

A Peculiar Linearly Cover Variant of Amensalism in Mathematical Ecology with Restricted Resources: Case(I)

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Abstract

This model consists of two first order nonlinear differential equations. Both are representing Amensal and Enemy Species respectively. In this Paper, The impact of natural growth rate(a) and Self inhibition(b) of Ammensal on constituted model are identified. Amensal Species has restricted resources with a linearly variant cover protection for safety from the attacks of enemy species. The graphs are illustrated whenever necessary in detail to observe keenly the nature of the interaction among the both species. The phase plane analysis is carried out to establish the stability of the model.

1. Introduction:

K.V.L.N.Acharyulu and N.Ch.PattabhiRamacharyulu [2–9] evaluated the local and global stability of an Ammensal-enemy eco-system based on the basic balancing quasi-linear equations. In previous studies, the author also did a detailed analysis on the stability of an Ammensal-enemy eco-system with restricted resources in different situations. Several authors [1,10–18] have opened new eras with effective ways to deal with various situations and study how different species interact with each other. The main goal of this study is to find out how the natural growth rate(a) and self-inhibition coefficient(b) affect how the two species interact with each other.

2. Basic Equations of the Considered Model:

The equations of a strange linearly cover variant of Amensalism with Limited Resources in Mathematical Ecology are considered as

$$dX/dt = aX - bX^2 - c(X - (d + eX))Y$$

$$dY/dt = fY - gY^2$$

Here x and y stand for Amensal and Enemy growth rates respectively. The natural growth rates of Amensal and enemy species are referred as a and f . c is the Amensal coefficient. b and g are the two species' decline rates due to natural resource restrictions. $d+ex$ is the Ammensal population, which protected from the attacks of enemy species by maintaining constant values for d and e .

3. Case(i): Effect of 'a' on Ammensal-Enemy Model

For investigating the interaction between Ammensal and Enemy Species, the following values of concerned parameters are considered.

Table (1):

Parameter	a	b	c	d	e	f	g	X_0	Y_0
Value	Changes	0.889	2.515	0.875	0.67	0.859	0.5	0.134	0.366

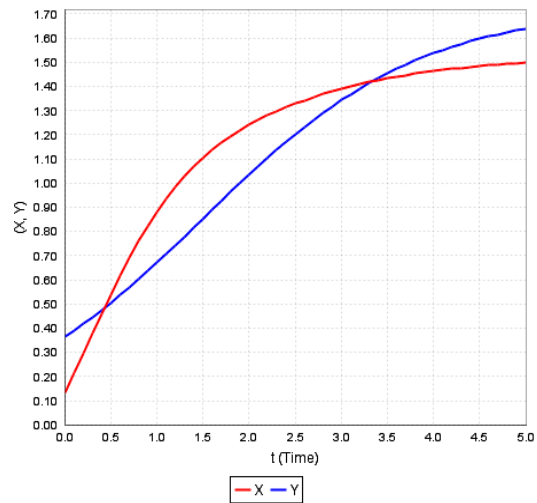
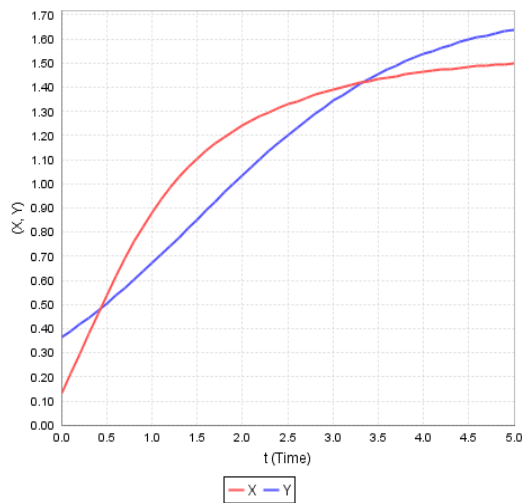
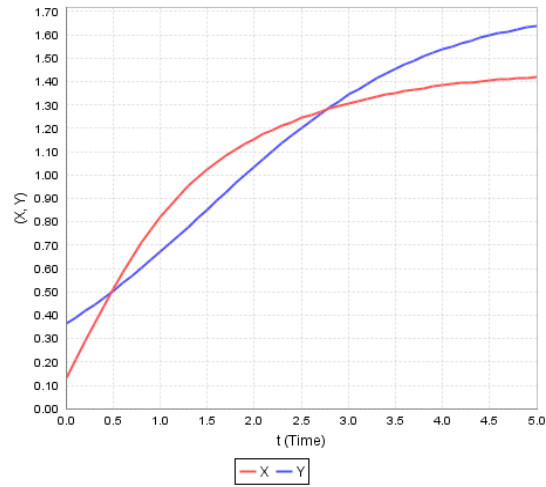
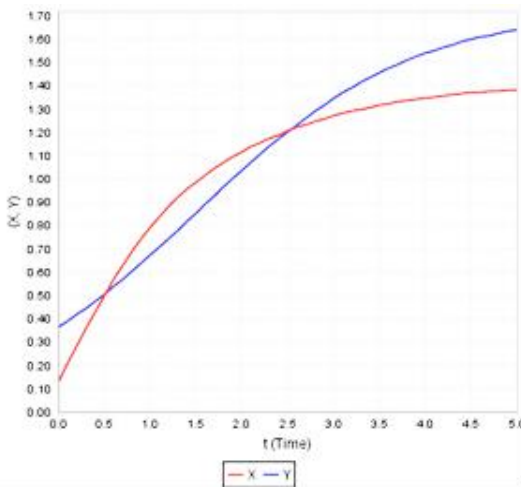
The impact of changing the parameter a on the model in view of possible Dominance time instincts.

