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HARVESTING MODEL FOR FISHERY RESOURCE WITH RESERVED AND UNRESERVED AREA OF PREY AND PREDATOR

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ABSTRACT

In this paper, a prey-predator fishery model with prey dispersal in a two-patch environment, one is assumed to be a free fishing zone and the other is a reserved zone where fishing and other extra activities are prohibited.Bird predators can move to both areas i.e. reserved and unreserved zone.Holling type II functional response is considered for bird predators. Harvesting is done in unreserved area of prey and predator zone. The existence and local stability of different equilibria are discussed.

Keywords: Prey-Predator; Stability; Harvesting;

I INTRODUCTION

Dynamics of interacting biological species has been studied in the past few decades from various angles. Many species become extinct and many others are at the verge of extinction due to several reasons like, over exploitation, over predation, environmental pollution, mismanagement of natural resources etc. To save these species, suitable measures such as restriction on harvesting, creating reserve zones/refuges etc have taken so that the species can grow in these regions without any external disturbances. The role of reserve zones/refuges in prey-predator dynamics creates a major interest to researchers .Recently, Dubey[2003] investigated the role of reserve zone on the dynamics of predator-prey system which indicates that reserve zone has a stabilizing effect on the predator-prey dynamics. Extensive and unregulated harvesting of marine fishes can lead to the depletion of several fish species. One potential solution to these problems is the creation of marine reserves where fishing and other extractive activities are prohibited. The idea of using marine reserves was first formally considered by Beverton and Holt. Marine protected and marine reserves areas have been promoted as conservation and fishery management tools to hedge marine life and sustain ecosystems. Marine reserves are also known as “no-take areas” or ‘no-take/no-harm areas’ .A ‘no-take’ aquatic reserve means that people are not permitted to fish by any method. Reserves are a special category of marine protected areas (MPAs).MPAs is defined as areas designated to enhance conservation of marine resources through legal protections from disturbance, harm and/or fishing. Marine reserves not only protect species inside the reserve area but they can also increase fish abundance in adjacent areas.

Kar and Chaudhuri[2007] investigated a dynamic reaction model in the case of a prey-predator type fishery system, where only the prey species is subject to harvesting, and taking taxation as a control instrument. Dubey analyzed a dynamic model for coexistence and stability behavior of predator-prey system in the habitat. Braza analyzed a two predator, one prey model in which one predator interferes significantly with other. Kar in their paper, offer some mathematical analysis of the dynamics of a two prey, one predator system in the presence of a time delay. A model of prey- predator with a generalized transmission function for unreserved zone has been analyzed by H. Mehta[2012].Recently Agarwal developed a simple two species prey predator model in which the prey disperse in a two patch environment.

In the above investigations, the dynamics of predator living in unreserved zone together with prey has not been studied explicitly. It was suggested that even if fishery is exploited continuously in the unreserved zone, fish populations can be maintained at an appropriate equilibrium level in the habitat. The model presented in this paper will be of great use in a National Park, Marine area of sea and Wildlife sanctuaries/Forest areas where prey-predator are living together. The prey species which are to be conserved can be protected from predators by creating an artificial boundary or shelter that will divide the habitat into two zones – one reserved and other unreserved. The entry of predators into reserved zone can be restricted by the artificial boundary that may be in the form of fencing of suitable mesh size through which prey can pass but predators cannot.

II THE MODEL

Consider a habitat where prey and predator species are living together. It is assumed that the habitat is divided into two zones, namely, reserved and unreserved zones. Each zone is supposed to be homogeneous. There is a bird predator feeding on both of them, that is, fishes of reserved as well as unreserved zones. It is assumed that the predator population is also harvested in unreserved zone. We suppose that the prey species migrate between the two zones randomly. The growth of prey in each zone in the absence of predator is assumed to be logistic. Keeping these in view, our model becomes

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\sigma_1 x+\sigma_2 y-\frac{m_1 xz}{\alpha+x}-q_1E_1x \\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y-\frac{m_2 yz}{\alpha+y} \qquad (1) \\ \frac{dz}{dt} &= -dz+\frac{K_1m_1xz}{\alpha+x}+\frac{K_2m_2yz}{\alpha+y}-q_2E_2z \end{aligned}$$

Here x (t) and y(t) are the respective biomass densities of the prey species inside the unreserved and reserved areas &z (t) is the biomass density of predator at time t.

