A Computational Study on Assignment Problem with Ramanujan Primes: Case (IV)

K.V.L.N.Acharyulu[1] & P.Prasanna Anjaneyulu[2]

[1]Associate Professor,

Dept. of Mathematics,Bapatla Engineering College,Bapatla.

[2]Associate Professor,

Department of S & H, VasireddyVenkatadri Institute of Technology, Nambur, India.

Abstract

The main objective of this paper is to discuss a special case in Assignment Problem. It is built by using Ramanujan Primes as cost assignments. Some cases receive in-depth investigation. Few fruitful outcomes have been established. The generalised optimum assignments are obtained in this study. Wherever possible, the representing graphs in various cases are illustrated.

1.INTRODUCTION:

This technique was familiar to Denes konig and Jeno, two Hungarian mathematicians. The Hungarian approach is the most comprehensive source of combinatorial optimization techniques for solving a wide range of difficult assignment problems. In 1955, Harold Kuhn developed and published the algorithm. He revealed that the algorithm's name was Hungarian algorithm. In 1957, James Munkres investigated that algorithm and discovered that it is strongly polynomial. Many mathematicians[1-37] have investigated the applicability of a few operations research approaches, which are useful in tracing an optimal solution that meets all requirements. The Hungarian Method is one such successful optimization techniques.

2. RAMANUJAN PRIMES:

Ramanujan started arriving at a holistic viewpoint. i.e the function $\pi(x) - \pi\left(\frac{x}{2}\right) \geq 1,2,3,4,5, \ldots \text{forall} x \geq 2,11,17,29,41, \ldots \text{respectively},$Where $\pi(x)$is the prime-counting functionwhich is equal to the number of primes less than or equal to x. The definition of Ramanujan primes is the inverse of this result: The n^{th}Ramanujan prime is the least integer R_n for which $\pi(x) - \pi\left(\frac{x}{2}\right) \geq n, forall x \geq Rn$. It is noted that the integer R_n is necessarily a prime number: $\pi(x) - \pi\left(\frac{x}{2}\right)$ and $\pi(x)$must increase by obtaining another prime at $x = R_n$. Since $\pi(x) - \pi\left(\frac{x}{2}\right)$ can increase by at most 1, $\pi(Rn) - \pi\left(\frac{Rn}{2}\right) = n$. Bounds and an Asymptotic formula are valid for all n≥1, the bounds $2n \ln 2n < Rn < 4n \ln 4n$ hold. If n>1, then also $p_{2n} < Rn < p_{3n}$ where p_n is the n^{th} prime number. As n tends to infinity, R_n is asymptotic to the 2nth prime, i.e., $R_n \sim p_{2n}$ ($n \to \infty$).

3. BASIC ASSIGNMENT MODEL:

3.1 Case(A).:

The mathematical model of assignment problem in case (i) is defined as

$$Min / Max\ Z = \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij} x_{ij}$$

Subject to the constraints:

$$\sum_{i=1}^{5} x_{ij} = 1 \text{ for j=1,2,3,4 and 5}$$

$$\sum_{j=1}^{5} x_{ij} = 1 \text{ for i=1,2,3,4 and 5}$$

x_{ij} = either 0 or 1 for all i,j

Here x_{ij} denotes the assignment of i^{th} resource to j^{th} activity with the successive numbers of Ramanujan primes column wise.

