International Journal of Advanced Technology in Engineering and Science
Vol. No.5, Issue No. 04, April 2017
www.ijates.com
ijates
ISSN 2348 - 7550

MIXED CONSTRAINT FUZZY TRANSPORTATION PROBLEM

Nidhi Joshi

PTU, Jalandhar, Research Scholar, College: Chandigarh Group of Colleges, Mohali, Punjab (India)

ABSTRACT

In present paper, authors have proposed a new algorithm for obtaining the best feasible solution of mixed constraint fuzzy transportation problem. The method is illustrated using a numerical example. Most of the transportation problems in real life have mixed constraints. These problems cannot be truly solved using general methods. For solving such mixed constraint fuzzy transportation problems proposed algorithm can be used.

MATHEMATICS SUBJECT CLASSIFICATION:

90C08, 90C90

Keywords: Fuzzy transportation problem; mixed constraints; Ranking function; Triangular fuzzy number.

I. INTRODUCTION

The transportation problem was initially developed by Hitchcock [1] in 1941. In conventional transportation problems it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh [2] may represent this data. So, fuzzy decision making method is needed here. Fuzzy transportation problem (FTP) is the problem of minimizing fuzzy valued objective functions with fuzzy source and fuzzy destination parameters. The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problem. At the same time in real life, most of the problems have mixed constraints but one use the FTPs for optimal solutions with equality constraints. However, the FTPs with mixed constraints are not addressed much in the literature. A literature review about mixed constraint FTPs shows no efficient method for finding its optimal solution. The More-for-less (MFL) paradox in a FTP occurs when it is possible to ship more „total goods‟ for less (or equal) "total fuzzy cost‟ while shipping the same amount or more from each origin and to each destination, keeping all shipping fuzzy costs non-negative. The mixed constraints transportation problems under crisp data have comprehensively been studied by many researchers in the past [3], [4], [5]. In 1974, Bridgen [6] considered the transportation problem with mixed constraints. H. Isermann [7] studied transportation problem with mixed constraint and develop its solution technique in 1979. Gupta et al. [8] and Arsham [9] in 1992 obtained the more-for-less solution, for the TPs with mixed constraints by relaxing the constraints and introducing new slack variables. Adalkha et al. [10] in 2007 developed a simple heuristic algorithm to identify the demand destinations and the supply points to ship more for less in fixed charge transportation problem. Pandian and Natrajan [11] have developed fourier method for solving transportation problems with mixed constraints in 2010. In 2012, Joshi and Gupta [12] discussed the identification of more-for-less paradox in the linear fractional transportation problem using objective matrix. Pandian and Anuradha [13] have introduced path method for finding a MFL optimal solution to Transportation problem in 2013. In 2014 Osuji George et al. [14] discussed an efficient statistical algorithm for computing paradox in linear transportation problem if paradox does exist. Kavitha and Anuradha [15] in 2015 proposed a new algorithm for finding the cost sensitivity analysis which determines the interval of perturbation to keep the current more-for less (MFL) optimal solution to the transportation problem remaining optimal.

In the present paper authors have attempted to find the best feasible solution for fuzzy transportation problem with mixed constraints using Improved VAM method. In literature maximum work is done for the transportation problem with mixed constraints under crisp data but in real life problem there is existence of only fuzzy data. In this paper, we are using Triangular Fuzzy numbers and for the comparison of triangular fuzzy numbers we are using the ranking function.

The present paper is organized as follows: In Section-2 basic definitions, arithmetic operations, properties of triangular fuzzy numbers and ranking function are reviewed. Section-3 is Mathematical formulation of fuzzy transportation problem with mixed constraints. In Section-4 the Proposed Algorithm is provided. In Section-5 Numerical example is solved explaining the algorithm. Section-6, presents the significance and conclusion of the present study. In section -7 future works is given.

II. PRELIMINARIES

In this section, basic definitions, arithmetic operations, properties of triangular fuzzy numbers and ranking function are presented.

2.1 Fuzzy set

A Fuzzy $\tilde{A}$ is defined by $\tilde{A} = \{ (x, \mu\tilde{A}(x)): x \in A, \mu\tilde{A}(x) \in [0,1]\}$. In the pair $(x,\mu\tilde{A}(x))$,the first elements x belongs to the classical set A, the second element $\mu\tilde{A}(x)$,belongs to the interval [0,1], called membership function.

2.2 Fuzzy number

A fuzzy set $\tilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number

i. $\tilde{A}$ must be a normal fuzzy set;

ii. ${}^{\alpha}\tilde{A}$ must be closed interval for every $\alpha \in [0, 1]$;

iii. The support of $\tilde{A}$, ${}^{0+}\tilde{A}$, must be bounded.