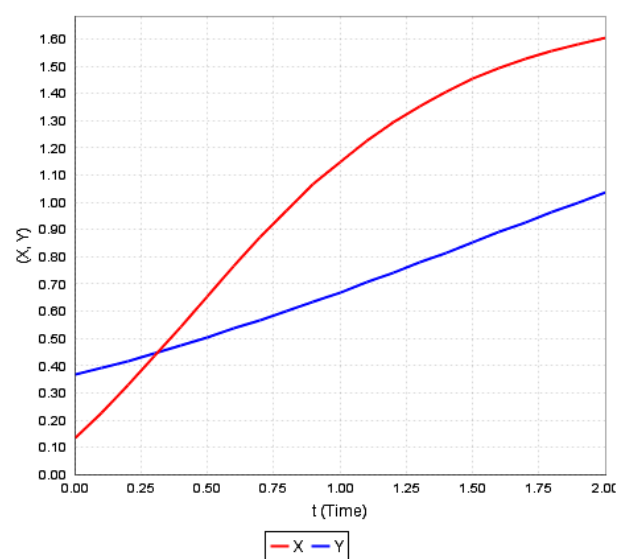
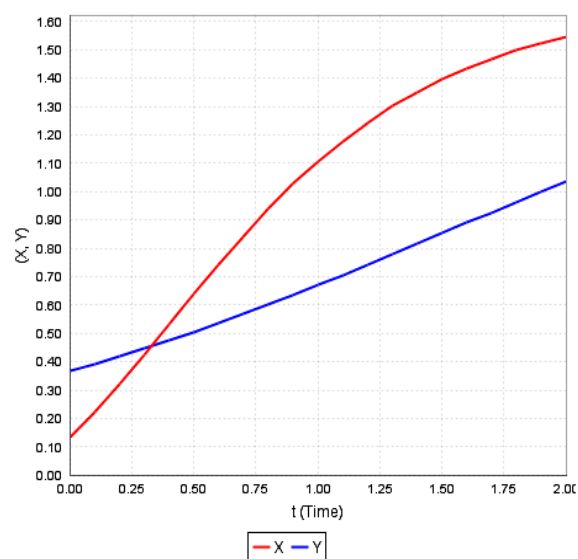
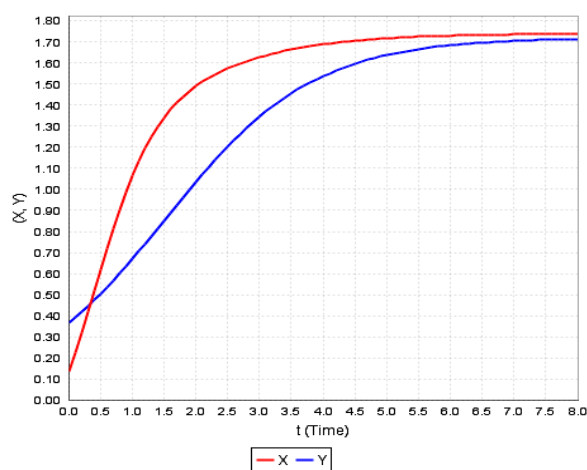
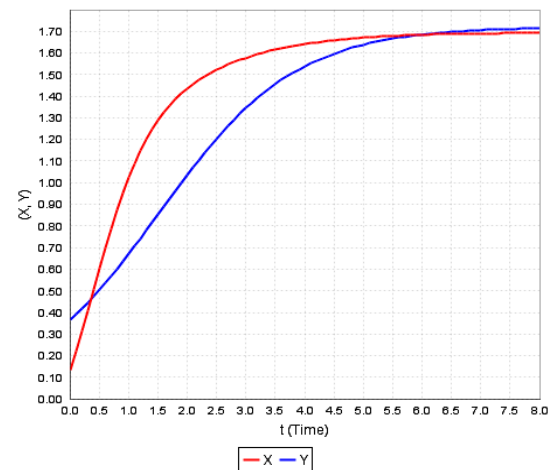
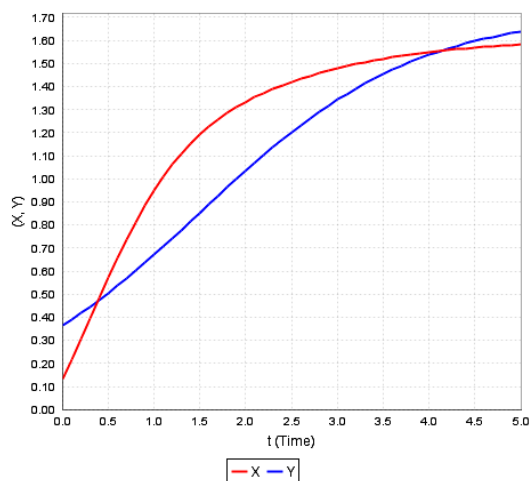
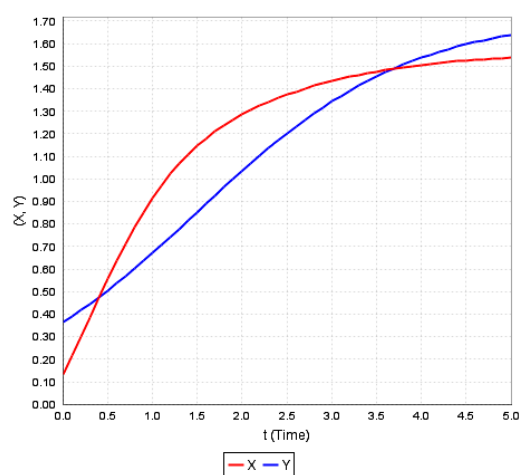