σ_1 & σ_2 : migration rates from the unreserved area to reserved area and the reserved area to the unreserved area respectively,

E_1 & E_2 : the efforts applied to harvest the fishes respectively,

q_1 & q_2 :Catchability coefficient of prey and predator respectively,

d: death rate of predator,

m_1 & m_2 : capturing rates of prey in reserved and unreserved zonerespectively,

K_1 & K_2 : conversion rates of prey in unreserved andreserved zone respectively,

We observe that if there is no migration of fish population from the reserved area to the unreserved area $(\sigma_2 = 0)$

$$r - \sigma_1 - q_1 E_1 < 0 \text{ then } \frac{dx}{dt} < 0 \tag{2}$$

Then, prey species will extinct in reserved area

Similarly if there is no migration of fish population from unreserved area to reserved area i.e.

$$(\sigma_1 = 0) \text{ and } s - \sigma_2 < 0 \text{ then } \frac{dy}{dt} < 0 \tag{3}$$

Then, prey species will extinct from unreserved area

To protect the prey (fishes) species from extinction, migration of prey species from both the patches is necessary.

Throughout our analysis, we assume that

$$r - \sigma_1 - q_1 E_1 > 0 \text{ and } s - \sigma_2 > 0$$

Now we take $m_1 = m_2 = m$ and $K_1 = K_2$ and $\alpha_1 = K_1 m$

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1 - \frac{x}{K}\right) - \sigma_1 x + \sigma_2 y - \frac{mxz}{\alpha + x} - q_1 E_1 x \\ \frac{dy}{dt} &= sy\left(1 - \frac{y}{L}\right) + \sigma_1 x - \sigma_2 y - \frac{myz}{\alpha + y} \\ \frac{dz}{dt} &= -dz + \alpha_1\left(\frac{xz}{\alpha + x} + \frac{yz}{\alpha + y}\right) - q_2 E_2 z \end{aligned} \tag{4}$$

Lemma 1. All the solutions of the system (4) which initiate in R_3^+ are uniformly bounded

Proof. Let $W = x + y + \frac{m}{\alpha_1} z$ and η> 0,be a constant. Then

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$$\frac{dw}{dt}+\eta W=\left[rx\left(1-\frac{x}{K}\right)-\sigma_1 x+\sigma_2 y-\frac{mxz}{\alpha+x}-q_1E_1x+\eta x+sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y-\frac{myz}{\alpha+y}+\eta y\right]$$

$$-\frac{m}{\alpha_1}d\,z+\frac{mxz}{\alpha+x}+\frac{myz}{\alpha+y}-\frac{q_2E_2mz}{\alpha_1}+\eta\frac{m}{\alpha_1}z$$

$$\leq-\frac{r}{K}\left[x-\frac{K}{2r}(r+\eta-q_1E_1)\right]^2+\frac{r}{K}.\frac{K^2}{4r^2}(r+\eta-q_1E_1)^2-\frac{s}{L}\left[y-\frac{L}{2s}(s+\eta)\right]^2+\frac{s}{L}.\frac{L^2}{4s^2}(s+\eta)^2$$
$$-\left[\frac{m}{\alpha_1}[\eta-(q_2E_2-d)]z\right]$$

Taking $\eta>q_2E_2-d$

$$\frac{dw}{dt}+\eta w\leq\frac{K}{4r}(r+\eta-q_1E_1)^2+\frac{L}{4s}(\eta+s)^2=N \text{ (say)}$$

By the theory of differential inequality we have

$$0\leq W\leq\frac{N}{\eta}(1-e^{-\eta t})+w_0e^{-\eta t} \quad (5)$$

as $t\to\infty, W\to\frac{N}{\eta}$

This proves the lemma.

III EXISTENCE OF EQUILIBRIA

Following are the possible equilibria

I. $P_0(0,0,0)$ which always exist

II. $P_1(x_1,y_1,0)$, where x_1 is theroot of

$$ax^3+bx^2+cx+d=0 \quad (6)$$

Where

$$a=\frac{sr^2}{L\sigma_2^2K},\quad b=\frac{-2rs(r-\sigma_1-q_1E_1)}{L\sigma_2^2K}$$

$$c=\frac{s(r-\sigma_1-q_1E_1)^2}{L\sigma_2^2}-\frac{(s-\sigma_2)r}{K\sigma_2}$$

$$d=\frac{(s-\sigma_2)(r-\sigma_1-q_1E_1)}{\sigma_2}-\sigma_1$$

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By Descartes rule of sign the above equation has unique positive solution if the following inequalities hold.