Table-1: Tabular Form of 5x5 Assignment Problem with Ramanujan primes

5x5	I	II	III	IV	V
A	2	47	101	167	233
B	11	59	107	179	239
C	17	67	127	181	241
D	29	71	149	227	263
E	41	97	151	229	269

Table-2: Hungarian Method with 5x5 Assignment Problems in Minimization Case with cycle-1

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization (5x5)	C1	(A,I), (B,III), (C,IV), (D,II)	P_{12},P_{13},P_{14},P_{15}, P_{52},P_{53},P_{54}, P_{55}	4	(A,IV), (B,III), (C,V), (D,II), (E,I)	627

Table-3: Hungarian Method with 5x5 Assignment Problems in Minimization Case with cycle-2

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization (5x5)	C2	(A,IV), (B,III), (C,V), (D,II), (E,I)	*	*	(A,IV), (B,III), (C,V), (D,II), (E,I)	627

Table-4: Hungarian Method with 5x5 Assignment Problems inMaximization Case with cycle-1

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization Type (5x5)	C1	(A,V), (C,I), (D,III), (E,II)	P_{11},P_{12},P_{13},P_{14}, P_{21},P_{22},P_{23},P_{24},	4	(A,V), (B,I), (C,III), (D,IV), (E,II)	695

Table-5: Hungarian Method with 5x5 Assignment Problems inMaximization Case with cycle-2

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization Type (5x5)	**C2**	**(A,V), (B,I), (C,III), (D,IV), (E,II)**	*	*	**(A,V), (B,I), (C,III), (D,IV), (E,II)**	**695**

Table-6: Bottle Neck Method With 5x5 Assignment Problem in Minimization/Maximization

Objective Function Type	Optimal Assignment	Total Assignment Cost
Minimization(5x5)	**(A,V),(B,IV),(C,III),(D,II), (E,I)**	**651**
Maximization(5x5)	**(A,V),(B,IV),(C,III),(D,II), (E,I)**	**651**

3.2Case(B).:

The mathematical model of assignment problem in case (i) is defined as

$$Min / Max\ Z = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij} x_{ij}$$

Subject to the constraints:

$$\sum_{i=1}^{5} x_{ij} = 1 \text{ for j=1,2 and 3}$$

$$\sum_{j=1}^{5} x_{ij} = 1 \text{ for i=1,2 and 3}$$

x_{ij} = either 0 or 1 for all i,j

Here x_{ij} denotes the assignment of i^{th} resource to j^{th} activity with the successive numbers of Ramanujan primes column wise.

Table-7: Tabular Form of 3x3 Assignment Problem with Ramanujan primes

3x3	I	II	III
A	2	29	59
B	11	41	67
C	17	47	71

Table-8: Hungarian Method with 3x3 Assignment Problems in Minimization Case with cycle-1

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization (3x3)	C1	(A,II), (B,I), (C,III)	*	*	(A,II), (B,I), (C,III)	111

Table-9: Hungarian Method with 3x3 Assignment Problems in Maximization Case with cycle-1

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization Type (*3x3*)	C1	(A,III), (C,I)	P_{11},P_{12},P_{21},P_{22}	2	(A,III), (B,I), (C,II)	117

Table-10: Hungarian Method with 3x3 Assignment Problems in Maximization Case with cycle-2

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines In Cycle Wise	Optimal Assignment	Total Assignment Cost
Maximization Type (*3x3*)	**C2**	**(A,III), (B,I), (C,II)**	*	*	**(A,III), (B,I), (C,II)**	**117**

