

STUDY OF IMBIBITION PHENOMENON ARISING IN IMMISCIBLE PHASE FLOW THROUGH HOMOGENEOUS POROUS MEDIA

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ABSTRACT

The present paper deals with the phenomenon of imbibitions in two immiscible phase flow through homogeneous porous media. When a porous medium is filled with some fluid which preferentially wets the medium then there is a spontaneous flow of the resident fluid from the medium. Such a phenomenon arises due to the difference in the wetting abilities of the fluid which is called counter-current Imbibition. The governing equation to this phenomenon is a partial differential equation. The solution is obtained by using Finite Difference Method.

Key Words : , *Homogeneous Porous media, Imbibition, Immiscible, S.O.R.*

I. INTRODUCTION

The present paper deals with the phenomenon of imbibitions in two immiscible phase flow through homogeneous porous media [1]. When two fluids are together in contact with a rock surface, the angle, measured through water is called the contact angle. In the whole study, two fluids such as water and oil are considered and immiscible flow occurs due to water and oil. If contact angle is acute, the porous medium is described as being water wet, whereas if contact angle is obtuse, it is oil wet. The wet ability, as defined by the angle, is measured of which fluid preferentially adheres to the rock. The solution of governing equation is obtained by a Successive over Relaxation method [2].

Many Authors, Verma [3], Scheidegger [4], have discussed this phenomenon in different ways. Some of them have used Numerical technique to obtain the solution. Verma [5] has obtained an approximate solution to determine the saturation distribution for Imbibitions phenomenon by considering the presence of heterogeneity in the medium marginally.

II. STATEMENT OF THE PROBLEM

Consider a finite cylindrical piece of homogeneous porous matrix of length L . It is assumed that it is fully saturated with an oil. Also it is completely bordered by an impervious surface except for one end which is uncovered to an adjacent formation of injected water. It is assumed that injected water is favorably more wetting than that of native liquid (oil) and due to this arrangement there is a unstructured linear

flow of water into the medium and a counter flow of the resident fluid (oil) from the medium, which is known as the phenomenon of linear counter-current Imbibition.[1]

The governing equations to this phenomenon is a partial differential equation, which is solved by Successive over Relaxation Method.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

According to Darcy's law, the equations of leakage velocity of flowing fluids are written as:

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \tag{1}$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \tag{2}$$

Where V_w and V_o are leakage velocity of water and oil respectively, k is the permeability of the homogeneous medium, k_w and k_o are relative permabilities of water and oil respectively, P_w and P_o are the pressures and μ_o and μ_w are viscosities of water and oil respectively.

The equations of continuity for the flowing phase are:

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \tag{3}$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \tag{4}$$

Where ϕ is the porosity of the medium and S_w and S_o are water and oil saturation respectively.

For imbibition phenomenon an analytic condition is given by

$$V_w = -V_o \tag{5}$$

$$P_c = P_o - P_w \tag{6}$$

From equations (1),(2) and (5) , we have

$$\frac{k_o}{\mu_o} \frac{\partial P_o}{\partial x} + \frac{k_w}{\mu_w} \frac{\partial P_w}{\partial x} = 0 \tag{7}$$

From (6) and (7),we have

$$\begin{aligned} \frac{k_o}{\mu_o} \left(\frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x} \right) + \frac{k_w}{\mu_w} \frac{\partial P_w}{\partial x} &= 0 \\ \therefore \frac{\partial P_w}{\partial x} &= \frac{-k_o/\mu_o}{(k_o/\mu_o + k_w/\mu_w)} \frac{\partial P_c}{\partial x} \end{aligned} \tag{8}$$

Substituting (1) into (3) , we get

$$\phi \frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x} \left(\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \right) = 0 \tag{9}$$

From equations (8) and (9) , we get

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left(k \frac{k_o k_w}{k_o \mu_w + k_w \mu_o} \frac{\partial P_c}{\partial S_w} \frac{\partial S_w}{\partial x} \right) = 0 \tag{10}$$

$$\text{Setting } H = \frac{k_o k_w}{k_o \mu_w + k_w \mu_o}$$

It is called co-efficient of saturation which is assumed to be a constant.