$$\frac{s(r-\sigma_1-q_1E_1)^2}{L\sigma_2}<\frac{(s-\sigma_2)r}{K} \tag{7}$$

$$(s-\sigma_2)(r-\sigma_1-q_1E_1)<\sigma_1\sigma_2$$

y is given by

$$y=\frac{1}{\sigma_2}\left[\frac{rx^2}{K}-(r-\sigma_1-q_1E_1)x\right] \tag{8}$$

Which exists provided $x_1^*>\frac{K}{r}(r-\sigma_1-q_1E_1)$

It is observed that existence of $P_1(x_1,y_1,0)$ does not depend on harvesting of predator population but on prey harvesting in unreserved area.

III. Non-zero point $P_2(x_2,y_2,z_2)$

Value of *x* is is given by

IV.

$$ax^4+bx^3+cx^2+dx+e=0 \tag{9}$$

Where

$$a=\left(\frac{dr}{K}+\frac{q_2E_2r}{K}\right)(2\alpha_1-d-q_2E_2),$$

$$b=[(2\alpha_1-d-q_2E_2)\left(\frac{(q_2E_2-d)r}{K}+(q_1E_1+\sigma_1-r)q_2E_2\right)+$$
$$\left(\frac{dr}{K}+\frac{q_2E_2}{K}\right)(\alpha_1-d-q_2E_2)\alpha];$$

$$c=[(2\alpha_1-d-q_2E_2)(q_2E_2(-q_1E_1-\sigma_1+r)+(d-\alpha_1)\alpha\sigma_2)+$$
$$(\alpha_1-d-q_2E_2)\alpha\left(-\frac{(d+q_2E_2)r}{K}+q_2E_2(q_1E_1+\sigma_1-r)\right)];$$

$$d=[(2\alpha_1-d-q_2E_2)(d\alpha^2\sigma_2+\alpha^2q_2E_2+\sigma_2\alpha)$$
$$+\alpha(\alpha_1-d-q_2E_2)((-q_1E_1-\sigma_1+r)q_2E_2+(d-\alpha_1)\alpha\sigma_2)];$$

$$e=[(d\alpha^2\sigma_2+\alpha^2q_2E_2)(\alpha_1-d-q_2E_2)\alpha-(d+q_2E_2)\sigma_2\alpha^2]$$

If $\alpha_1 < d + q_2E_2 < 2\alpha_1$ then $a > 0$ and $e < 0$

Product of 4 roots is negative. Which insures the existence of at least one positive root x of equation (9)

Value of *y* and *z* is given by

$$y = \frac{1}{\sigma_2}\left[q_1E_1 + \sigma_1 + \frac{mz}{\alpha + x} - r\left(1 - \frac{x}{K}\right)\right]x \tag{10}$$

$$z = \frac{\alpha_1}{m(\mathrm{d} + q_2E_2)}\left[rx\left(1 - \frac{x}{K}\right) + sy\left(1 - \frac{y}{L}\right) - q_1E_1x\right] \tag{11}$$

y and *z* are positive provided $x < K(1 - q_1E_1)$ & $y < L$ (12)

Substituting the value of *z* in (10), we will get *y* in the form of *x*.

$$\Rightarrow y = \frac{1}{\sigma_2}\left[Lx + (1/(\alpha M + x(\alpha_1 + M)))\left[-\alpha_1x(\alpha + x)L - (Mx - d\alpha - \alpha q_2E_2)(\alpha\sigma_2 + Lx)\right]\right]$$

$$L = q_1E_1 + \sigma_1 - r + \frac{rx}{K}$$

$$M = \alpha_1 - d - q_2E_2$$

Substituting the value of *x* and *y* in we shall get value of *z*

IV STABILITY ANALYSIS

The stability of the various equilibrium state is given as

Jacobian Matrix is given by

$$J = \begin{bmatrix} r - \frac{2rx}{K} - \sigma_1 - q_1E_1 - \frac{mz\alpha}{(\alpha + x)^2} & \sigma_2 & -\frac{mx}{\alpha + x} \\ \sigma_1 & s - \frac{2sy}{L} - \sigma_2 - \frac{mz\alpha}{(\alpha + y)^2} & -\frac{my}{\alpha + y} \\ \frac{\alpha_1 z\alpha}{(\alpha + x)^2} & \frac{\alpha_1\alpha z}{(\alpha + y)^2} & -d + \frac{\alpha_1 x}{\alpha + x} + \frac{\alpha_1 y}{\alpha + y} - q_2E_2 \end{bmatrix}$$

