V_4-Vertex Magic labeling for Hexagonal Mesh and Honeycomb Graph

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Abstract

Let V_4 *be* an abelian group under multiplication. Let $g : E(G) \to V_4 - \{1\}$. The vertex magic labeling on V_4 is defined as the vertex labeling $g^*: V(G) \to V_4$ such that $g^*(v) = \prod_u g(uv)$ where the product is taken over all edges uv of G incident at v is a constant. A graph is said to be $V_4 -$ magic if its admits a vertex magic labeling on V_4. In this paper we investigate the results on Torus graph, Hexagonal Mesh and Honeycomb graph.

Keyword: $T_{m,n}, HX_n, HC(n)$

AMS subject classification (2010): 05C78

1. Introduction

Laid foundation by Euler in the 18th Century, Graph Theory grew wider by Sedlack, Kong, Lee and Sun. Sedlack introduced Magic Labeling Bloom and Golomb connected Graph labeling to a wide range of applications such as Coding theory, Communication design, Radar, Circuit design, Astronomy, Network and X-ray crystallography.

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Let V_4 be an abelian group under multiplication. Let $g: E(G) \to V_4 - \{1\}$. The vertex magic labeling on V_4 is defined as the vertex labeling $g^*: V(G) \to V_4$ such that $g^*(v) = \prod_u g(uv)$ where the product is taken over all edges uv of G incident at v is a constant. A graph is said to be V_4- magic if its admits a vertex magic labeling on V_4.

The result is verified for Torus graph, Hexagonal Mesh and Honeycomb graph.

2. Preliminaries:

Torus Graph

Cartesian product of two cycles is a Torus graph.

Hexagonal Mesh

A Hexagonal Mesh of dimension n is denoted by HX_n and has $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. These are six vertices of degree three called the corner vertices. The centre of HX_n is at a distance $n - 1$ from each corner vertex. Hexagonal Mesh has 2n-1 vertical lines. Let the middle line be X_0, lines on the left side of X_0 are $X_1, X_2, \dots, X_{n-1}$ and the lines on the right side of X_0 are $X_{-1}, X_{-2}, \dots, X_{-n+1}$.

Labeling for vertices on X_0 are $v_{1,1}, v_{1,2}, \dots, v_{1,2n-1}$

Labeling for vertices on X_1 are $v_{2,1}, v_{2,2}, \dots, v_{2,2n-2}$ and so on

Vertices on X_{n-1} are $v_{n,1}, v_{n,2}, \dots, v_{n,n}$.

Labeling for vertices on X_{-1} are $v_{-1,1}, v_{-1,2}, \dots, v_{-1,2n-1}$ and so on.

Finally labeling for vertices on X_{-n+1} are $v_{-n+1,1}, v_{-n+1,2}, \dots, v_{-n+1,n}$.