Now using the condition for capillary pressure depending upon phase saturation as [6]

$$P_c = -\beta S_w \tag{11}$$

Equation (10) can be written as

$$\phi \frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x} \left[kH\beta \frac{\partial s_w}{\partial x} \right] = 0 \tag{12}$$

With $s_w(0, t) = s_0, s_w(L, t) = s_1, s_w(L, t) = 0, 0 \leq x \leq L$

$$\frac{\partial s_w}{\partial t} - kD\beta \frac{\partial^2 s_w}{\partial x^2} = 0 \tag{13}$$

Let $\xi = \frac{x}{L}, T = \frac{kD(s_w)\beta}{L^2} t$

$$\frac{\partial s_w}{\partial T} - \frac{\partial^2 s_w}{\partial \xi^2} = 0 \tag{14}$$

With $s_w(0, T) = s_0, s_w(1, T) = s_1, s_w(1, T) = 0, 0 \leq \xi \leq 1$

IV. MATHEMATICAL SOLUTION

Using S.O.R. method [6], we have

$$s_{w_{i,j+1}} = s_{w_{i,j}} + \frac{k}{2h^2} (s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}} + s_{w_{i+1,j+1}} - 2s_{w_{i,j+1}} + s_{w_{i-1,j+1}})$$

Let $r = \frac{k}{h^2}$

$$(1+r)s_{w_{i,j+1}} = s_{w_{i,j}} + \frac{r}{2} (s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}} + s_{w_{i+1,j+1}} + s_{w_{i-1,j+1}})$$

$$\lambda_i = s_{w_{i,j}} + \frac{r}{2} (s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}})$$

$$s_{w_{i,j+1}} = (1 - \omega)s_{w_{i,j}} + \omega \left[\frac{r}{2(1+r)} (s_{w_{i+1,j}} + s_{w_{i-1,j+1}}) + \frac{\lambda_i}{(1+r)} \right]$$

Choose $k = 0.01, h=0.1, \omega = 1.9, s_0 = 1, s_1 = 0$

$$s_{w_{i,j+1}} = -0.9s_{w_{i,j}} + 1.9 \left[0.25 (s_{w_{i+1,j}} + s_{w_{i-1,j+1}}) + \frac{\lambda_i}{2} \right]$$

Numerical calculations for different values of saturation at different time and different length are shown in the following table.

| T→ | T=0.01 | T=0.02 | T=0.003 | T=0.04 |
|-----|----------|----------|----------|----------|
| ξ ↓ | Sw | | | |
| 0 | 1 | 1 | 1 | 1 |
| 0.1 | 0.475 | 0.736844 | 0.737498 | 0.802634 |
| 0.2 | 0.225625 | 0.474377 | 0.543561 | 0.574843 |
| 0.3 | 0.107172 | 0.284407 | 0.382552 | 0.415526 |
| 0.4 | 0.050907 | 0.163156 | 0.256252 | 0.289048 |
| 0.5 | 0.024181 | 0.090829 | 0.164475 | 0.192241 |
| 0.6 | 0.011486 | 0.049475 | 0.101924 | 0.122827 |
| 0.7 | 0.005456 | 0.026508 | 0.061376 | 0.075883 |
| 0.8 | 0.002591 | 0.01402 | 0.036098 | 0.045359 |

| | | | | |
|-----|----------|----------|----------|----------|
| 0.9 | 0.001231 | 0.007338 | 0.020569 | 0.025344 |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 |

