



# Analysis of the Performance of Zero Forcing and the MMSE Equalizer on a MIMO System in 5G Communication

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## Abstract

One of the main settings in wireless communication research is multiple communication antennas. MIMO is the best example of how quickly wireless communication is currently progressing. The effects of fading and interference on wireless transmission can be minimized by using an equalizer. It causes an issue for wireless communication signal recovery due to fading and interference. The MIMO system makes use of many transmit and receive antennas to benefit from multipath propagation in a busy environment. Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) equalizer performance for 22 and 44 MIMO wireless channels is examined in this article. Simulation data can be delivered to the RF processing lab using MATLAB Toolbox 2015a. Several benefits and drawbacks of the system are explained as the Bit Error Rate (BER) features for different communication antennas are simulated in the MATLAB toolbox. According to the simulation results, equalizer-based zero-forcing receivers are useful for noise-free channels and effective at reducing ISI, however MMSE is a better option than ZF in terms of BER characteristics.

**Keywords—** ISI, Bit Error Rate (BER), BPSK, Maximal Ratio Combining (MRC), 2×2 MIMO channel, MIMO system, MMSE equalizer, Signal to Noise Ratio (SNR), ZF equalizer.

## I INTRODUCTION

Many Input The term "Multiple Output" (MIMO) refers to a wireless communication antenna configuration technique that makes use of several antennas on both the transmitter and reception sides. In order to minimize errors and maximize data speed, the antennas at both ends of the communications circuit are merged. The MIMO system employs several antennas to send numerous concurrent signals. Due to the use of many antennas at the transmitter and receiver, or MIMO techniques, it is possible to achieve higher data rates compared to single antenna systems. With the use of space-time codes, a point-to-point (for single user) MIMO communication system can achieve significant increases in data transmission rates and reliability (diversity gain oriented). This method gives wireless systems a larger channel capacity and can expand the channel's capacity linearly with the many antennas and link range without needing more bandwidth or power. For a number of services, including video, high-quality audio, and mobile integrated service digital network, data transmission at high bit rates is crucial in mobile communication systems. The channel impulse response can spread over

multiple symbol periods and generate inter-symbol interference when data is transmitted through mobile radio channels at greater data transmission speeds (ISI). The performance of ZF and MMSE equalization techniques is discussed in this study by taking into account both the 2x2 MIMO channel scenario with 2 transmitting and 2 receiving antennas and the 4x4 transmitting and 4 receiving antenna scenarios (resulting in a 4x4 MIMO channel). Take into account the BPSK modulation approach for the channel in this study along with the flat fading Rayleigh multipath.

With high data rates, the ultimate goal is to enable global personal and multimedia communication that is independent of location or mobility. A dynamic equalization's primary goal is to eliminate the ISI. To lessen this interference, equalizers are used. It is evident from the theoretical analysis of the MIMO system that each transmit antenna sends out a variety of signals to enable the receiver to receive the transmitted signals with ease. The receiver must solve a system of linear equations to demodulate the message because all signals are only broadcast once from all components.

This essay is structured as follows: MIMO System explanation is in Section II. ZF Equalizer explanation is in Section III. In Part IV, the MMSE Equalizer is explained. The simulation model is found in Section V. Discussion and Simulation Outcome are included in Section VI. The Conclusion is then presented in Section VII.

**II MIMO SYSTEM**

Multiple-Input Multiple-Output (MIMO) is a wireless technology known as multiple-input multiple-output (MIMO), as depicted in Fig. 1, boosts an RF radio's data capacity by using many transmitting and receiving antennas. The same data is delivered via several antennas over the same path and bandwidth in a MIMO system. Since each signal takes a different route to the receiving antenna as a result, the data obtained is more trustworthy. Moreover, a factor based on the quantity of broadcast and receive antennas affects the data rate.

