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FUZZY INVENTORY MODEL WITH LOGARITHMIC DEMAND RATES FOR EXPONENTIALLY DETERIORATING ITEMS

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ABSTRACT

The objective of this model is to discuss the inventory model for both the crisp and fuzzy environments where exponential deteriorating items for Logarithmic demand rate are used. Mathematical model has been developed for determining the optimal order quantity, the optimal cycle time and optimal total inventory cost in fuzzy environment. For defuzzification, graded unit preference integration method is used. Numerical examples are given to validate the proposed model.

Keywords: Fuzzy Inventory system, Logarithmic Demand, Deterioration.

AMS CLASSIFICATION NO: $90B05$

1.0. INTRODUCTION

The effect of deterioration of physical goods can't be neglected in any inventory system because almost all the physical goods deteriorate over time. A number of mathematical models have been presented for these deteriorating items.

Inventory model for single period with deterministic demand have been considered in the past. Variation in the demand rate plays an important role in the inventory management. Therefore, decisions of inventory are to be made because of the present and future demands. The constant demand is valid, only when the phase of the product life cycle is matured and also for finite periods of time. Ghare and Scharder [5] developed an EOQ model with an exponential decay and a deterministic demand. Wee[11] developed EOQ models to allow deterioration and an exponential demand pattern. The assumption of the constant deterioration rate was relaxed by Covert and Philip[3], who used a two-parameter Weibull distribution to represent the distribution of time to deterioration. Dave and Patel [4] were the first to depart from the restrictive assumption of constant demand over an infinite planning horizon. Sachan[8] extended the model of Dave and Patel [4] to allow for shortage after correcting some of its approximation errors. Sharma and Kumar [9] presented an inventory model for exponentially decaying inventory with known demand. In this model, demand is a function of selling price and rate of deterioration is a function of time.

In the crisp inventory models, all the parameters in the total cost are known and have definite values. But in the practical situation it is not possible. Hence fuzzy inventory models fulfill that gap. Different fuzzy inventory models occur due to various fuzzy cost parameters in the total cost. Researchers related to this area are: Bellman and Zadeh [1], Zimmermann [12], [13], Chen and Ouyang [2], Vijayan and Kumaran [10], Mahata and Goswami [6], [7], etc.

In the present paper, a Fuzzy economic order quantity model is developed for exponential deteriorating items for Logarithmic demand rate. The time horizon is classified into two intervals. In the 1st interval the given stock is decreased to zero level due to the combined effect of deterioration and demand and in the next interval the shortages are allowed. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment trapezoidal fuzzy number is utilized in developing the model. The main aim is to estimate the fuzzy optimal order quantity and fuzzy optimal total cost of the system under study. The parameters are fuzzified to the tapezoidal fuzzy numbers. The fuzzy model is defuzzified by using signed distance method. Numerical examples are discussed to illustrate the procedure of solving the model.

1.1. DEFINITIONS AND PRELIMINARIES

Definition 1.1.1: Fuzzy set

A fuzzy set $\tilde{A}$ in a universe of discourse x is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.

Definition **1.1.2:**(Convex Fuzzy Set) A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} \subseteq X$ is called convex fuzzy set if all $\tilde{A}_x$ are convex sets for every element $x_1, x_2 \in A_\alpha$ for every $\alpha \in [0,1]$ $\lambda\, x_1 + (1-\lambda)\, x_2 \in A_\alpha \; \forall \lambda \in [0,1]$ .Otherwise the fuzzy set is called non convex fuzzy set.

