-\varepsilon)(x_2^*-1+\frac{b_2}{e_2}-\frac{d_2}{e_2}),$$

$$Q_4=\frac{e_2}{d_2}(x_2^*-\frac{a_2}{e_2}-\frac{d_2}{e_2}\varepsilon)(x_2^*-1+\frac{b_2}{e_2}-\frac{d_2}{e_2}),$$

$$R_4=(\frac{e_2}{d_2}-\frac{b_2}{d_2}-\varepsilon)(x_2^*-1)(x_2^*-1+\frac{b_2}{e_2}-\frac{a_2}{e_2}),$$

$$S_4=\frac{e_2}{d_2}\{x_2^*-\frac{a_2}{e_2}-\frac{d_2}{e_2}\varepsilon\}\{(x_2^*-1)(x_2^*-1+\frac{b_2}{e_2}-\frac{a_2}{e_2})\},$$

$$U_4=(1+\varepsilon)(1-\frac{b_2}{e_2}+\frac{d_2}{e_2}),$$

$$V_4=(1-\frac{b_2}{e_2}+\frac{d_2}{e_2})[(1+\varepsilon)(x_2^*-\frac{a_2}{e_2})+\varepsilon(\frac{2b_2}{e_2}-1-\frac{d_2}{e_2})+\frac{b_2^2}{e_2d_2}-\frac{b_2}{d_2}],$$

$$W_4=(1+\varepsilon)(2x_2^*-\frac{a_2}{e_2}+\frac{b_2}{e_2}-1),$$

$$Z_4=(2x_2^*-\frac{a_2}{e_2}+\frac{b_2}{e_2}-1)\{(1+\varepsilon)(x_2^*-\frac{a_2}{e_2})+\varepsilon(\frac{2b_2}{e_2}-1-\frac{d_2}{e_2})+\frac{b_2^2}{e_2d_2}-\frac{b_2}{d_2}\},$$

Putting $a_2=d_2=\lambda+2\mu,\ b_2=\lambda+\mu,\ \ e_2=\mu,\ x_2^*=\frac{\rho v^2}{e_2}$ in equation (29), the roots α_2^2 and α_3^2 become

$$\alpha_2^2=1-\frac{v^2}{(1+\varepsilon)v_T^2},\quad \alpha_3^2=1-\frac{v^2}{v_S^2},$$

where

$$v_S^2=\frac{\mu}{\rho}\quad\text{and}\quad v_T^2=\frac{\lambda+2\mu}{\rho},$$

and the equation (30) reduces to

$$(2-\frac{v^2}{v_S^2})^2=4[(1-\frac{v^2}{v_S^2})(1-\frac{v^2}{(1+\varepsilon)v_T^2})]^{1/2} \tag{31}$$

which agrees with those obtained by Lockett [17].

(b) In absence of rotational, magnetic and initial stress parameter (Transversely isotropic thermoelastic case)

If we neglect rotation, magnetic field and initial stress parameters, i.e.

$$\Omega^*=0,\ \Omega_0=0,\ H=0,\ \ p_o=0\ ,$$

then the equation (27) reduces to

$$(1+\varepsilon_{33})\alpha^4+\{(1+\varepsilon_{33}+\frac{e_1}{d_1})x^*+\frac{b_1^2-e_1^2-a_1d_1}{e_1d_1}-\varepsilon_{33}(\frac{a_1}{e_1}-\frac{2b_1}{e_1}N+\frac{d_1}{e_1}N^2)\}\alpha^2$$
$$+(x^*-1)\{\frac{e_1}{d_1}(x^*-\frac{a_1}{e_1})-\varepsilon_{11}\}=0, \tag{32}$$

where

$$a_1=c_{11},\ b_1=c_{13}+c_{44},\ d_1=c_{33},\ e_1=c_{44},\ \ x_1^*=\frac{\rho v^2}{e_1}.$$