Illustration: HX_4

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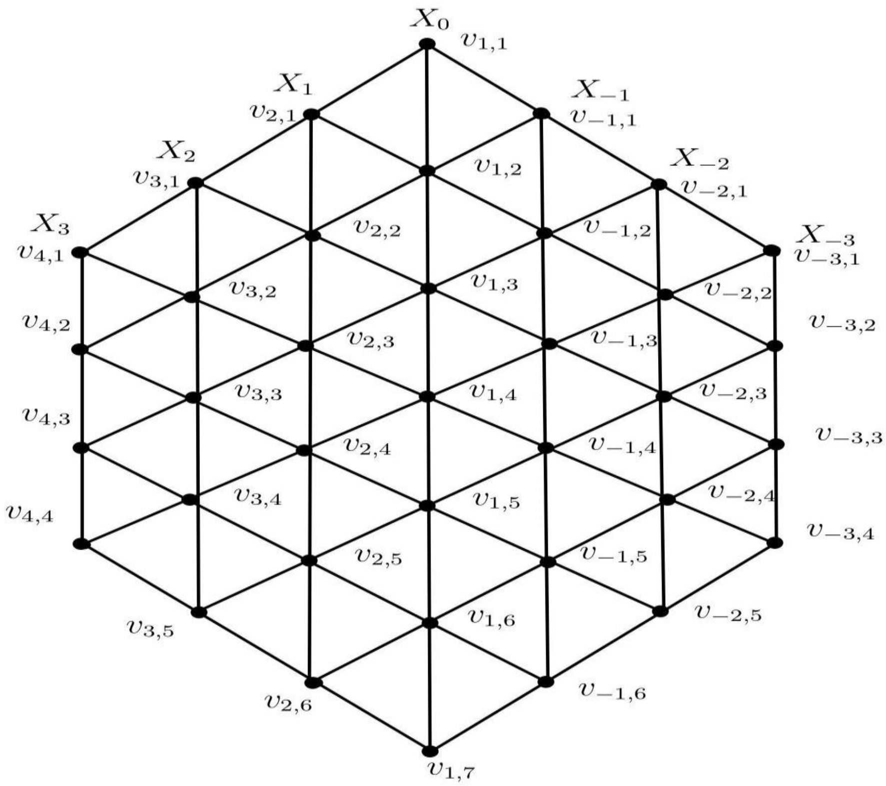

Honeycomb Graph

One dimension of Honeycomb graph is a Hexagon denoted by HC(1). Let this be layer L_1 . $HC(2)$ is constructed by attaching 6 Hexagons on the boundary edges of HC(1). Boundary edges of HC(2) is L_2. L_2 is connected with L_1 through 6 edges.

L_n is connected with L_{n-2} through $6(n-1)$ edges.

(i.e) $HC(n)$ is constructed by attaching L_n to L_{n-1} by 6(n-1) edges. There are $6n^2$ vertices and $9n^2-3n$ edges in HC(1). Each vertex in HC(n) is denoted by v_{pq} where p denotes the line number in which the vertex exists and q denotes the position of the vertex in the line.

Illustration: $HC(3)$

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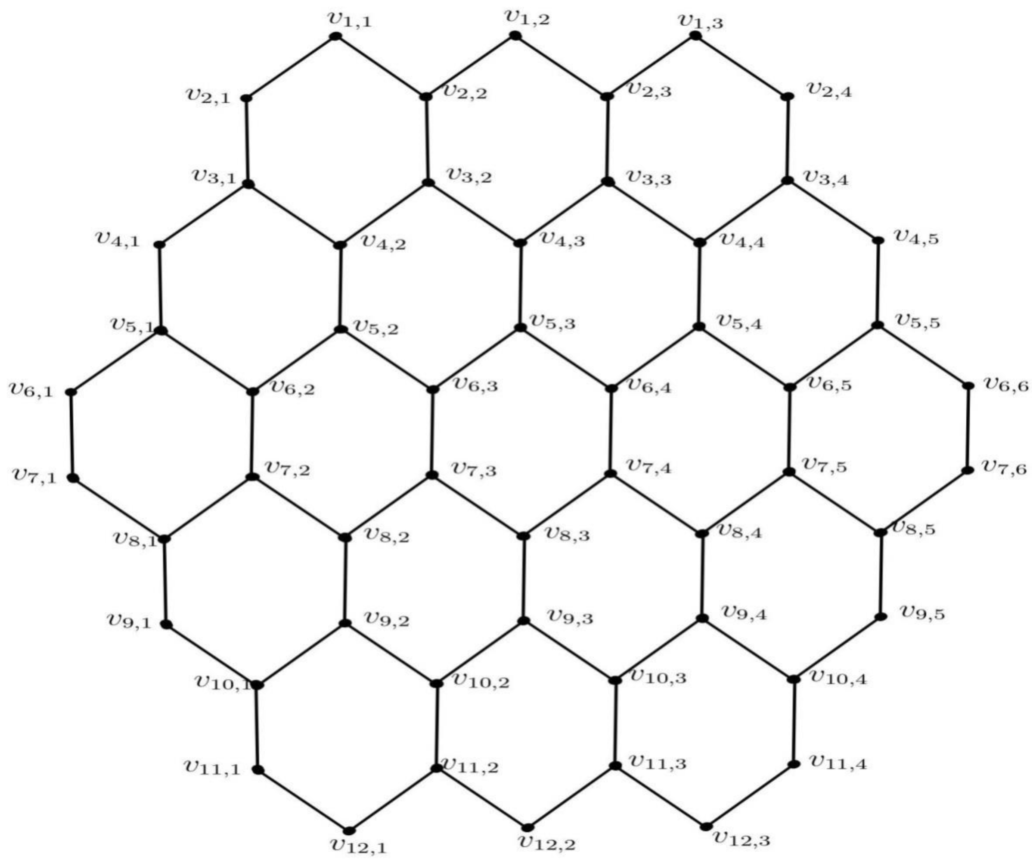