Theorem 2 Equilibrium point $P_0(0,0,0)$ is always unstable

Proof:

$$J_0 = \begin{bmatrix} r-\sigma_1-q_1E_1 & \sigma_2 & 0 \\ \sigma_1 & s-\sigma_2 & 0 \\ 0 & 0 & -d-q_2E_2 \end{bmatrix}$$

As the one of the eigen value

$$\lambda_1 = -d - q_2E_2 < 0$$

Other two eigen values are given by

$$\lambda^2 - \{(r-\sigma_1-q_1E_1)+(s-\sigma_2)\}\lambda + (r-\sigma_1-q_1E_1)(s-\sigma_2) - \sigma_1\sigma_2 = 0$$

As $(r-\sigma_1-q_1E_1)+(s-\sigma_2) > 0$ (Assumptions)

So all the eigen values of above characteristics equation are not negative.

Therefore equilibrium point $P_0(0,0,0)$ is always unstable.

Hence the theorem.

Remark: It is concluded that even if the system is exploited continuously in the unreserved zone, the prey or the predator population persists and are not extinct for sufficiently large time.

Theorem 2. Equilibrium point $P_1(x_1, y_1, 0)$ is always asymptotically locally stable

$$2\alpha < d + q_2E_2;$$

$$\text{provided}\left(r-\frac{2rx}{K}-\sigma_1-q_1E_1\right)+\left(s-\frac{2sy}{L}-\sigma_2\right)<0;$$

$$\left(r-\frac{2rx}{K}-\sigma_1-q_1E_1\right)\left(s-\frac{2sy}{L}-\sigma_2\right)>\sigma_1\sigma_2,$$

Proof:

$$J_1(x,y,0) = \begin{bmatrix} r-\frac{2rx}{K}-\sigma_1-q_1E_1 & \sigma_2 & -\frac{mx}{\alpha+x} \\ \sigma_1 & s-\frac{2sy}{L}-\sigma_2 & -\frac{my}{\alpha+y} \\ 0 & 0 & -d-q_2E_2+\frac{\alpha_1 x}{\alpha+x}+\frac{\alpha_1 y}{\alpha+y} \end{bmatrix}$$

One of the Eigenvalue of J_1 is

$$\lambda_1 = -d - q_2E_2 + \frac{\alpha_1 x}{\alpha + x} + \frac{\alpha_1 y}{\alpha + y}$$

It will be negative if

$$-d - q_2E_2 + \frac{\alpha_1 x}{\alpha + x} + \frac{\alpha_1 y}{\alpha + y} < 0$$

$$i.e \ \frac{\alpha_1 x}{\alpha + x} + \frac{\alpha_1 y}{\alpha + y} < d + q_2E_2$$

$$i.e \ 2\alpha < d + q_2E_2$$

Other two eigen values are given by

$$\begin{vmatrix} r - \frac{2rx}{K} - \sigma_1 - q_1E_1 - \lambda & \sigma_2 \\ \sigma_1 & s - \frac{2sy}{L} - \sigma_2 - \lambda \end{vmatrix} = 0$$

i.e

$$\lambda^2 - \left\{(r - \frac{2rx}{K} - \sigma_1 - q_1E_1) + (s - \frac{2sy}{L} - \sigma_2)\right\}\lambda + (r - \frac{2rx}{K} - \sigma_1 - q_1E_1)(s - \frac{2sy}{L} - \sigma_2) - \sigma_1\sigma_2 = 0$$

Its two eigens values are negative if

$$(r - \frac{2rx}{K} - \sigma_1 - q_1E_1) + (s - \frac{2sy}{L} - \sigma_2) < 0$$

$$(r - \frac{2rx}{K} - \sigma_1 - q_1E_1)(s - \frac{2sy}{L} - \sigma_2) > \sigma_1\sigma_2$$

Thus $P_1(x_1, y_1, 0)$ will be stable provided following conditions hold.

$$2\alpha < d + q_2E_2; \ (r - \frac{2rx}{K} - \sigma_1 - q_1E_1) + (s - \frac{2sy}{L} - \sigma_2) < 0; (r - \frac{2rx}{K} - \sigma_1 - q_1E_1)(s - \frac{2sy}{L} - \sigma_2) > \sigma_1\sigma_2$$

Remark: It is concluded that predator harvesting and prey harvesting in unreserved zone will helpful in maintaining the stability of $P_1(x_1, y_1, 0)$.