2.3 Triangular Fuzzy Number

$\tilde{A} = (a_1, a_2, a_3)$, is interpreted as membership function of triangular fuzzy number and holds the following conditions:

(i) a_1 to a_2 is increasing function.

(ii) a_2 to a_3 is decreasing function.

(iii) $a_1 \leq a_2 \leq a_3.$

ijates
ISSN 2348 - 7550

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & a_1 \le x \le a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases}$$

2.3.1 Properties and Arithmetic Operations of Triangular Fuzzy Numbers

i. Two triangular fuzzy numbers $\tilde{A}_1 = (m_1, n_1, p_1)$ and $\tilde{A}_2 = (m_2, n_2, p_2)$ are said to be equal Iff $m_1= m_2$, $n_1= n_2$, $p_1 =p_2$.

ii. If $\tilde{A} = (m_1, n_1, p_1)$ and $\tilde{B} = (m_2, n_2, p_2)$ two triangular fuzzy numbers then the arithmetic operations on $\tilde{A}$ and $\tilde{B}$ are as follows:

Addition $\tilde{A} + \tilde{B} = (m_1+m_2, n_1+n_2, p_1+p_2)$

Subtraction $\tilde{A} - \tilde{B} = (m_1-m_2, n_1-n_2, p_1-p_2)$

Multiplication $\tilde{A} * \tilde{B}$= (min $(m_1m_2, m_1p_2, p_1m_2, p_1p_2)$, n_1n_2, max $(m_1m_2, m_1p_2, p_1m_2, p_1p_2)$).

2.4 Ranking Function

A ranking function is defined as

$$\mathfrak{R}: F(R) \rightarrow R$$

where $F(R)$ is set of fuzzy numbers defined on real numbers mapping each fuzzy number to real number.

III. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS

Let p be the number of sources and q the number of destinations. Suppose that the cost of transporting one unit of the commodity from source i to the destination j is $\tilde{c}_{ij}$. Let $\tilde{a}_i$ be the quantity of the commodity available at source i and $\tilde{b}_j$ be the quantity required at destination j. Thus, $\tilde{a}_i \geq 0$ and $\tilde{b}_j \geq 0$ for all i and j.

If $\tilde{x}_{ij}$ is the quantity transported from source i to destination j then the transportation problem is written as

$$\text{Minimize} \quad Z= \sum_{i=1}^{p}\sum_{j=1}^{q}\tilde{c}_{ij}\tilde{x}_{ij}$$

Subject to

$$\sum_{j=1}^{q} x_{ij} \le | = | \ge \tilde{a}_i, \qquad \forall\ i= 1,2,3\ldots\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} x_{ij} \le | = | \ge \tilde{b}_j, \qquad \forall\ j= 1,2,3\ldots\ldots\ldots\ldots q$$

$$\tilde{x}_{ij} \ge 0$$

The above formulation represents a Linear Programming Problem (LPP) with p x q variables and $p + q$ constraints.

Remark 1: If all constraints are of equal (=) sign, then the problem becomes the transportation problem with equality constraints.

IV. ALGORITHM

Step 1: Balance the given transportation problem if either (total supply>total demand) or (total supply<total demand).

Step 2: Obtain the TOC matrix.

Step 3: Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell cost in the same row or column.

Step 4: Select the rows or columns with the highest three penalty costs.

Step 5: Compute three transportation costs for selected three rows or columns in step 4 by assigning as many units as possible to the lowest cost square in the row or column selected.(If the assignment unit contains $\lesssim$ sign, then assign as lowest unit as possible. If the assignment unit is of $\gtrsim$ sign, then assign the possible maximum value.)

We will follow the following Table 1 to assign the supply and demand unit.

Table 1: Chart to assign Supply and Demand units

SUPPLY ($\tilde{a}_i$)	DEMAND ($\tilde{b}_j$)	ASSIGNMENT
=	=	Min ($\tilde{a}_i, \tilde{b}_j$)
=	$\lesssim$	Min ($\tilde{a}_i, \tilde{b}_j$)
=	$\gtrsim$	$\tilde{a}_i$
$\lesssim$	$\lesssim$	0
$\lesssim$	=	Min ($\tilde{a}_i, \tilde{b}_j$)
$\lesssim$	$\gtrsim$	$\tilde{a}_i$
$\gtrsim$	$\gtrsim$	Min ($\tilde{a}_i, \tilde{b}_j$)
$\gtrsim$	=	$\tilde{b}_j$
$\gtrsim$	$\lesssim$	$\tilde{b}_j$

Step 6: Select minimum transportation cost of three allocations in step 5(breaking ties arbitrarily or choosing the lowest-cost cell).