3.Main results:

Theorem 3.1: Torus graph $T_{m,n}$ is V_4-magic when $m, n \geq 2$.

Proof:

Let G be a Torus graph $T_{m,n}$

Let $V(G) = \{u_{pq} : 1 \leq p \leq m; 1 \leq q \leq n\}$ be the vertex set of G.

Let $E(G) = \left\{u_{pq}u_{p+1,q} : 1 \leq p \leq m, 1 \leq q \leq n\right\} \cup$

$$\cup \left\{u_{pq}\ u_{pq+1} : 1 \leq p \leq\ m, 1 \leq q \leq n\right\}$$

$[u_{p,n+1} = u_{p_1};\ \ u_{m+1,q} = u_{1,q}\ ]$

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Let us define a function $g: E(G) \rightarrow V_4 - \{1\}$ such that

$g(u_{pq}\, u_{p+1,q}) = -i$ for $1 \le p \le m, 1 \le q \le n$

$g(u_{pq}\, u_{pq+1}) = i$ for $1 \le p \le m, 1 \le q \le n$

Then $g^*: V(G) \rightarrow V_4 - \{1\}$ is

$g^*(u_{pq}) = 1$ for $1 \le p \le m, 1 \le q \le n$.

This labeling holds for all cases whether m, n are either even or odd.

Thus Torus graph becomes a V_4- vertex magic graph as it satisfies V_4 vertex magic Labeling for $m, n \ge 2$.