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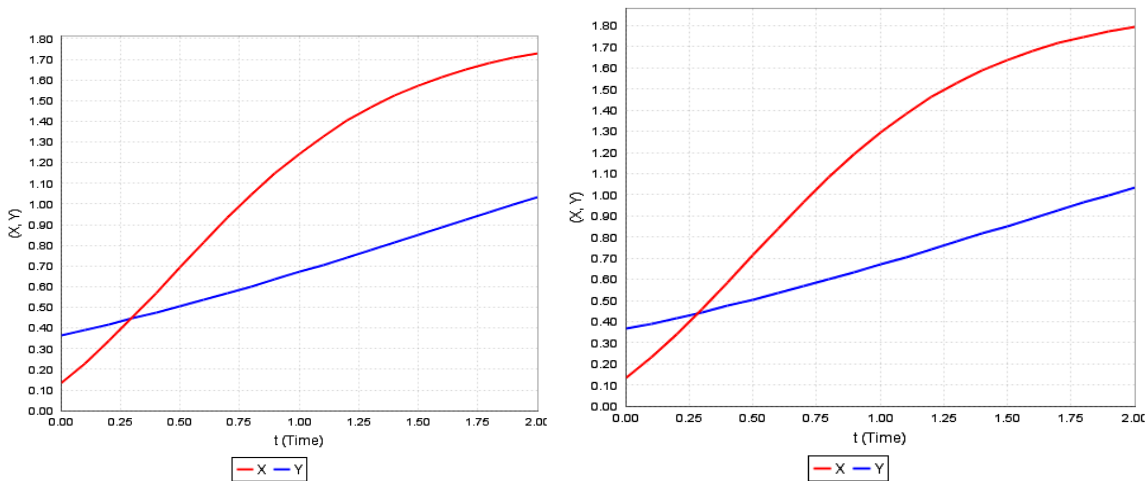
S.NO	a	t_1^*	t_2^*
1	0	0.5	2.5
2	0.1	0.5	2.7
3	0.2	0.5	3.0
4	0.3	0.4	3.3
5	0.4	0.4	3.7
6	0.5	0.3	4.0
7	0.6	0.3	4.5
8	0.7	0.3	5.9
9	0.8	0.3	*
10	0.9	0.275	*
11	1.0	0.275	*
12	1.1	0.275	*

