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Study of Butterfly Pattern and Orthogonal Property of PDN

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ABSTRACT

PDN is an emerging new topic in Interconnection network for research.We can try to study various properties of PDN network. In this paper we will discuss about PDN interconnection network and will derive butterfly pattern from itsAdjacency Matrix. Also, we will discuss about orthogonal property of PDN interconnection networkandwill discuss whether the derived Adjacency matrix is orthogonal or not.

Keyword: *Adjacency Matrix, Hadamard Product, Interconnection Network, Orthogonal, PDN (Perfect Difference Network),*

1. INTRODUCTION

1.1. PDN (Perfect Difference Network)

Perfect Difference Network [1][2] is the network architecture, in which the diameter is always 2, i.e., everyi^{th} node needs to visit only two links to communicate with other nodes$i \pm 1$ & $i \pm S_j \ (mod\ n)$, for $2 \le j \le \delta$. In a Perfect Difference Network, the total number of nodes is $\delta^2 + \delta + 1$, i.e., if $\delta = 2$ then the total number of nodes in PDN is 7 and if $\delta = 3$, then number of nodes in PDN is 13. Also, the degree of every node in a PDN is 2δ i.e., if δ = 2 then degree of every node in a PDN is 4 and similarly for other prime or power of prime numbers. The design of Perfect Difference Network is done in such a way where each node is connected via directed links to every other node. The links in PDN architecture are bidirectional.[2]

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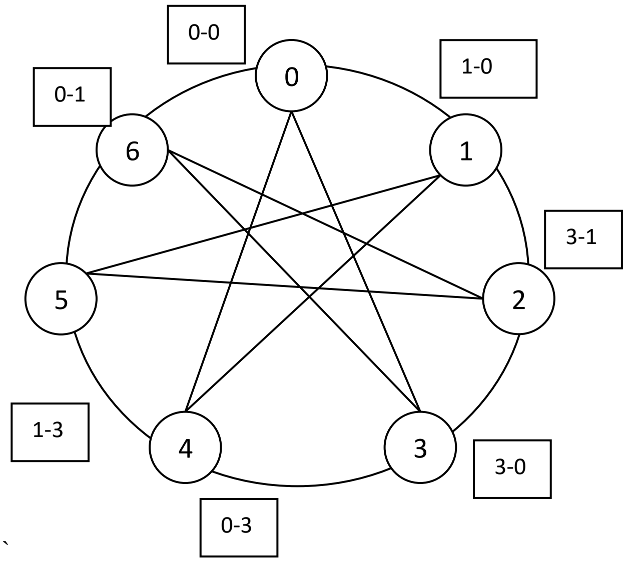

Figure1. Geometric Model of Perfect Difference Network with $\delta = 2$ and PDS = {0, 1, 3}.

1.2. Adjacency Matrix of PDN

Suppose G is a simple directed graph with m nodes, and suppose the nodes of G have been ordered and are called $v_1, v_1, v_1, \dots \dots, v_1$. Then the adjacency matrix A = (a_{ij}) of the graph G is the $m \times m$ matrix defined as follows:

$$a_{ij} = \begin{cases} 1 & if\, v_i is adjacent to\, v_j, that is, if there is an edge\,(v_i, v_j) \\ & 0 \quad otherwise \end{cases}$$

Such a matrix A, which contains entries of only 0 and 1, is called a bit matrix or a Boolean matrix.[3][6]

$$L = \begin{bmatrix} 0 & 1 & 01 & 1 & 01 \\ 1 & 0 & 10 & 1 & 10 \\ 0 & 1 & 01 & 0 & 11 \\ 1 & 0 & 10 & 1 & 01 \\ 1 & 1 & 01 & 0 & 10 \\ 0 & 1 & 10 & 1 & 01 \\ 1 & 0 & 11 & 0 & 10 \end{bmatrix}$$