Also, the equation (28) reduces to

$$P_5\alpha_2^2\alpha_3^2+Q_5\alpha_2^2+R_5\alpha_3^2+S_5+\alpha_2\alpha_3[U_5\alpha_2^2\alpha_3^2+V_5\alpha_2^2+W_5\alpha_3^2+Z_5]=0, \tag{33}$$

where

$$P_5 = \left(\frac{e_1}{d_1} - \frac{b_1}{d_1} - \varepsilon_{33} N\right)\left(x_1^* - 1 + \frac{b_1}{e_1} - \frac{d_1}{e_1} N\right),$$

$$Q_5 = \frac{e_1}{d_1}\left(x_1^* - \frac{a_1}{e_1} - \frac{d}{e}\varepsilon_{11}\right)\left(x_1^* - 1 + \frac{b_1}{e_1} - \frac{d_1}{e_1} N\right),$$

$$R_5 = \left(\frac{e_1}{d_1} - \frac{b_1}{d_1} - \varepsilon_{33} N\right)(x^* - 1)\left(x^* - \frac{a_1}{e_1} + \left(\frac{b_1}{e_1} - 1\right)N\right),$$

$$S_5 = \frac{e_1}{d_1}\left\{x^* - \frac{a_1}{e_1} - \frac{d_1}{e_1}\varepsilon_{33} N^2\right\}\left\{(x^* - 1)\left(x^* - \frac{a_1}{e_1} + \left(\frac{b_1}{e_1} - 1\right)N\right)\right\},$$

$$U_5 = (1 + \varepsilon_{33})\left(1 - \frac{b_1}{e_1} + \frac{d_1}{e_1} N\right),$$

$$V_5 = \left(1 - \frac{b_1}{e_1} + \frac{d_1}{e_1} N\right)\left[(1 + \varepsilon_{33})\left(x^* - \frac{a_1}{e_1}\right) + \frac{b_1^2}{e_1 d_1} - \frac{b_1}{d_1} + \varepsilon_{33}\left\{\left(\frac{2b_1}{e_1} - 1\right)N - \frac{d_1}{e_1} N^2\right\}\right],$$

$$W_5 = (1 + \varepsilon_{33})\left\{(1 + N)x^* - \frac{a_1}{e_1} + \left(\frac{b_1}{e_1} - 1\right)N\right\},$$

$$Z_5 = \left\{(1 + N)x^* - \frac{a_1}{e_1} + \left(\frac{b_1}{e_1} - 1\right)N\right\}\left[\left\{(1 + \varepsilon_{33})\left(x^* - \frac{a_1}{e_1}\right) + \frac{b_1^2}{e_1 d_1} - \frac{b_1}{d_1}\right.\right.$$
$$\left.\left. + \varepsilon_{33}\left\{\left(\frac{2b_1}{e_1} - 1\right)N - \frac{d_1}{e_1} N^2\right\}\right\}\right],$$

The reduced equations (32) and (33) agree with those obtained by Chakraborty and Pal [37]

V. CONCLUSION

Theoretical analysis of the Rayleigh wave in a rotating and initially stressed transversely isotropic magneto-thermoelastic solid half-space is presented. The velocity equation for Rayleigh wave is obtained and found frequency-dependent. A special case of small reduced frequency is also considered, where the velocity equation is found to be independent of frequency. In absence of rotation, magnetic and initial stress parameters, the velocity equation (25) is reduced for isotropic thermoelastic case and transversely isotropic thermoelastic case as particular cases. These theotetical results can be verified numerically for a realistic model with relevant data. The present theoretical approach can be applied in the case of piezothermoelastic composite materials or magneto-electro-elastic composites.

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Appendix I

$$X_1 = \frac{i\rho p C_E^*}{K_{33}} - \frac{K_{11}s^2}{K_{33}} + \rho p^2\left(\frac{1}{d}+\frac{1}{e}\right) + s^2\left(\frac{b^2-e^2-ad}{ed}\right) + \rho\Omega^2\left(\frac{1}{d}+\frac{1}{e}\right) + \frac{ipT_0\beta_{33}^*\beta_{33}}{d\,K_{33}}$$