Definition 1.1.3:A fuzzy set $[a_\alpha, b_\alpha]$ where $0 \le \alpha \le 1$ and $a < b$ defined on *R*, is called a fuzzy interval if its membership function is

$$\mu_{[a_\alpha, b_\alpha]} = \begin{cases} \alpha, & a \le x \le b \\ 0, & Otherwise \end{cases}$$

Definition 1.1.4:A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-a}{b-a}, & a\le x\le b\\ 1, & b\le x\le c\\ \frac{d-x}{d-c}, & c\le x\le d\\ 0\ , & Otherwise\end{cases}$$

Definition 1.1.5: If $\tilde{A}=(a,b,c,d)$ is a trapezoidal fuzzy number then the signed distance of $\tilde{A}$ is defined as

$$d(\tilde{A},0)=\tfrac{1}{2}\int_0^1 (A_L(\alpha)+A_R(\alpha))\,d\alpha$$

where $A_\alpha=[A_L(\alpha),A_R(\alpha)]=[a+(b-a)\alpha,d-(d-c)\alpha],\alpha\in[0,1]$

is α –cut of fuzzy set $\tilde{A}$ which is a closed interval.

The model is developed on the basis of the following assumptions and notations.

2.0.ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

i. Logarithmic Demand rate is considered.

ii. Single inventory will be used.

iii. Lead time is zero.

iv. Shortages are allowed.

v. Replenishment rate is infinite but size is finite.

vi. Time horizon is finite.

vii. There is no repair of deteriorated items occurring during the cycle.

Following notations are made for the given model:

$I(t)$ = On hand inventory at time t.

$R(t)=\log(1+t)$ the demand rate at $t>0$.

The distribution of the time to deterioration of an item follows the exponential distribution $\theta(t)$

where $\theta(t)=\begin{cases}\theta\, e^{\theta t}\ , & for\ t>0\\ 0\ \ \ , & otherwise\end{cases}$,where θ $(0<\theta<1)$ is called as deterioration rate.

$I(0)=Q$=Inventory at time $t=0$.

$T=$ Duration of a cycle.

$p_c=$ The purchasing cost per unit item.

d_c = The deterioration cost per unit item.

h_c = The holding cost per unit item.

b_c = The shortage cost per unit item.

3. FORMULATION

The aim of this model is to optimize the total cost incurred and to determine the optimal ordering level. In the interval $[0, t_1]$ the stock will be decreased due to the effect of deterioration and demand. At time t_1, the inventory level reaches zero and in the next interval the shortage are allowed.

If $I(t)$ be the on hand inventory at time $t \geq 0$, then at time $t+\Delta t$, the on hand inventory in the interval $[0, t_1]$ will be

$$I(t+\Delta t) = I(t) - \theta(t)\, I(t).\Delta t - \log(1+t)\, \Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.1) $$\frac{dI}{dt} = -\theta\, e^{\theta t} I(t) - \log(1+t) \text{ for } 0 \leq t \leq t_1 .$$

In the end interval , $[t_1, T]$

$$I(t+\Delta t) = I(t) - \log(1+t).\Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$, we get,

(3.2) $$\frac{dI}{dt} = -\log(1+t), t_1 \leq t \leq T .$$

Now solving equation **(3.1)**with boundary condition $I(0) = Q$

(3.3) $$I(t) = \frac{2-\theta t}{4}\left[\frac{\theta t^2 + 8t - 2\theta t}{4} - \frac{\theta t^2 + 4t - \theta + 4}{2}\log(1+t) + 2Q\right] \text{ for } 0 \leq t \leq t_1 .$$

On solving equation **(3.2)** with boundary condition $I(t_1) = 0$

(3.4) $$I(t) = (t - t_1) + (1+t_1)\log(1+t_1) - (1+t)\log(1+t) \quad \text{for } t_1 \leq t \leq T .$$

Form equation **(3.3)**, we obtain the initial inventory level using the condition $I(t_1) = 0$.