V. GRAPHICAL REPRESENTATION:

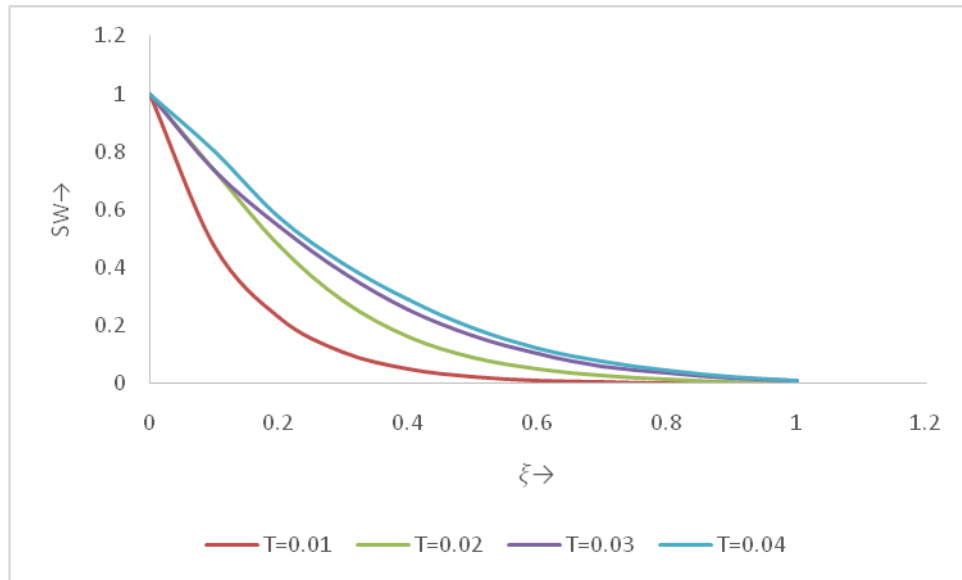


Figure -I : Length → Saturation

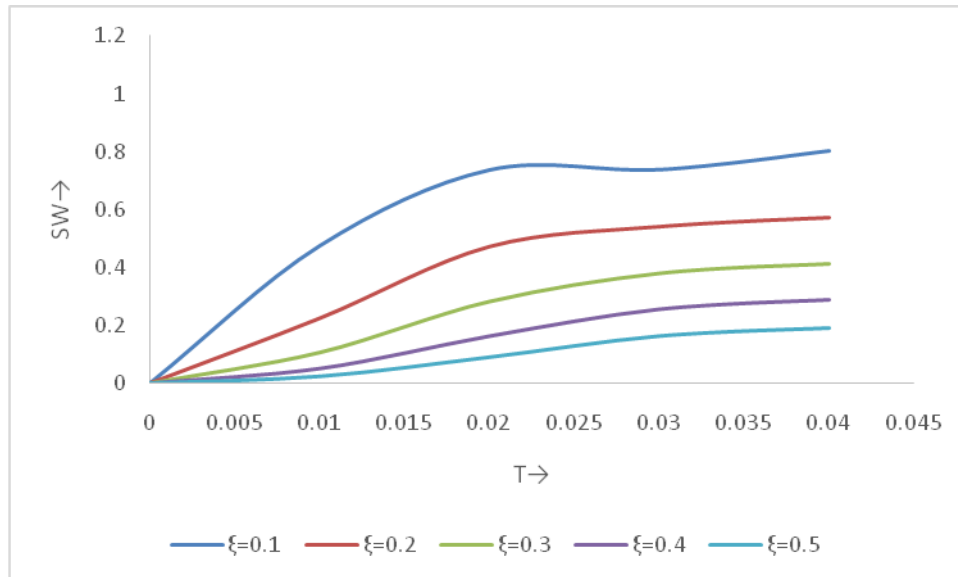


Figure-II: Time → Saturation

VI. CONCLUSION:

From figure-I, we can say that as length increases the saturation decreases. Keeping time constant as *length* increases saturation decreases. From figure -II, it is clear that as time increases saturation increases. Keeping length constant, for different values of time the saturation increases as time increases.



REFERENCES

- [1.] Priti V. Tandel. P.H. Bhathawala, Imbibition in Double Phase Flow Through Porous Media, International Journal on Recent and Innovation Trends in Computing and Communication, Volume: 3 Issue: 9, pp. 5431 - 5433
- [2.] S.S.Shastry: Introductory Methods Of Numerical Analysis(4th Edition).
- [3.] Verma, A.P.(1969), Imbibition in Cracked Porous Media, Cand. J.Physics.,47,2519.
- [4.] Scheidegger, A.E. (1960). The physics of flow through porous media, University of Toronto Press, 229-231,224,231,216.
- [5.] Verma, A.P.(1968), Motion of Immiscible Liquids in a cracked Heterogeneous Porous Medium with Capillary Pressure , Rev. Roumaine Sci.Techn.,Mecan.Appl.,13(2), 277-292.
- [6.] Mehta, M.N. and Verma, A.P. (1977), Indian Journal of Double Pure and Applied Mathematics, vol.8, No.5,523