Step 7: Eliminate the row or column that has just been completely satisfied by the assignment just made.

Step 8: Repeat step 3-6 until all requirements have been meet.

Step 9: Compute total transportation cost for the feasible allocations using the original balanced-transportation cost matrix.

V. NUMERICAL EXAMPLE

Consider the following fuzzy transportation problem given below. The A clothing group owns factories in three towns that distribute to four dress shops (P, Q, R, S) as shown in Table 2.

Table 2: Tableau representation of Numerical problem

	P	Q	R	S	Supply
Town 1	(9, 12, 15)	(1, 4, 7)	(6, 9, 12)	(2, 5, 8)	= (50, 55, 60)
Town 2	(5, 8, 11)	(0, 1, 2)	(3, 6, 9)	(3, 6, 9)	$\gtrsim$ (35, 40, 45)
Town 3	(0, 1, 2)	(1, 2, 3)	(1, 4, 7)	(4, 7, 10)	$\lesssim$(25, 30, 35)
Demand	= (35, 40, 45)	= (15, 20, 25)	$\lesssim$ (45, 45, 45)	$\lesssim$ (15, 20, 25)	

Using step 2 the TOC matrix obtained from Table 2 is shown in Table 3 given below

Table 3: Tableau representation of TOC Matrix

	P	Q	R	S	Supply
Town 1	(17, 19, 21)	(1, 3, 5)	(10, 10, 10)	(1, 1, 1)	= (50, 55, 60)
Town 2	(10, 14, 18)	(0, 0, 0)	(5, 7, 9)	(4, 6, 8)	≥ (35, 40, 45)
Town 3	(0, 0, 0)	(2, 2, 2)	(1, 3, 5)	(6, 8, 10)	≤(25, 30, 35)
Demand	= (35, 40, 45)	= (15, 20, 25)	≤ (45, 45, 45)	≤ (15, 20, 25)	

Using Table 3 the first allocation made with the minimum transportation cost of three allocations (using step 3-6) we obtain Table 4 given below

Table 4: Tableau representation of the first allocation using proposed algorithm

(17, 19, 21)	(1, 3, 5)	(10, 10, 10)	(1, 1, 1)	= (50, 55, 60)
(10, 14, 18)	(0, 0, 0)	(5, 7, 9)	(4, 6, 8)	≥ (35, 40, 45)
(25, 30, 35) (0, 0, 0)	(2, 2, 2)	(1, 3, 5)	(6, 8, 10)	≤(25, 30, 35)
= (35, 40, 45)	= (15, 20, 25)	≤ (45, 45, 45)	≤ (15, 20, 25)	

Now eliminating the third row the second allocation is made similar to that.

Similarly all other allocations can be made using the proposed algorithm and the final table obtained is shown in Table 5 below.

Table 5: Tableau representation of best feasible solution

	P	Q	R	S	Supply
Town 1	(9, 12, 15)	(1, 4, 7)	(35, 35, 35) (6, 9, 12)	(15, 20, 25) (2, 5, 8)	= (50, 55, 60)
Town 2	(10, 10,10) (5, 8, 11)	(15, 20, 25) (0, 1, 2)	(10, 10, 10) (3, 6, 9)	(3, 6, 9)	≥ (35, 40, 45)
Town 3	(25, 30, 35) (0, 1, 2)	(1, 2, 3)	(1, 4, 7)	(4, 7, 10)	≤(25, 30, 35)
Demand	*= (35, 40, 45)*	*= (15, 20, 25)*	≤ (45, 45, 45)	≤ (15, 20, 25)	

After applying proposed algorithm,

X_{13} = (210, 315, 420), X_{14} = (30, 100, 200), X_{21} = (50, 80, 110), X_{22} = (0, 20, 50), X_{23} = (30, 60, 90), X_{31} = (0, 30, 70) and the best feasible solution obtained for this problem is X_0 = (320, 605, 940).

VI. SIGNIFICANCE & CONCLUSION

1) This is a new method for solving transportation problem of More-For-Less (MFL) solution with mixed constraints.
2) The algorithm does not require any deep knowledge and understanding of complex concepts like linear programming or goal and parametric programming, etc.

VII. FUTURE WORK

1) In present paper authors have obtained the best feasible solution but this problem can be extended to obtain the optimal solution for mixed constraint fuzzy transportation problem.
2) Algorithm can be converted to a computer program for verifying the results and to make the calculation work easy and effective.

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