(3.5) $$Q = \frac{\theta t_1^{\,2} + 4t_1 - \theta + 4}{4}\log(1+t_1) - \frac{\theta t_1^{\,2} + 8t_1 - 2\theta t_1}{8} .$$

The total inventory holding during the time interval $[0, t_1]$ is given by,

(3.6) $$I_T = \int_0^{t_1} I\, dt = \frac{4\theta t_1^{\,3} + (36 - 12\theta)t_1^{\,2} - (24 - 4\theta)}{48}\log(1+t_1)$$

$$+\frac{20\,\theta\,t_1^{\,3}+(3\theta-18)t_1^{\,2}+(108-18\theta)\,t_1}{72}$$

From of equation **(3.4)** amount of shortage during the time interval $[t_1, T]$ is

(3.7) $$B_T=\int_{t_1}^{T} I\,dt=T^2-Tt_1+\frac{T-t_1-(T+1)^2}{2}\log(1+T)+\frac{2(1+t_1)T+1-t_1^2}{2}\log(1+t_1)$$

The total number of deteriorated units during the inventory cycle is given by,

(3.8) $$D=Q-\int_0^{t_1} R(t)\,dt=\frac{\theta\,t_1^{\,2}-\theta}{4}\log(1+t_1)-\frac{\theta\,t_1^{\,2}-2\theta\,t_1}{8}$$

Using the above equations into consideration the different costs will be as follows.

1. **Purchasing cost per cycle**

(3.9) $$p_c\;I(0)=p_c\left\{\frac{\theta\,t_1^{\,2}+4t_1-\theta+4}{4}\log(1+t_1)-\frac{\theta\,t_1^{\,2}+8t_1-2\theta\,t_1}{8}\right\}$$

2. **Holding cost per cycle**

(3.10) $$h_c\int_0^{t_1} I(t)dt\;=h_c\left\{\frac{4\theta\,t_1^{\,3}+(36-12\theta)t_1^{\,2}-(24-4\theta)}{48}\log(1+t_1)\right.$$

$$\left.+\frac{20\,\theta\,t_1^{\,3}+(3\theta-18)t_1^{\,2}+(108-18\theta)\,t_1}{72}\right\}$$

3. **Deterioration cost per cycle**

(3.11) $$d_c\;D=d_c\left\{\frac{\theta\,t_1^{\,2}-\theta}{4}\log(1+t_1)-\frac{\theta\,t_1^{\,2}-2\theta\,t_1}{8}\right\}$$

4. **Shortage cost per cycle**

(3.12) $$-b_c\int_{t_1}^{T} I(t)dt$$

$$=-b_c\left[T^2-Tt_1+\frac{T-t_1-(T+1)^2}{2}\log(1+T)+\frac{2(1+t_1)T+1-t_1^2}{2}\log(1+t_1)\right]$$

The average total cost per unit time of the model will be

(3.13)

$$C(t_1)=\frac{1}{T}\left[\;p_c\left\{\frac{\theta\,t_1^{\,2}+4t_1-\theta+4}{4}\log(1+t_1)-\frac{\theta\,t_1^{\,2}+8t_1-2\theta\,t_1}{8}\right\}\right.$$

$$+h_c\left\{\frac{4\theta\, t_1^{\,3}+(36-12\theta)t_1^{\,2}-(24-4\theta)}{48}\log(1+t_1)+\frac{20\,\theta\, t_1^{\,3}+(3\theta-18)t_1^{\,2}+(108-18\theta)\,t_1}{72}\right\}$$

$$+d_c\left\{\frac{\theta\, t_1^{\,2}-\theta}{4}\log(1+t_1)-\frac{\theta\, t_1^{\,2}-2\theta\, t_1}{8}\right\}$$

$$-b_c\left\{T^2-Tt_1+\frac{T-t_1-(T+1)^2}{2}\log(1+T)+\frac{2(1+t_1)T+1-t_1^2}{2}\log(1+t_1)\right\}\Bigg]$$

As it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.13), Matlab Software has been used to determine optimal t_1^* and hence the optimal $I(0)$, the minimum average total cost per unit time can be determined.