Fig(1) to Fig(12): when $b=0.889, c=2.515, d=0.875, e=0.67, f=0.859, g=0.5, X_0=0.134, Y_0=0.366$

Vs change in b





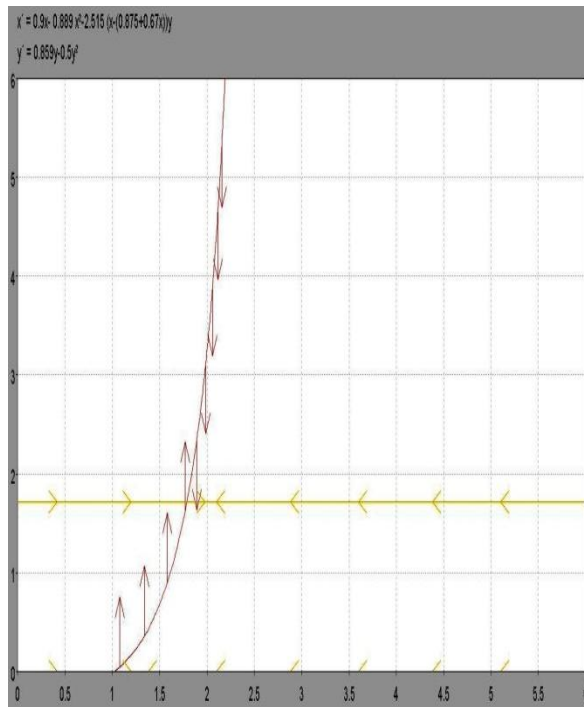


4. Phase Plane Analysis:

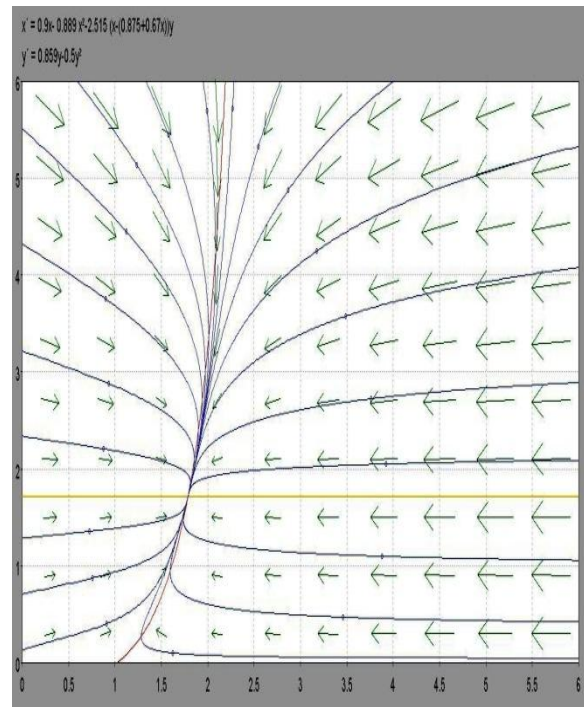
Phase plane analysis has been carried out to analyze the model for establishing stability with equilibrium points which are associated with Eigen values.

When $a=Changes, b=0.889, c=2.515, d=0.875, e=0.67, f=0.859, g=0.5$

S.No	Values of Parameters	Equilibrium Point	Jacobian matrix	Eigenvalues	eigenvectors
1	0.1	(1.4472, 1.718)	$\begin{bmatrix} -3.899 & 0.99952 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.899$	$E_1 = (0.31234, 0.94997)^T$ $E_2 = (0, 1)^T$
2	0.3	(1.524, 1.718)	$\begin{bmatrix} -3.8356 & 0.93576 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.835$	$E_1 = (0.2999, 0.95397)^T$ $E_2 = (0, 1)^T$
3	0.5	(1.6062, 1.718)	$\begin{bmatrix} -3.7817 & 0.86755 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.7817$	$E_1 = (0.28456, 0.95866)^T$ $E_2 = (0, 1)^T$
4	0.7	(1.694, 1.718)	$\begin{bmatrix} -3.7378 & 0.7947 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.737$	$E_1 = (0.2661, 0.9639)^T$ $E_2 = (0, 1)^T$
5	0.9	(1.7876, 1.718)	$\begin{bmatrix} -3.7041 & 0.71704 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.7041$	$E_1 = (0.24438, 0.96968)^T$ $E_2 = (0, 1)^T$



Phase Plane Figure(1)



Phase Plane Figure(2)

5.Conclusions: The change of Ammensal growth rate significantly influences on the Ammensal-Enemy Species. Initially Ammensal Species is dominated by Enemy Species, in a course time Ammensal dominates enemy species. Meanwhile dominance reversal time occurred two times at t_1^* and t_2^* . Here t_1^* gradually decreases and t_2^* Gradually increases.