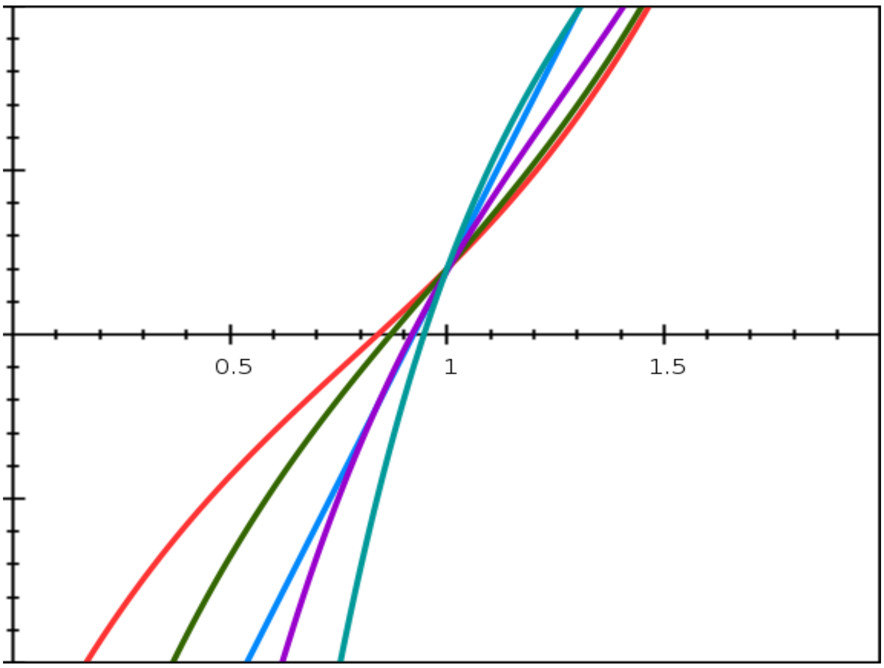
Table-11: Bottle Neck Method With 3x3 Assignment Problem in Minimization/Maximization

Objective Function Type	Cycle	Assigned Zero Positions	Positions Of Uncovered Elements	Minimum No. Of Lines in Cycle Wise	Optimal Assignment	Total Assignment Cost
Minimization (3x3)	**C1**	**(A,II),(B,I)**	**P_{22},P_{23}, P_{32},P_{33}**	**2**	**(A,III),(B,II), (C,I)**	**117**
	C2	**(A,III),(B,II), (C,I)**	*	*		
Maximization (3x3)	**C1**	**(A,III),(C,I)**	**P_{11},P_{12}, P_{21},P_{22}**	**2**	**(A,III),(B,II), (C,I)**	**117**
	C2	**(A,III),(B,II), (C,I)**	*	*		

Based on the sizes of the assignment problems, the polynomials are derived and illustrated as below.

Table-12: Polynomials at different Cases

S.No	Cases	Polynomial
1	n=1,2	$26x-24$
2	n=1,2,3	$28.5x^2-59.5x+33$
3	n=1,2,3,4	$7.8333x^3-18.5x^2+26.666x-14$
4	n=1,2,3,4,5	$-0.375x^4+11.5833x^3-31.625x^2+45.4167x-23$
5	n=1,2,3,4,5,6	$0.3083x^5-5x^4+37.7916x^3-101x^2+129.9x-60$
6	n=1,2,3,4,5,6,7	$-0.104x^6+2.495x^5-23.229x^4+114.3541x^3-270.166x^2+313.65x-135$



Grapgh-1

4.Conclusions:

In this special case study on the assignment problem with Ramanujan primes, the following observations are made.

(i).The movement of uncovered elements changes in a systematic way, cycle by cycle and size by size.

(ii).The minimum number of lines required to cover all assigned zeros and other remaining zeros plays a significant role in many cycles as the system approaches optimality.

(iii).The Hungarian method and the Bottle neck method successfully derive the possible Optimum Assignments and Total cost values in the cases of Minimization and Maximization of this model.

(iv).The deviation between the Polynomials is gradually reduced in different cases.

Reference:

[1].Alexander Schrijver, Theory of Linear and Integer Programming. John Wiley & Sons,1998

[2].Billy E. Gillett, Introduction to operations Research, Tata McGraw-Hill Publishing Company limited, New York,1979.

[3].Bland, Robert G. , "New Finite Pivoting Rules for the Simplex Method". Mathematics of Operations Research. 2 (2): 103–107. 1977.

[4].George B. Dantzig and Mukund N. Thapa.,Linear programming 1: Introduction. Springer-Verlag,1997.

[5].George B. Dantzig and Mukund N. Thapa., Linear Programming 2: Theory and Extensions. Springer-Verlag,2003.

[6]. S.D.Sharma, Operations Research, KedarNath Ram Nath& Co. ,1999.