4.0 FUZZY MODEL AND SOLUTION PROCEDURE

We consider the model in fuzzy environment. Due to fuzziness, it is not easy to define all the parameters precisely. We use the following variables

$\tilde{p}_c$: fuzzy purchasing cost,

$\tilde{h}_c$: fuzzy carrying cost,

$\tilde{d}_c$: fuzzy deterioration cost,

$\tilde{b}_c$: fuzzy shortage cost,

Suppose $\tilde{p}_c=(p_1,p_2,p_3,p_4)$, $\tilde{h}_c=(h_1,h_2,h_3,h_4)$, $\tilde{d}_c=(d_1,d_2,d_3,d_4)$, $\tilde{b}_c=(b_1,b_2,b_3,b_4)$ are nonnegative trapezoidal fuzzy numbers.

The total average cost per unit time is given by

(4.1) $$\tilde{C}_{avg}(t_1)=(\alpha\otimes\tilde{p}_c)\oplus(\beta\otimes\tilde{h}_c)\oplus(\gamma\otimes\tilde{d}_c)\oplus(\mu\otimes\tilde{b}_c)$$

$$where\;\; \alpha=\frac{1}{T}\left\{\frac{\theta\, t_1^{\,2}+4t_1-\theta+4}{4}\log(1+t_1)-\frac{\theta\, t_1^{\,2}+8t_1-2\theta\, t_1}{8}\right\}$$

$$\beta=\frac{1}{T}\left\{\frac{4\theta\, t_1^{\,3}+(36-12\theta)t_1^{\,2}-(24-4\theta)}{48}\log(1+t_1)+\frac{20\,\theta\, t_1^{\,3}+(3\theta-18)t_1^{\,2}+(108-18\theta)\,t_1}{72}\right\}$$

$$\gamma=\frac{1}{T}\left\{\frac{\theta\, t_1^{\,2}-\theta}{4}\log(1+t_1)-\frac{\theta\, t_1^{\,2}-2\theta\, t_1}{8}\right\}$$

$$\mu=\frac{1}{T}\left\{-T^2+Tt_1-\frac{T-t_1-(T+1)^2}{2}\log(1+T)-\frac{2(1+t_1)T+1-t_1^2}{2}\log(1+t_1)\right\}$$

(4.2) Now $\tilde{C}_{avg}(t_1)=(\tilde{C}_{avg_1}(t_1),\tilde{C}_{avg_2}(t_1),\tilde{C}_{avg_3}(t_1))$

$$\tilde{C}_{avg_1}(t_1)=(\alpha\ p_1+\beta\ h_1+\gamma\ d_1+\mu\ b_1\ )$$

$$\tilde{C}_{avg\,2}(t_1)=(\alpha\ p_2+\beta\ h_2+\gamma\ d_2+\mu\ b_2\ )$$

$$\tilde{C}_{avg_3}(t_1)=(\alpha\ p_3+\beta\ h_3+\gamma\ d_3+\mu\ b_3\ )$$

$$\tilde{C}_{avg_4}(t_1)=(\alpha\ p_4+\beta\ h_4+\gamma\ d_4+\mu\ b_4)$$

Now on defuzzifying the fuzzy total average cost $\tilde{C}_{avg}(t_1)$ *using Signed distance method we have*

(4.3)

$$d(\tilde{C}_{avg}(t_1),0)=\frac{1}{4}(\alpha\ p_1+\alpha\ p_2+\alpha\ p_3+\alpha\ p_4)+\frac{1}{4}(\beta\ h_1+\beta\ h_2+\beta\ h_3+\beta\ h_4)$$
$$+\frac{1}{4}(\gamma\ d_1+\gamma\ d_2+\gamma\ d_3+\gamma\ d_4)+\frac{1}{4}(\mu\ b_1+\mu\ b_2+\mu\ b_3+\mu\ b_4)$$
$$=\frac{\alpha}{4}(\ p_1+p_2+p_3+p_4)+\frac{\beta}{4}(\ h_1+h_2+h_3+h_4)$$
$$+\frac{\gamma}{4}(d_1+d_2+d_3+d_4)+\frac{\mu}{4}(b_1+b_2+b_3+b_4)=F(t_1)\ (say)$$