6.Case(ii): The influence of b on Ammensal-Enemy Model:

For investigating the interaction between Ammensal and Enemy Species, the following values of concerned parameters are considered.

Table(3):

a	b	c	d	e	f	g	X_0	Y_0
1.1100	Changes	2.515	0.875	0.67	0.859	0.5	0.134	0.366

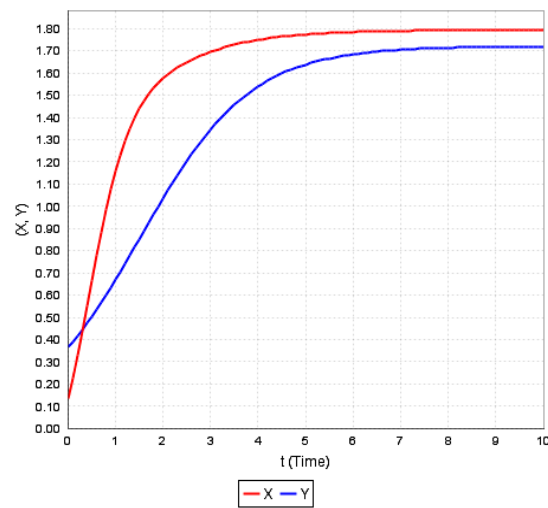
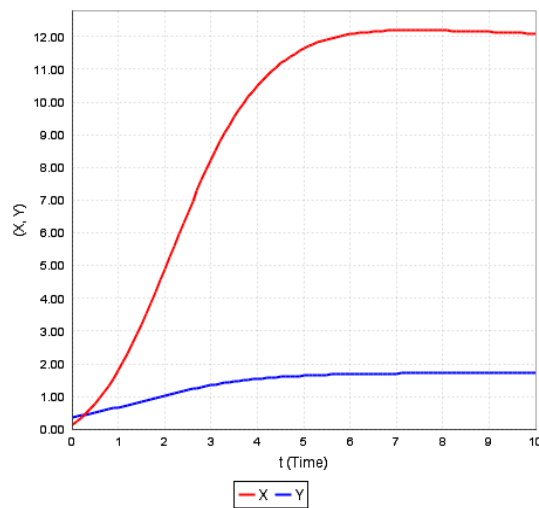
The impact of changing the parameter b on the model in view of possible Dominance time instincts .

