

Higher Order Squeezing in Stimulated Mode in Seven Wave Mixing

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ABSTRACT

Higher order squeezing in stimulated mode in seven-wave mixing optical processes has been studied. It is found to be dependent on coupling constant “g” and phase values of the field amplitude with photon number of the fundamental as well as harmonic mode under short time approximation.

Keywords: Higher order squeezing, Nonclassicality, Multiwave mixing.

1. INTRODUCTION

Squeezed states of electromagnetic field correspond to a non-classical state [1, 2] in which reduction of quantum fluctuation below the coherent state level takes place. A large number of methods have been proposed in literature to generate and detect squeezed states theoretically and experimentally [3-10]. Due to low noise property of squeezed states potential applications of squeezed light have been reported in recent past in the processing of quantum information, for example, for quantum teleportation [11-13], quantum cryptography [14] and quantum information networking [15].

In the present work, we have reported the generation of higher order squeezing state in stimulated mode in seven-wave mixing optical process.

2. HIGHER ORDER SQUEEZING

Higher order squeezing is defined in various ways. Hong and Mandel [16, 17] and Hillery [1] have introduced the notion of higher order squeezing of quantized electromagnetic field as generalization of normal squeezing. Amplitude-cubed squeezing is defined in terms of operators Z_1 and Z_2 as

$$Z_1 = \frac{1}{2}(A^3 + A^{\dagger 3}) \quad \text{and} \quad Z_2 = \frac{i}{2}(A^3 - A^{\dagger 3}) \quad (1)$$

Where Z_1 and Z_2 are the real and imaginary parts of the cube of field amplitude, respectively. A and A^\dagger are slowly varying operators defined by $A = ae^{i\omega t}$ and $A^\dagger = a^\dagger e^{-i\omega t}$.

The operators Z_1 and Z_2 obey the commutation relation

$$[Z_1, Z_2] = i / 4 (9N_A^2 + 9N_A + 6) \quad (2)$$

This leads to the uncertainty relation

$$\Delta Z_1 \Delta Z_2 \geq 1 / 4 \left\langle (9N_A^2 + 9N_A + 6) \right\rangle \quad (3)$$

where N_A is the usual number operator.

Amplitude-cubed squeezing is said to exist in Z_1 variable if

$$(\Delta Z_1)^2 < 1 / 4 \left\langle 9N_A^2 + 9N_A + 6 \right\rangle \quad (4)$$

Or the squeezing f is

$$f = (\Delta Z_1)^2 - 1 / 4 \left\langle 9N_A^2 + 9N_A + 6 \right\rangle < 0 \quad (5)$$

3. SEVEN-WAVEMIXINGPROCESS

A multiwave mixing process can be viewed in the optics as a process involving multi photon interaction. In this process, the interaction is looked upon as a process which involves the absorption of two pump photons, each having frequency ω_1 and emission of two probe photons of frequency ω_2 and signal photons of frequency ω_3 where

$$2\omega_1 = 2\omega_2 + \omega_3$$

The Hamiltonian for this process is given as follows ($\hbar=1$)

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \omega_3 c^\dagger c + g \left(a^2 b^\dagger c^\dagger + a^\dagger b^2 c^3 \right) \quad (6)$$

in which g is a coupling constant. $A = a \exp(i\omega_1 t)$, $B = b \exp(i\omega_2 t)$ and $C = c \exp(i\omega_3 t)$ are the slowly varying operators at frequencies ω_1 , ω_2 and ω_3 , a (a^\dagger), b (b^\dagger) and c (c^\dagger) are the usual annihilation (creation) operators, respectively. The Heisenberg equation of motion for fundamental mode A is given as