To minimize the average total cost per unit time, the optimal value of t_1 can be obtained by solving the following equation

(4.4)

$$\frac{d(F(t_1))}{dt_1}=\frac{\alpha'}{4}(\ p_1+p_2+p_3+p_4)+\frac{\beta'}{4}(\ h_1+h_2+h_3+h_4)$$
$$+\frac{\gamma'}{4}(d_1+d_2+d_3+d_4)+\frac{\mu'}{4}(b_1+b_2+b_3+b_4)$$

Thus minimum value of the total cost $C_{avg}(t_1)$ denoted by $C^*_{avg}(t_1)$

(4.5)

$$C^*_{avg}(t_1)=\frac{1}{4}(\alpha\ p_1+\beta\ h_1+\gamma\ d_1+\mu\ b_1\ )+\frac{1}{4}(\alpha\ p_2+\beta\ h_2+\gamma\ d_2+\mu\ b_2\ )$$
$$+\frac{1}{4}(\alpha\ p_3+\beta\ h_3+\gamma\ d_3+\mu\ b_3\ )+\frac{1}{4}(\alpha\ p_4+\beta\ h_4+\gamma\ d_4+\mu\ b_4)$$

5.0. COMPUTATIONAL ALGORITHM:

Step-1: Start.

Step-2: Initialize the value of the variables θ,T,h_c,p_c,d_c,b_c .

Step-3: Evaluate $C(t_1)$.

Step-4: Evaluate $\frac{\partial C(t_1)}{\partial t_1}$.

Step-5: Solve the equation $\frac{\partial C(t_1)}{\partial t_1} = 0$.

Step-6: Choose the solution from Step-5.

Step-7: Evaluate $\frac{\partial^2 C(t_1)}{\partial t_1^2}$.

Step-8: If the value of Step-7 is greater than zero then this solution is optimal (minimum) and go to Step-10.

Step-9: Otherwise go to Step-6.

Step-10: End.

6.0 NUMERICAL EXAMPLES

To illustrate the proposed method, let us consider the following input data:

Crisp Model:

The values of the parameters are considered as follows:

$\theta = 0.2, T = 1\,Year, h_c = \$4/unit/year, p_c = \$15/unit, d_c = \$9/unit, b_c = \$10/unit$

According to equation (3.13), we obtain the optimal $t_1^* = 0.1438$. From equation (3.13), we have the minimum average total cost per unit time as $C^* = 2.947$.

Fuzzy Model:

We can apply the fuzzy inventory model with fuzzy order quantity to find the optimal fuzzy total average cost. First, we represent the case of vague value as the type of trapezoidal fuzzy number.

Suppose

$\tilde{p}_c = (p_1, p_2, p_3, p_4) = (10,15,15,20)$,

$\tilde{h}_c = (h_1, h_2, h_3, h_4) = (2,4,4,6)$,

$\tilde{d}_c = (d_1, d_2, d_3, d_4) = (6,8,8,10)$, $\tilde{b}_c = (b_1, b_2, b_3, b_4) = (8,10,10,12)$, $\theta = 0.2, T = 1\,Year$.

Then by using (4.5) the fuzzy total average cost is minimized and optimal $t_1^* = 0.1837$ and hence the average optimal cost $C(t_1^*) = 3.102$.

7.0 CONCLUSION

Here we have derived a fuzzy inventory control model for deteriorating items. In particular deterioration is considered to be of exponential type. In this model, shortages are allowed. An optimal replenishment policy is derived with minimization of average total cost under the influence of Logarithmic demand. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, all related inventory parameters were assumed to be trapezoidal fuzzy numbers. The optimum results of fuzzy model are defuzzified using signed distance method. The result is illustrated through numerical example.

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