Illustration: $T_{6,8}$

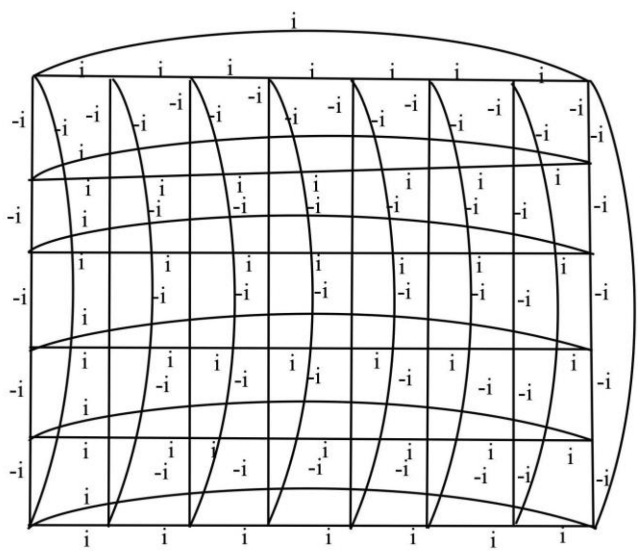

Illustration: $T_{7,8}$

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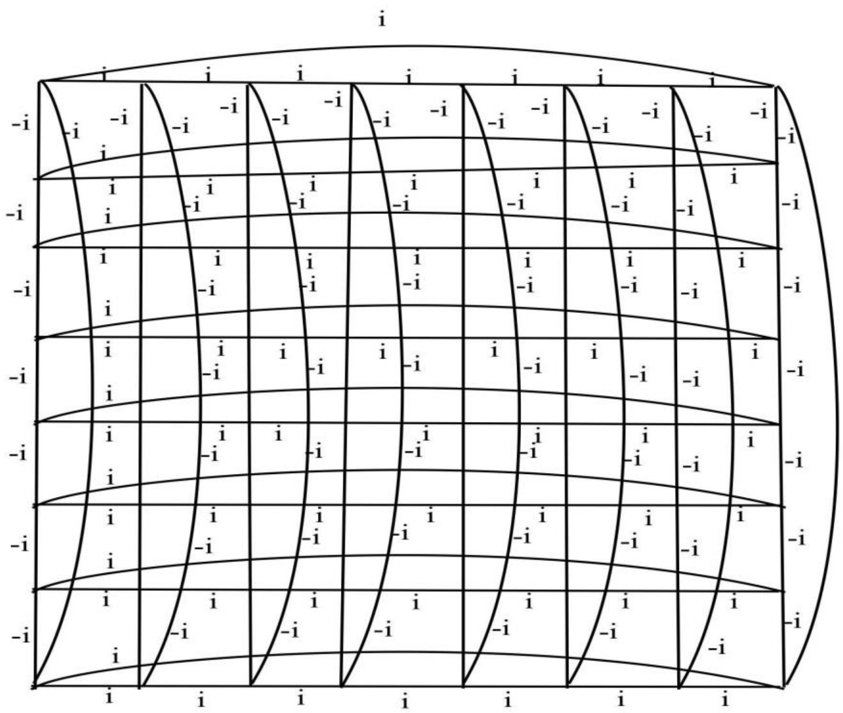

Remark:

Torus graph can be labelled either with $\{i\}$ or $\{-i\}$ or {-1} for all edges. Torus graph can also be labelled by using $\{i, -i, -1, -1\}$ for the edges meeting at each vertex. Thus getting the magic number "1" under multiplication at each vertex. Hence Torus graph becomes V_4- magic graph in all cases.

Theorem 3.2: For $n \geq 2,$ Hexagonal Mesh HX_n becomes a V_4-magic graph.

Proof: :

Let G be the Hexagonal Mesh graph HX_n where n is the dimension.

Define a mapping $g: E(G) \rightarrow \mathrm{V}_4 - \{1\}$ such that

$$g(v_{p,1}\ v_{p+1,1}) = \begin{cases} i \quad if\ \ p\ is\ odd\ ; 1 \le p \le n-1 \\ -i \quad\ if\ \ p\ is\ even; 1 \le p \le n-1 \end{cases}$$

$g(v_{p,1}v_{-p,1}) = i$ when p=1

$$g(v_{-p,1}\ v_{-(p+1),1}) = \begin{cases} -i \quad if\ \ p\ is\ odd\ ; 1 \le p \le n-1 \\ i \quad\ if\ \ p\ is\ even; 1 \le p \le n-1 \end{cases}$$

$g(v_{p,q}\ v_{p,q+1}) = -1$, for $1 \le p \le n-1; 1 \le q \le n+1$ if n is even & $1 \le q \le\ n+2$ (if n is odd)

$g(v_{-p,q}\ v_{-p,q+1}) = -1$ when $1 \le p \le n-2, 1 \le q \le n+1$ if n is even & $1 \le q \le\ n+2$ (if n is odd)

$When$ p=n $\qquad g(v_{p,q}\ v_{p,q+1}) = \begin{cases} -i\ if\ \ n\ is\ odd \\ i\ if\ \ n\ is\ even \end{cases}$

When $p = -(n-1)$

$$g(v_{p,q}\ v_{p,q+1}) = \begin{cases} -i\ if\ \ n\ is\ odd \\ i\ if\ \ n\ is\ even \end{cases}$$

$g(v_{p,q}\ v_{p-1,q+1}) = -1$ when $2 \le p \le n-2, 1 \le q \le n+1$ if n is even & $1 \le q \le\ n+2$ (if n is odd)