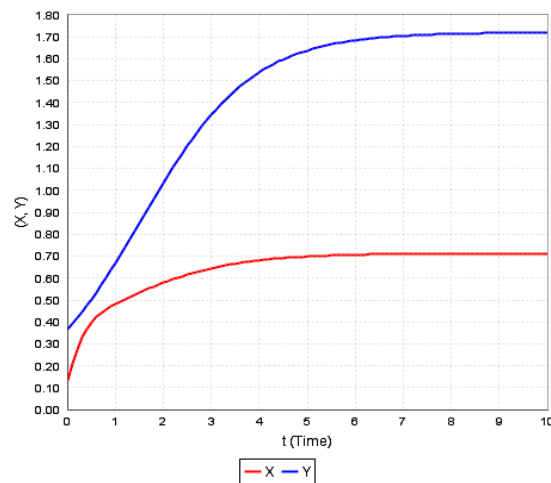
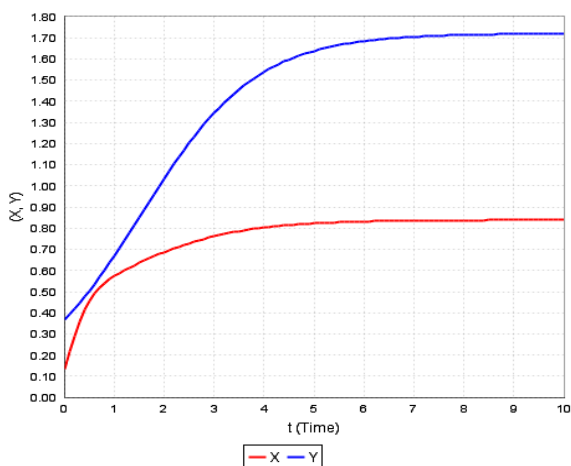
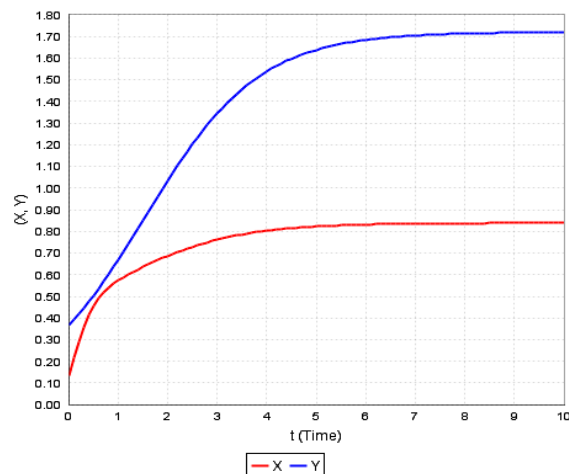
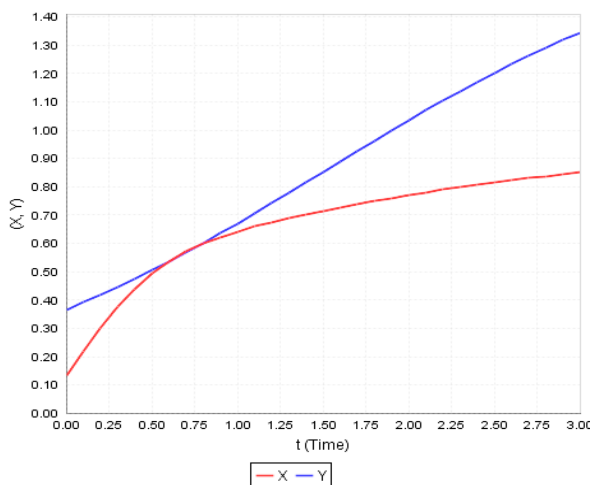
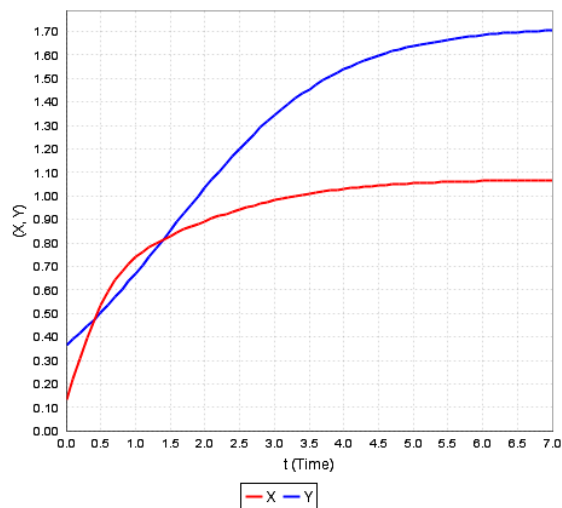
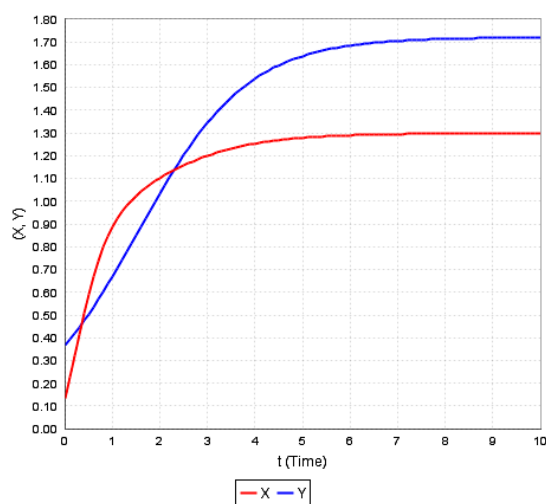
Table(4):