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III. Non-zero point $P_2(x_2, y_2, z_2)$

Variational matrix is

$$J_2(x,y,z)=\begin{bmatrix} r-\frac{2r}{K}x-\sigma_1-q_1E_1-\frac{mz\alpha}{(\alpha+x)^2} & \sigma_2 & \frac{-mx}{\alpha+x} \\ \sigma_1 & s-\frac{2sy}{L}-\sigma_2-\frac{mz\alpha}{(\alpha+y)^2} & \frac{-my}{\alpha+y} \\ \frac{\alpha\alpha_1 z}{(\alpha+x)^2} & \frac{\alpha\alpha_1 z}{(\alpha+y)^2} & -d+\frac{\alpha_1 x}{\alpha+x}+\frac{\alpha_1 y}{\alpha+y}-q_2E_2 \end{bmatrix}$$

i.e

$$J_2(x,y,z)=\begin{bmatrix} -\frac{r}{K}x-\sigma_2\frac{y}{x}-\frac{mz\alpha}{(\alpha+x)^2}+\frac{mz}{\alpha+x}-\lambda & \sigma_2 & \frac{-mx}{\alpha+x} \\ \sigma_1 & -\sigma_1\frac{x}{y}-\frac{sy}{L}-\frac{mz\alpha}{(\alpha+y)^2}+\frac{mz}{(\alpha+y)}-\lambda & \frac{-my}{\alpha+y} \\ \frac{\alpha\alpha_1 z}{(\alpha+x)^2} & \frac{\alpha\alpha_1 z}{(\alpha+y)^2} & 0-\lambda \end{bmatrix}$$

Its character equation isgiven by

$$\lambda^3+c_1\lambda^2+c_2\lambda+c_3=0$$

Where

$$C_1=\frac{sy}{L}+\frac{\sigma_1 x}{y}-\frac{mz}{\alpha+y}+\frac{mz\alpha}{(\alpha+y)^2}+\frac{rx}{K}+\frac{\sigma_2 y}{x}-\frac{mz}{\alpha+x}+\frac{mz\alpha}{(\alpha+x)^2}$$

$$C_2=\frac{m\alpha\alpha_1 yz}{(\alpha+y)^3}-\sigma_2\sigma_1+\frac{m\alpha\alpha_1 xz}{(\alpha+x)^3}+\left(\frac{rx}{K}+\frac{\sigma_1 y}{x}-\frac{mz}{\alpha+x}+\frac{mz\alpha}{(\alpha+x)^2})\right)\left(\frac{sy}{L}+\frac{\sigma_1 x}{y}-\frac{mz}{\alpha+y}+\frac{mz\alpha}{(\alpha+y)^2}\right)$$

$$C_3=\frac{\sigma_2 m\alpha\alpha_1 yz}{(\alpha+y)(\alpha+x)^2}+\frac{m\alpha\alpha_1\sigma_1 xz}{(\alpha+x)(\alpha+y)^2}+\frac{m\alpha\alpha_1 xz}{(\alpha+x)^3}\left(\frac{sy}{L}+\frac{\sigma_1 x}{y}-\frac{mz}{\alpha+y}+\frac{mz\alpha}{(\alpha+y)^2}\right)$$
$$+\frac{m\alpha\alpha_1 yz}{(\alpha+y)^3}\left(\frac{rx}{K}+\frac{\sigma_2 y}{x}+\frac{mz\alpha}{(\alpha+x)^2}-\frac{mz}{\alpha+x})\right)$$

by the Routh Hurwitz Criteria, equilibrium point $P_2(x_2, y_2, z_2)$ is locally asymptotically stable provided

$C_1 > 0,\ \ C_3 > 0 \,\&\, C_1C_2 - C_3 > 0$ at this equilibrium point.

V CONCLUSIONS

In this paper, we have analyzed a prey-predator fishery model with preydispersal in a two-patch environment, one is assumed to be a free fishing zoneand the other is a reserved zone where fishing and other extractive activities areprohibited. We have discussed the local stability of the system and various thresholds are given for the stability of various equilibria.It is observed that proper harvesting of prey (in unreserved area) and predator species will helpful in co-existence of all the species. Appropriate migration of prey species from unreserved to reserve and vice versa is necessary to save system from extinction.

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