Figure 2. Adjacency Matrix of PDN

1.3. Butterfly Pattern

With the above method, we have derived Adjacency matrices of various network architecture and it has given us unique patterns in each and every matrix. We have named these patterns as ***Butterfly Pattern*** because butterfly carries various patterns on their wings and here, we also got variety of patterns when represents them in Geometrical form, named as Geometrical Representation of Matrices.[4]

i/j	0	1	2	3	4	5	6
0	0	1	0	1	1	0	1
1	1	0	1	0	1	1	0
2	0	1	0	1	0	1	1
3	1	0	1	0	1	0	1
4	1	1	0	1	0	1	0
5	0	1	1	0	1	0	1
6	1	0	1	1	0	1	0

Figure 3. Butterfly Pattern of PDN

2. PDN HAS LADDER-LIKE BUTTERFLY PATTERN

On observing Adjacency Matrix of PDN architecture from Fig. 3, we see it has a unique pattern. Let say this pattern as Geometrical Butterfly Pattern. PDN architecture has this own unique pattern and is similar for all PDN architecture where number of nodes will be greater than 7. Here we have shown each this pattern for PDN having 7 number of nodes. Here in Fig. 4, we have presented geometrical pattern of PDN mentioned above in graphical form. This looks like a Ladder;thus, we have named it as Ladder-Like Butterfly Pattern.

Figure 4. Ladder-Like Butterfly Pattern of PDN

3. ORTHOGONALITY OF A MATRIX

In linear algebra, an orthogonal matrix, or orthonormal matrix, is a real square matrix whose columns and rows are orthonormal vectors.

One way to express this is $Q^T.Q = Q.Q^T = I$, where Q^T is the transpose of Q and I is the identitymatrix.[5]

For PDN,

$$L = \begin{bmatrix} 0 & 1 & 01 & 1 & 01 \\ 1 & 0 & 10 & 1 & 10 \\ 0 & 1 & 01 & 0 & 11 \\ 1 & 0 & 10 & 1 & 01 \\ 1 & 1 & 01 & 0 & 10 \\ 0 & 1 & 10 & 1 & 01 \\ 1 & 0 & 11 & 0 & 10 \end{bmatrix}$$

Transpose Matrix of L will be,

$$L^T = \begin{bmatrix} 0 & 1 & 01 & 1 & 01 \\ 1 & 0 & 10 & 1 & 10 \\ 0 & 1 & 01 & 0 & 11 \\ 1 & 0 & 10 & 1 & 01 \\ 1 & 1 & 01 & 0 & 10 \\ 0 & 1 & 10 & 1 & 01 \\ 1 & 0 & 11 & 0 & 10 \end{bmatrix}$$

3.1. Lemma: Is Adjacency Matrix of PDN architecture satisfies the property of orthogonality?

Proof: In order to prove the orthogonal property of PDN we have to prove that $L^T.L = L.L^T = I$, where L is the Adjacency Matrix of PDN architecture.

We will apply multiplication between L andL^T by using two ways of multiplication. First, by Mathematical Multiplication between matrices and other by applying Hadamard Product between matrices.

3.1.1. Applying Mathematical Multiplication between L andL^T

If we apply Mathematical multiplication of Matrices[6] between L and its transpose, we will get the following:

$$L.L^T = \begin{bmatrix} 4 & 4 & 44 & 4 & 44 \\ 4 & 4 & 44 & 4 & 44 \\ 4 & 4 & 44 & 4 & 44 \\ 4 & 4 & 44 & 4 & 44 \\ 4 & 4 & 44 & 4 & 44 \\ 4 & 4 & 44 & 4 & 44 \\ 4 & 4 & 44 & 4 & 44 \end{bmatrix} \neq I$$

Here, we can see all elements are same that is 4. Also, we know that for PDN degree of a node is 4 and here also we are getting 4 for each element. We can further write it as,

$$L.L^T = 4\begin{bmatrix} 1 & 1 & 11 & 1 & 11 \\ 1 & 1 & 11 & 1 & 11 \\ 1 & 1 & 11 & 1 & 11 \\ 1 & 1 & 11 & 1 & 11 \\ 1 & 1 & 11 & 1 & 11 \\ 1 & 1 & 11 & 1 & 11 \\ 1 & 1 & 11 & 1 & 11 \end{bmatrix} = J \neq I$$

Here, J is all-onesMatrix[7]and is not equal to Identity Matrix. Hence,this does not prove that Adjacency Matrix has Orthogonal property.