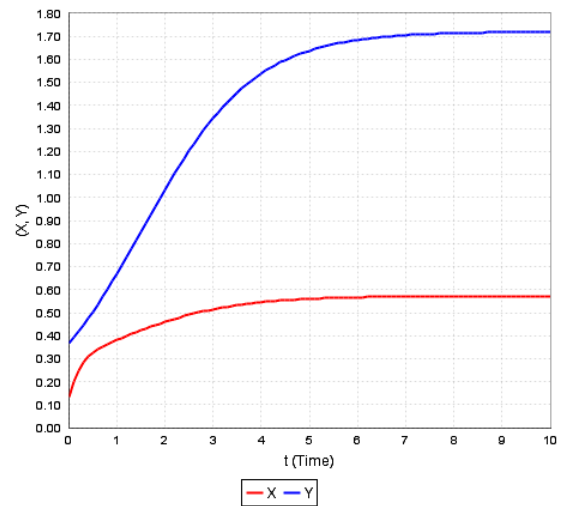
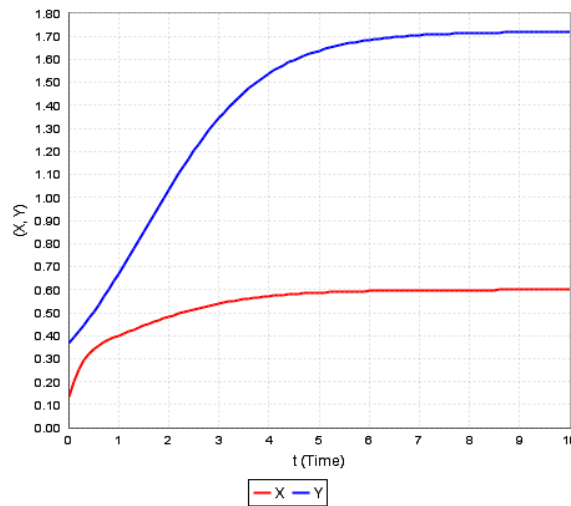
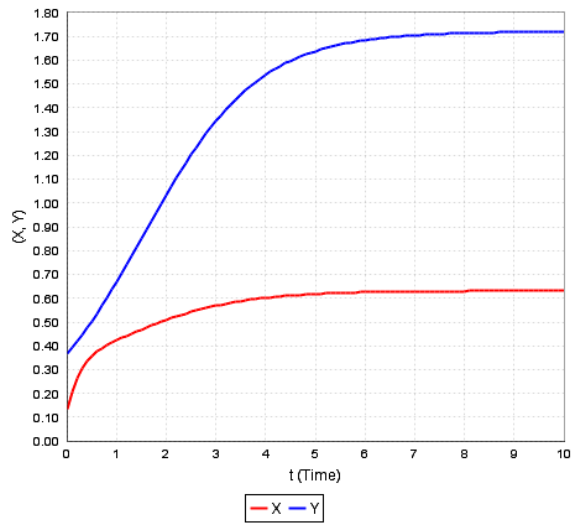
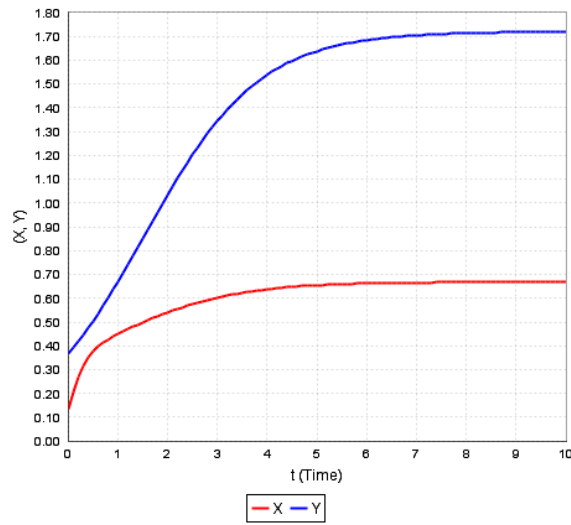
S.No	b	t_1^*	t_2^*
1	0	0.275	*
2	1	0.312	*
3	2	0.354	2.3
4	3	0.417	1.4
5	4	0.7	0.7
6	5	*	*
7	6	*	*
8	7	*	*
9	8	*	*
10	9	*	*
11	10	*	*
12	11	*	*

Fig(13) to Fig(24): when $a=1.1100, c=2.515, d=0.875, e=0.67, f=0.859, g=0.5, X_0=0.134, Y_0=0.366$