[7].K.V.L.N.Acharyulu and NaguVadlana,Influence of G.P on Networks - A Scientific study on Case (I), International Journal of Computer Networking, Wireless and Mobile Communications, Vol. 3, Issue 2, pp. 83-92, 2013.

[8].K.V.L.N.Acharyulu and Maddi.N.MuraliKrishna,Impact of A.P on Networks - A Computational study on Case (I), International Journal of Computer Networking, Wireless and Mobile Communications, Vol. 3, Issue 2, pp. 55-793-102, 2013.

[9].K.V.L.N.Acharyulu and Maddi.N.Murali Krishna, Some Remarkable Results in Row and Column both Dominance Game with Brown's Algorithm, International Journal of Mathematics and Computer Applications Research, Vol. 3, No.1, pp.139-150, 2013.

[10].K.V.L.N.Acharyulu,Maddi. N. Murali Krishna, SateeshBandikalla&NaguVadlana,(2013). A Significant Approach On A Special Case Of

Game Theory, International Journal of Computer Science Engineering and Information Technology Research, Vol. 3, Issue 2, pp. 55-78, 2013.

[11].K.V.L.N.Acharyulu and Maddi.N.MuraliKrishna,A Scientific Computation On A Peculiar Case of Game Theory in Operations Research, International Journal of Computer Science Engineering and Information Technology Research,Vol. 3 , No.1, pp.175-190, 2013.

[12].K.V.L.N.Acharyulu and NaguVadlana,Impact of G.P on Networks - A Computational Study on Case (II),International Journal of Computer Science Engineering and Information Technology Research, Vol. 3, Issue 3, Aug 2013, 241-250,2013.

[13].K.V.L.N.Acharyulu, Maddi.N.Murali Krishna & P. PrasannaAnjaneyulu ;A ScientificStudy On A Network With Arithmetic Progression On Optimistic Time Estimate, ActaCienciaIndica, Volume 40, No 2, 177-188, 2014.

[14].KanduriVenkata Lakshmi Narasimhacharyulu&I.Pothuraju ,A Peculiar Case In Game Theory- A Computational Study, International Journal of Scientific and Innovative Mathematical Research (IJSIMR), Volume 2, Issue 3, PP 269-280,2014.

[15].K.V.L.N.Acharyulu&I.Pothuraju ,A Special case in Network –G.P on optimistic time estimate, ActaCienciaIndica, Volume 40, No 3, 315-321, 2014.

[16].K.V.L.N.Acharyulu,Ch.ChandraSekaraRao&I.Pothuraju, AScientific Approach with Computational Study on Case(I), International Journal of Scientific andInnovative Mathematical Research (IJSIMR), Volume 2, Issue 12,PP 989-998,2014.

[17].K.V.L.N.Acharyulu&I.Pothuraju, Geometric Progression in Operations Research (PERT) –A Special Case Study, International Journal of Scientific and Innovative Mathematical Research (IJSIMR), Volume 2, Issue 1,PP 83-93,2014.

[18].K.V.L.N.Acharyulu, Maddi.N.Murali Krishna & P. PrasannaAnjaneyulu; Arithmetic Progression in Operations Research(PERT)-A Special case study,ActaCiencia Indica,Vol.40, No 3, 425-434, 2014.

[19].N.SeshagiriRao, K.Kalyani and K.V.L.N.Acharyulu, Threshold results for host –Mortal Commensal ecosystem with limited resources, Global Journal of Pure and Applied Mathematics, Volume10, No.6, PP:787-791,2014.

[20].K.V.L.N.Acharyulu, “A Special Case Study On 10x10 Symmetric Problem in Game Theory–Brown’s Algorithm”, ActaCienciaIndica, Volume.43, No.2, pp.141-148,2017.

[21].K.V.L.N.Acharyulu, “A Case Study On The Influence of Optimistic Time Estimate On A Network With Arithmetic Progression”, International Journal of Advance research in science and Engineering, Volume.6, No.10, PP.1198-1205,2017.