$g(v_{p,q}\ v_{p-2,q}) = -1$ when $p = 1\ \ , q = 2,3, \dots, n+1$ (if n is even) & $1 \le q \le n+2$ (if n is odd)

$g(v_{p,q}\ v_{p-1,q}) = -1$ when $p = n\ , q = 2,3, \dots, n$

$g(v_{p,q}\ v_{p-2,q-1}) = -1$ when $p = 1, q = 2,3, \dots, n+1$ (if n is even) & $q = 2,3, \dots, n+2$ (if n is odd)

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Then $g^*: V(G) \rightarrow \mathrm{V}_4 - \{1\}$ is

$g^*(v_{pq}) = 1$ for all $p = 1,2,\dots,n; p = -1,-2,\dots,-(n-1); q = 1,2,\dots n$

Thus G satisfies V_4- vertex magic labeling.

Hence Hexagonal Mesh HX_n is V_4- magic for $n \geq 2$. □

Illustration: HX_2

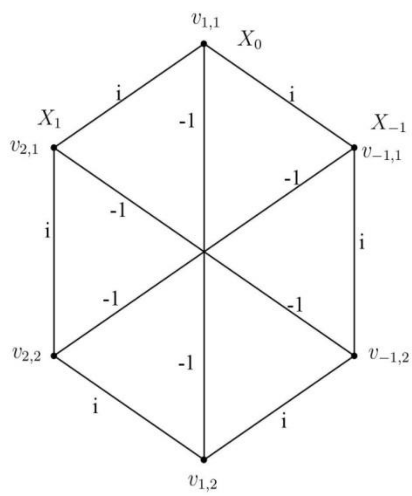


Illustration: HX_3

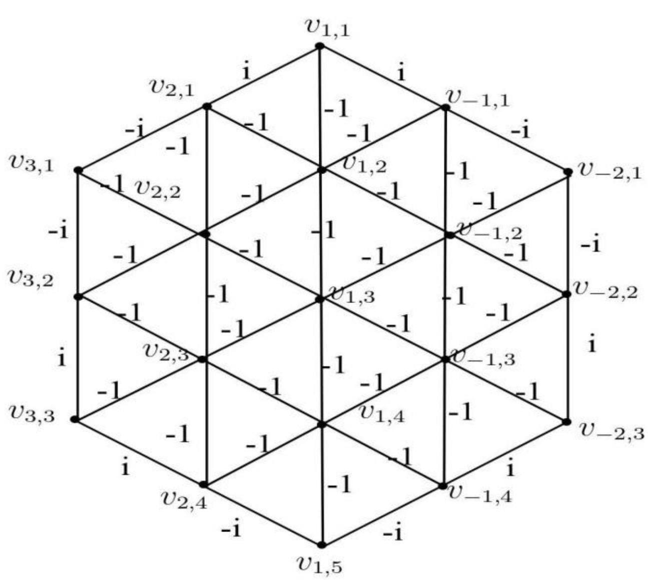
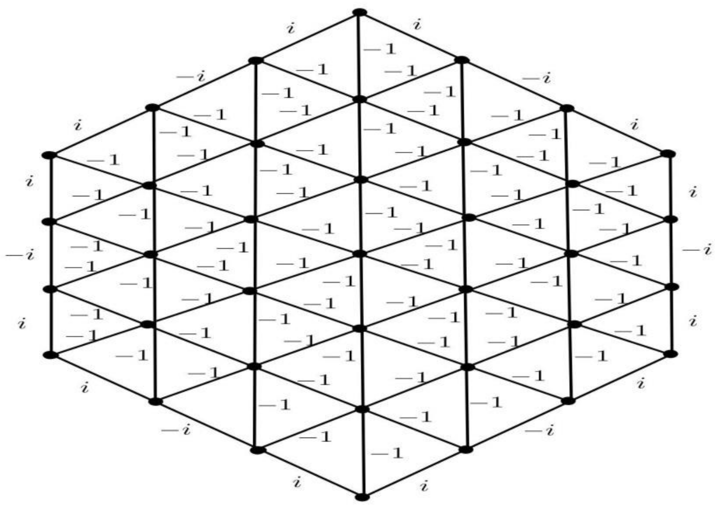


Illustration: HX_4



Remark:

Hexagonal Mesh can also be labelled by replacing $\{i\}$ by $\{-i\}$ and $\{-i\}$ by $\{i\}$ to satisfy V_4- vertex magic labeling.