3.1.2. Applying Hadamard Product between L and L^T

In mathematics, the Hadamard product [8] is a binary operation that takes two matrices of the same dimensions and produces another matrix of the same dimension as the operands, where each element i, j is the product of elements i, j of the originaltwo matrices.

For two matrices A and B of the same dimension m × n, the Hadamard product $A \circ B$ $(or A \odot B)$ is a matrix of the same dimension as the operands, with elements given by

$$(A \circ B)_{ij} = (A \odot B)_{ij} = (A)_{ij}(B)_{ij}$$

For matrices of different dimensions (m × n and p × q, where m ≠ p or n ≠ q), the Hadamard product is undefined.

If we apply Hadamard Product (multiplication) between L and its transpose we will get the following:

$$L \circ L^T = \begin{bmatrix} 0 & 1 & 01 & 1 & 01 \\ 1 & 0 & 10 & 1 & 10 \\ 0 & 1 & 01 & 0 & 11 \\ 1 & 0 & 10 & 1 & 01 \\ 1 & 1 & 01 & 0 & 10 \\ 0 & 1 & 10 & 1 & 01 \\ 1 & 0 & 11 & 0 & 10 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 01 & 1 & 01 \\ 1 & 0 & 10 & 1 & 10 \\ 0 & 1 & 01 & 0 & 11 \\ 1 & 0 & 10 & 1 & 01 \\ 1 & 1 & 01 & 0 & 10 \\ 0 & 1 & 10 & 1 & 01 \\ 1 & 0 & 11 & 0 & 10 \end{bmatrix}$$

$$L \circ L^T = \begin{bmatrix} 0.0 & 1.1 & 0.01.1 & 1.1 & 0.01.1 \\ 1.1 & 0.0 & 1.10.0 & 1.1 & 1.10.0 \\ 0.0 & 1.1 & 0.01.1 & 0.0 & 1.11.1 \\ 1.1 & 0.0 & 1.10.0 & 1.1 & 0.01.1 \\ 1.1 & 1.1 & 0.01.1 & 0.0 & 1.10.0 \\ 0.0 & 1.1 & 1.10.0 & 1.1 & 0.01.1 \\ 1.1 & 0.0 & 1.11.1 & 0.0 & 1.10.0 \end{bmatrix}$$

$$L \circ L^T = \begin{bmatrix} 0 & 1 & 01 & 1 & 01 \\ 1 & 0 & 10 & 1 & 10 \\ 0 & 1 & 01 & 0 & 11 \\ 1 & 0 & 10 & 1 & 01 \\ 1 & 1 & 01 & 0 & 10 \\ 0 & 1 & 10 & 1 & 01 \\ 1 & 0 & 11 & 0 & 10 \end{bmatrix} = L$$

$$L.L^T = L \neq I$$

Here, on applying Hadamard Product between L and L^T we get back L and is not equal to Identity Matrix. Hence, this also does not prove that Adjacency Matrix has Orthogonal property.

Therefore, from both cases we conclude that Adjacency Matrix of PDN is not Orthogonal.

4. CONCLUSION

As PDN is such aninterconnection Network in which research and studies are going on. We also tried to study its properties and, on that note, we have checked orthogonal property of it. A new pattern is observed from Adjacency matrix of PDN that will help us in studying further properties of it. As I have tried for orthogonal property, we can further try for other geometrical properties of it.

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