Vs change in b







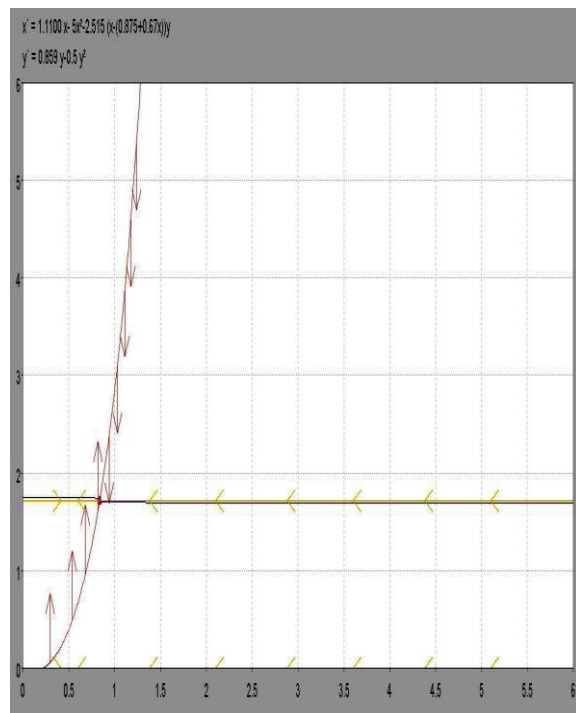
7. Phase Plane Analysis:

Phase plane analysis has been carried out to analyze the model with equilibrium points which are associated with Eigen values.

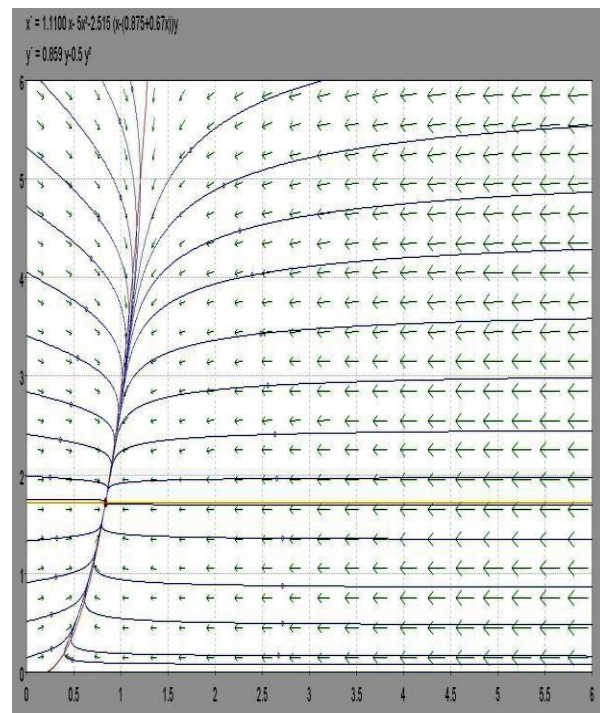
When $a=1.1100, b= \text{Changes}, c=2.515, d=0.875, e=0.67, f=0.859, g=0.5$

S.No	Values of b	Equilibrium Point	Jacobian matrix	Eigenvalues	eigenvectors
1	1	(1.7929, 1.718)	$\begin{bmatrix} -3.9016 & 0.7126 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.901$	$E_1 = (0.2280, 0.9736)^T$ $E_2 = (0, 1)^T$

2	2	(1.2982, 1.718)	$\begin{bmatrix} -6.743 & 1.3116 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -6.743$	$E_1 = (0.2348, 0.9720)^T$ $E_2 = (0, 1)^T$
3	3	(1.0712, 1.718)	$\begin{bmatrix} -6.743 & 1.3116 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -6.743$	$E_1 = (0.2175, 0.9760)^T$ $E_2 = (0, 1)^T$
4	4	(0.9335, 1.718)	$\begin{bmatrix} -7.784 & 1.4259 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -7.784$	$E_1 = (0.2016, 0.9794)^T$ $E_2 = (0, 1)^T$
5	5	(0.8385, 1.718)	$\begin{bmatrix} -8.7013 & 1.5047 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -8.7013$	$E_1 = (0.1884, 0.9820)^T$ $E_2 = (0, 1)^T$



Phase Plane Figure(3)



Phase Plane Figure(4)

8.Conclusions: The change of Ammensal growth rate significantly influences on the Ammensal –Enemy Species. Initially Enemy Species is dominated by Ammensal Species, in a course of time Enemy dominates Ammensal species. Meanwhile dominance reversal time occurred two times at t_1^* and t_2^* . Here t_1^* gradually increases and t_2^* Gradually decreases.

9. Overall Conclusions:

Change in the Parameter	Dominance Time	Nature
'a' increases	' t_1^* ' decreases ' t_2^* ' increases	Dominance is reversed
'b' increases	' t_1^* ' increases ' t_2^* ' decreases	Dominance is reversed

10. References

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