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + i [H, A] \quad (7)$$

By using the short-time approximation technique, we expand $A(t)$ by using Taylor's series expansion and retaining the terms up to $g^2 t^2$ as

$$A(t) = A - 2igtA^\dagger B^2 C^3 + g^2 t^2 \left[2AN_B^2 N_C^3 - 9A^\dagger A^2 N_B^2 N_C^2 - 18A^\dagger A^2 N_B^2 N_C - 6A^\dagger A^2 N_B^2 - 4A^\dagger A^2 N_B N_C^3 - 36A^\dagger A^2 N_B N_C^2 - 72A^\dagger A^2 N_B N_C - 24A^\dagger A^2 N_B - 2A^\dagger A^2 N_C^3 - 18A^\dagger A^2 N_C^2 - 36A^\dagger A^2 N_C - 12A^\dagger A^2 \right] \quad (8)$$

Where $N_A = A^\dagger A$, $N_B = B^\dagger B$ and $N_C = C^\dagger C$.

Using Equations (8) number of photons in mode A may be expressed as

$$N_{1A}(t) = A^\dagger(t)A(t) = A^\dagger A - 12g^2 t^2 A^\dagger A^2 (B^\dagger B^2 + 4B^\dagger B + 2) \tag{9}$$

$$N_{1A}^2(t) = N_{1A}(t)N_{1A}(t) = A^\dagger A^2 + A^\dagger A - 12g^2 t^2 (2A^\dagger A^3 + 4A^\dagger A^2) (B^\dagger B^2 + 4B^\dagger B + 2) \tag{10}$$

3.1 STIMULATED THIRD ORDER AMPLITUDE SQUEEZING

The real quadrature component for third order squeezing in fundamental mode is defined as

$$Y_{1A}(t) = \frac{1}{2} [A^3(t) + A^{\dagger 3}(t)]$$

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode A and harmonic mode for B and vacuum state for mode C i.e.

$$|\psi\rangle = |\alpha\rangle_A |\beta\rangle_B |0\rangle_C \tag{11}$$

Using Equations (8) and (11) the third-order amplitude of the fundamental mode is expressed as

$$A^3(t) = A^3 - 6g^2 t^2 (3A^\dagger A^4 + 3A^3) (B^\dagger B^2 + 4B^\dagger B + 2) \tag{12}$$

$$Y_{1A}(t) = \frac{1}{2} [A^3(t) + A^{\dagger 3}(t)] = \frac{1}{2} [A^3 + A^{\dagger 3} - 6g^2 t^2 (3A^\dagger A^4 + 3A^3 + 3A^{\dagger 4} A + 3A^{\dagger 3}) (B^\dagger B^2 + 4B^\dagger B + 2)] \tag{13}$$

Using Equations (11) and (13), we get the expectation values as

$$\langle \psi | Y_{1A}^2(t) | \psi \rangle = \frac{1}{4} [\alpha^6 + \alpha^{*6} + 2|\alpha|^6 + 9|\alpha|^4 + 18|\alpha|^2 + 6 - 6g^2 t^2 (15\alpha^6 + 15\alpha^{*6} + 6\alpha^6 |\alpha|^2 + 6\alpha^{*6} |\alpha|^2 + 12|\alpha|^8 + 84|\alpha|^6 + 270|\alpha|^4 + 252|\alpha|^2 + 36) (|\beta|^4 + 4|\beta|^2 + 2)] \tag{14}$$

and

$$\langle \psi | Y_{1A}(t) | \psi \rangle^2 = \frac{1}{4} [\alpha^6 + \alpha^{*6} + 2|\alpha|^6 - 12g^2 t^2 (3\alpha^6 + 3\alpha^{*6} + 3\alpha^6 |\alpha|^2 + 3\alpha^{*6} |\alpha|^2 + 6|\alpha|^8 + 6|\alpha|^6) (|\beta|^4 + 4|\beta|^2 + 2)] \tag{15}$$

Therefore

$$\left[\Delta Y_{1A}(t) \right]^2 = \frac{1}{4} \left[9|\alpha|^4 + 18|\alpha|^2 + 6 - 6g^2t^2(9\alpha^6 + 9\alpha^{*6} + 72|\alpha|^6 + 270|\alpha|^4 + 252|\alpha|^2 + 36) \left(|\beta|^4 + 4|\beta|^2 + 2 \right) \right] \quad (16)$$