$$X_2=(\frac{\rho p^2-es^2}{e})(\frac{\rho p^2-as^2}{d})+\rho\Omega^2(\frac{\rho\Omega^2-as^2-es^2-2\rho p^2}{ed})+$$
$$(\frac{i\rho pC_E^*}{K_{33}}-\frac{K_{11}s^2}{K_{33}})\{\rho p^2(\frac{1}{d}+\frac{1}{e})+s^2(\frac{b^2-e^2-ad}{ed})+\rho\Omega^2(\frac{1}{d}+\frac{1}{e})\}$$
$$+\frac{ipT_0\beta_{33}^*\beta_{33}}{dK_{33}}(\frac{\rho p^2-as^2}{e}+\frac{\rho\Omega^2}{e}-\frac{ds^2\beta_{11}^2}{e\,\beta_{33}^2}+\frac{2bs^2\beta_{11}}{e\,\beta_{33}})$$

$$X_3=(\frac{i\rho pC_E^*}{K_{33}}-\frac{K_{11}s^2}{K_{33}})\{(\frac{\rho p^2-es^2}{e})(\frac{\rho p^2-as^2}{d})+$$
$$\rho\Omega^2(\frac{\rho\Omega^2-as^2-es^2-2\rho p^2}{ed})\}-\frac{ipT_0s^2\beta_{11}^*\beta_{11}}{dK_{33}}(\frac{\rho p^2-es^2}{e}+\frac{\rho\Omega^2}{e})$$

$$Y_1=(1+\frac{e}{d})x^*+\frac{b^2-e^2-ad}{ed}-N'+\frac{i\rho v^2}{a\chi}(1+\varepsilon_{33})+\Omega_0(1+\frac{e}{d})$$

$$Y_2=(x^*-1)(x^*-\frac{a}{e})\frac{e}{d}+\Omega_0(\Omega_0-2x^*-\frac{a}{e}-1)\frac{e}{d}-N'\{(x^*+\Omega_0)(1+\frac{e}{d})+$$
$$\frac{b^2-e^2-ad}{ed}\}+\frac{i\rho v^2}{a\chi}\{(x^*+\Omega_0)(1+\frac{e}{d})+\frac{b^2-e^2-ad}{ed}+$$
$$\varepsilon_{33}(x^*-\frac{a}{e}+\frac{2b}{e}N-\frac{d}{e}N^2+\Omega_0)\}$$

$$Y_3=\frac{i\rho v^2}{a\chi}\{(x^*-1)(x^*-\frac{a}{e})\frac{e}{d}+\Omega_0(\Omega_0-2x^*-\frac{a}{e}-1)\frac{e}{d}-\varepsilon_{11}(x^*-1+\Omega_0)\}-$$
$$N'\frac{e}{d}\{(x^*-1)(x^*-\frac{a}{e})+\Omega_0(\Omega_0-2x^*-\frac{a}{e}-1)\}$$

Appendix II

$$\chi_j=ies\frac{L_1\alpha_j^2+E_1\alpha_j+L_2}{M_1\alpha_j^2+F_1\alpha_j+M_2},\quad (j=1,\,2),$$

$$\chi_3=ids\frac{L_1'\alpha_3^5+L_2'\alpha_3^3+E_1'\alpha_3^2+L_3'\alpha_3+G_1}{M_1'\alpha_3^3+F_1'\alpha_3^2+M_2'\alpha_3+G_1'}$$

$$\xi_j=(-s)\frac{L_3\alpha_j^3+F_1\alpha_j^2+L_4\alpha_j+G_2}{M_1\alpha_j^2+F_1\alpha_j+M_2},\ (j=1,\,2),$$

$$\xi_3=s\frac{L_4'\alpha_3^4+F_1\alpha_3^3+L_5'\alpha_3^2+G_2'\alpha_3+L_6'}{M_1'\alpha_3^3+F_1'\alpha_3^2+M_2'\alpha_3+G_1'}$$

$$\gamma_j=(-\frac{ies^2\alpha_1}{\beta_{33}})\frac{L_6\alpha_j^4+L_7\alpha_j^2+L_8}{M_1\alpha_j^2+F_1\alpha_j+M_2},\ (j=1,\,2)$$