Theorem 3.3: Honeycomb $HC(n)$ is a V_4- magic graph.

Proof:

Let G be HC(n) of dimension n.

Let $V(G) = \{v_{pq} : 1 \leq p \leq 4n; 1 \leq q \leq 2n\}$

Define a function $g: V(G) \to \mathrm{V}_4 - \{1\}$ such that

Boundary edges of $L_1, L_2 \ldots, L_{n-1}$ are labelled with "i" and their connecting edges are labelled with "-1".

Label the boundary edges of L_n with $\{i, -i\}$ where two edges incident and with $\{i, i\}$ or $\{-i, -i\}$ where there edges incident label the connecting edges between L_n and L_{n-1} with "-1".

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That induces $g^*: V(G) \to \mathrm{V}_4 - \{1\}$such that

$g^*(v_{pq}) = 1$ for all $p = 1,2, \dots ,4n; q = 1,2, \dots ,2n$

Thus G satisfies V_4- vertex magic labeling.

So Honeycomb HC(n) is a V_4- magic graph. □

Illustration: $\boldsymbol{HC(3)}$

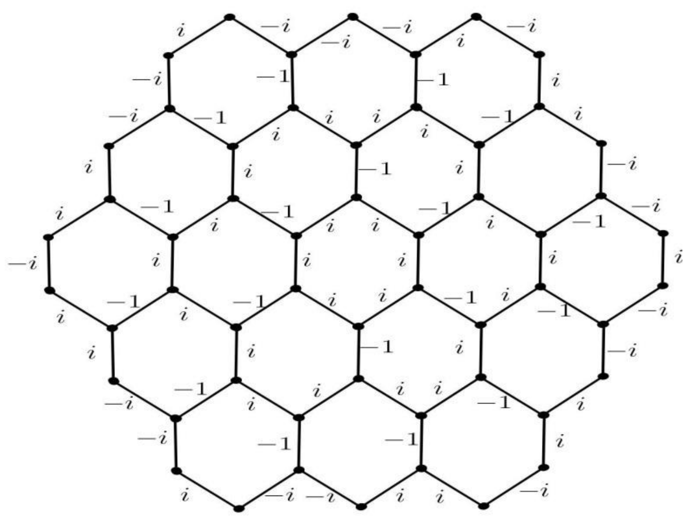

Illustration: $\boldsymbol{HC(4)}$

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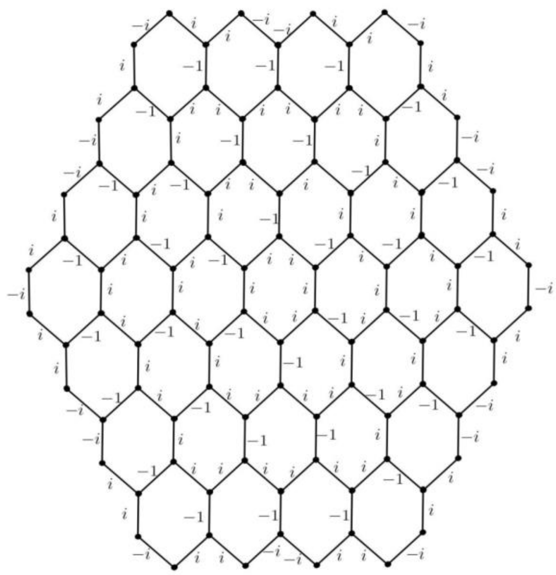

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