[22].K.V.L.N.Acharyulu,“Arithmetic Progression on Most Likely Time Estimate -A case study”,ActaCienciaIndica, Volume.43,No.2,pp.165-172,2017.

[23].K.V.L.N.Acharyulu, “Prime Problem In Game Theory – Brown’s Algorithm”,International Journal of Advance research in science and Engineering, Volume.6,No.10,PP.1206-1212,2017.

[24].K.V.L.N.Acharyulu, “A Problem in Game Theory with Fibonacci Numbers”, International Journal of Advance research in science and Engineering, Volume.6, No.11, PP.1954-1960,2017.

[25].K.V.L.N.Acharyulu,“ A Game with non zero Triangular numbers”,ActaCienciaIndica, Volume.43,No.4,pp.247-253,2017.

[26].K.V.L.N.Acharyulu,“Pessimistic Time Estimate With Arithmetic Progression”, International Journal of Advance research in science and Engineering, Volume.6,No.11, PP.1961-1967,2017.

[27].K.V.L.N.Acharyulu, “A Special symmetric Game Problem with Triangular Numbers-Brown’s Algorithm”, ActaCienciaIndica, Volume.43,No.4,pp.221-227,2017 .

[28].K.V.L.N.Acharyulu, B.SaiPrasanna and B.SriSatyaRajani,‘A Special Case Study in LPP’,International Journal of Management, Technology and Engineering, Volume.8,Issue10, pp.1512-1521,2018.

[29]K.V.L.N.Acharyulu,A.Bhargavi and G.Sravani ‘A Peculiar Problem in Linear Programming Problem’ ,International Journal of Management, Technology and Engineering, Volume.8,Issue10,pp.1532-1540,2018.

[30].K.V.L.N.Acharyulu, B.Jayasree&Sk.Mubeena, ‘A Generalized problem in Linear programming problem’,International Journal of Management, Technology and Engineering, Volume.8,Issue10,pp.1541-1548,2018.

[31].K.V.L.N.Acharyulu, P.Hema and T.Vimala ‘A Generalized problem in Linear programming problem’,International Journal of Management, Technology and Engineering, Volume.8,Issue10,pp.1549-1556,2018.Fbodigiri

[32].K.V.L.N.Acharyulu, O.Nagaraju&G.Srikanth, Special Case in Assignment Problem with Ramanujan Primes, Journal of the Gujarat Research Society, Vol. 21, Issue 3, pp.368-377,October 2019.

[33].K.V.L.N.Acharyulu,V.Saritha&K.YaminiDevi,A Peculiar Case in Assignment Problem With Triangular Numbers,Journal of the Gujarat Research Society, Vol. 21, Issue 3, pp.378-389,October 2019.

[34].K.V.L.N.Acharyulu , Ch. Sri Lakshmi &Y.Anusha, Generalized Case in Assignment Problem With Lucas Numbers in Journal of the Gujarat Research Society, Vol. 21, Issue 3, pp.390-400,October 2019.

[35].K.V.L.N.Acharyulu , H. Mounika& CH. Raja Rajeswari Devi, A Variety Case in Assignment Problem With Tribonacci Numbers, Journal of the Gujarat Research Society, Vol. 21, Issue 3, pp.401-413,October 2019.

[36].K.V.L.N.Acharyulu, D.Jaswanth&D.Chiranjeevi ,An Exclusive Case in Assignment Problem With Fibonacci Numbers,Journal of the Gujarat Research Society, Vol. 21, Issue 3, pp.414-426,October 2019.

[37].K.V.L.N.Acharyulu, Dr. N.Phani Kumar, G.Srikanth&O.Nagaraju SpecialAmmensal Model with Monad Coefficient - A Logical Study,South East Asian J. of Mathematics and Mathematical Sciences, Vol. 16, No. 1 (April), pp. 97-104,2020.