$$\gamma_3 = (-\frac{ipT_0\beta_{33}^*\alpha_3}{K_{33}})\frac{L_7'\alpha_3^3 + L_8'\alpha_3 + G_3}{M_1'\alpha_3^3 + F_1'\alpha_3^2 + M_2'\alpha_3 + G_1'}$$

$$L_1 = x^* - 1 + \frac{b}{e} - \frac{d}{e}N + \Omega_0, E_1 = \frac{2\Omega x^*}{p}(\frac{b}{e} - 1 - \frac{d}{e}N),$$

$$L_2 = (x^* - 1)[x^* - \frac{a}{e} + (\frac{b}{e} - 1)N + \Omega_0] + \Omega_0\{x^* - \frac{a}{e} + (\frac{b}{e} - 1)N + \Omega_0\} - 4\Omega_0 x^*,$$

$$L_3 = 1 - \frac{b}{e} + \frac{d}{e}N,$$

$$L_4 = (1+N)x^* - \frac{a}{e} + (\frac{b}{e} - 1)N + (1+N)\Omega_0,$$

$$L_6 = \frac{d}{e},$$

$$L_7 = \frac{d}{e}(x^* - \frac{a}{e} + \Omega_0) + x^* - 1 + \Omega_0 + (\frac{b}{e})^2,$$

$$L_8 = (x^* - 1)(x^* - \frac{a}{e}) + \Omega_0(\Omega_0 - 1 - 2x^* - \frac{a}{e}),$$

$$L_1' = 1, \quad L_2' = x^* - N' - \frac{a}{e} + \frac{b}{d}(\frac{b}{e} - 1) + \Omega_0 + \frac{iex^*}{a\chi}(1 + \varepsilon_{33}),$$

$$L_3' = -N'(x^* - \frac{a}{e} + \frac{b^2}{ed} - \frac{b}{d} + \Omega_0) + \frac{iex^*}{a\chi}\{(x^* - \frac{a}{e} + \frac{b^2}{ed} - \frac{b}{d} + \Omega_0) + \varepsilon_{33}(x^* - \frac{a}{e} +$$
$$(\frac{2b}{e} - 1)N - \frac{d}{e}N^2 + \Omega_0)\},$$

$$L_4' = 1 - \frac{b}{e}, \; L_5' = x^* - \frac{a}{e} - N'(1 - \frac{b}{e}) + \frac{iex^*}{a\chi}(1 - \frac{b}{e}) - \frac{iNdx^*}{a\chi}\varepsilon_{33} + \Omega_0,$$

$$L_6' - N'(x^* - \frac{a}{e} + \Omega_0) + \frac{iex^*}{a\chi}(x^* - \frac{a}{e} + \Omega_0) - \frac{iN^2dx^*}{a\chi}\varepsilon_{33},$$

$$L_7' = 1, \; L_8' = (x^* - \frac{a}{e} + \frac{b}{e}N + \Omega_0),$$

$$G_1 = -\frac{2\Omega x^*}{p}\{-N'(\frac{b}{d} - \frac{e}{d}) + \frac{iex^*}{a\chi}(\frac{b}{d} - \frac{e}{d} + N\varepsilon_{33})\},$$

$$G_2 = \frac{2N\Omega x^*}{p}, \; G_3 = -\frac{2\Omega N}{p}x^*, \; G_1' = \frac{2\Omega x^*}{p}(N' - \frac{iex^*}{a\chi}),$$

$$G_2' = \frac{2\Omega x^*}{p}(\frac{iex^*}{a\chi} - N'),$$

$$M_1 = N\frac{d}{e} - \frac{b}{e}, \; M_2 = N(x^* - 1 + \Omega_0),$$

$$M_1^{/} = \frac{b}{e},\ M_2^{/} = -N^{/}\frac{b}{e} + \frac{ibx^*}{a\chi} + i\varepsilon_{33}\frac{Ndx^*}{a\chi},$$

$$F_1 = \frac{2\Omega x^*}{p},\ F_1^{/} = -\frac{2\Omega x^*}{p},\ \ E_1^{/} = \frac{2\Omega x^*}{p}\left(\frac{e-b}{d}\right),$$