Using Equations (9), (10) and (16) a straightforward but strenuous calculation yields

$$\left[\Delta Y_{1A}(t) \right]^2 - \frac{1}{4} \langle 9N_{1A}^2(t) + 9N_{1A}(t) + 6 \rangle = -27g^2t^2 \left[|\alpha|^6 (\cos 6\theta + 2) + 10|\alpha|^4 + 14|\alpha|^2 + 2 \right] \left(|\beta|^4 + 4|\beta|^2 + 2 \right) \quad (17)$$

The right hand side of Equation (17) is negative, indicating that squeezing occurs in cube of field amplitude in the fundamental mode in seven wave mixing.

4. RESULT

The results show the presence of squeezing in third order of field amplitude in seven wave mixing in stimulated mode. Taking $|gt|^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing, the variations of S_Y is shown in Figure 1.

Degree of squeezing is shown as a function of $|\alpha|^2$. It is clear from Figure 1 that squeezing increase nonlinearly with $|\alpha|^2$ and $|\beta|^2$. Thus we can conclude that the degree of squeezing directly depends upon the photon number of the fundamental mode as well as on the harmonic mode.

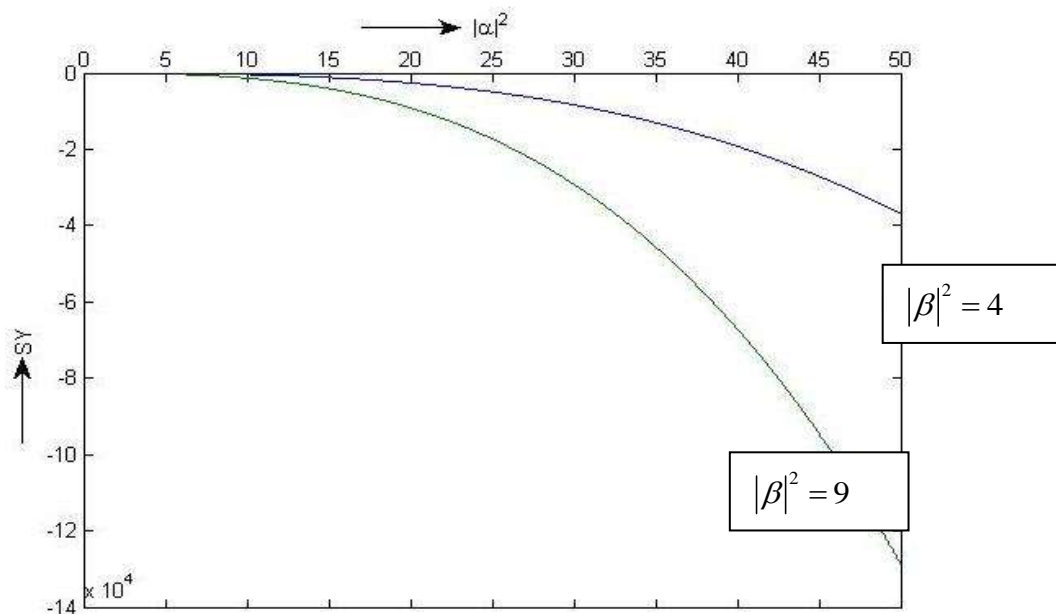


Figure.1. Dependence of stimulated third order amplitudesqueezing in 7wmxon $|\alpha|^2$.

5. CONCLUSION

It is shown that the selective phase values of field amplitude of fundamental mode lead to higher order squeezing up to third order in seven-wave mixing process. Again, from Equation (17) we can conclude that the degree of higher order squeezing present in the system can be tuned by varying the values of initial phase of the coherent state (θ), number of photons present in the radiation field prior to the interaction ($|\alpha|^2$) and the interaction time (t).

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