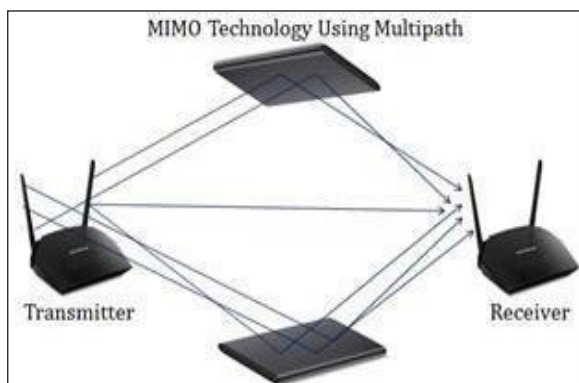


Fig. 1: MIMO Technology Using Multipath

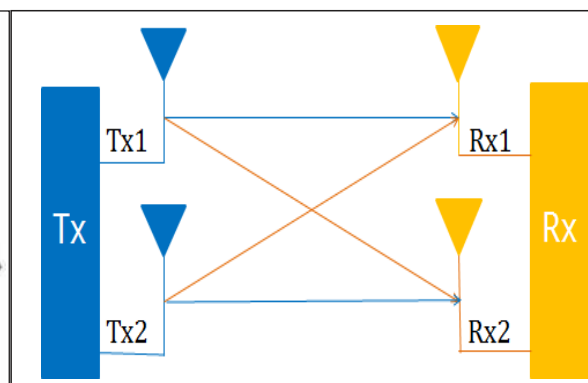


Fig. 2: 2 Transmit 2 Receive (2x2) MIMO Channel

A MIMO technology that concentrates on radio wave phenomena includes multipath as one of its components. It receives the received signal at various times and from various angles since the broadcast information is interrupted at various locations as a result of bounces off of walls, ceilings, and other objects. Multipath initially

involved interference, which led to a slow decline in wireless signals as a result of the interference.

MIMO uses its benefits to improve connection reliability, performance, and receiver diversity as well as receiver diversity and receiver signal-capturing power. MIMO thereby combines data streams that arrive from various paths and at various times.

Often, higher speeds correspond to more antennas. A wireless adapter with three antennae has a 600 Mbps maximum speed. The speed of a two-antenna adaptor is 300 Mbps. For the router to achieve the maximum speed possible, multiple antennas and complete support for all 802.11n capabilities are required..

#### A. *2x2 MIMO Channel*

The two transmitting antennas that are available in a 2x2 MIMO channel might likely be used in the following ways:

Consider that we have a transmission sequence, for example  $\{x_1, x_2, x_3, x_4, \dots\}$ .

- In a typical transmission, we would send  $x_1$  during the first time slot,  $x_2$  during the next,  $x_3$ , and so on.
- We may divide the symbols into groups of two as we now have two transmitting antennas. Send  $x_1$  and  $x_2$  in the first time slot using the first and second antennas. Send  $x_3$  and  $x_4$  from the first and second antenna in the second time slot, then  $x_5$  and  $x_6$  in the third, and so on.
- Keep in mind that since we are grouping two symbols and sending them in a single time slot, the number of time slots required to finish the transmission is simply  $n/2$ . Hence, the data rate has been doubled.

With two transmitting antennas and two receiving antennas in the MIMO transmission method, this type of interpretation is quite straightforward.

### III ZERO FORCING EQUALIZER

In order to recover the signal, a technique known as zero forcing equalizer applies inversely to the received signal. This particular type of graphical equalization method is popular in communication networks. Robert Lucky was the first to put up this kind of equalizer.

There are many practical uses for it. In order to accurately determine the channel at each antenna and recover two or more streams from the received signal, for instance, a thorough understanding of the MIMO system in 802.11n is required.

Using the ZF equalization technique will enable one to set the Inter Symbol Interference (ISI) value for a noise-free channel to zero. When ISI is important compared to noise, this will be helpful. Where  $f$  is a channel frequency response, the Zero Forcing Equalizer  $C(f)$  is denoted by  $C(f)$

$= 1/F(f)$ .  $F(f)C(f) = 1$  and a flat frequency response are displayed by the channel and equalizer together.

#### A. *Zero Forcing (ZF) Equalizer for 2x2 MIMO Channel*

If we take a look at a 2x2 MIMO channel, the signal received on the first receive antenna is,

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 = [h_{11} \ h_{12}][x_1 \ x_2] + n_1 \quad (1)$$

On the second receive antenna, the signal is received as follows:



$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}][x_1 \ x_2] + n_2 \quad (2)$$

Where,

$y_1, y_2$  are the received symbol on the first and second antenna respectively,

$h_{1,1}$  is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$  is the channel from 2nd transmit antenna to 1st receive antenna,

$h_{2,1}$  is the channel from 1st transmit antenna to 2nd receive antenna,

$h_{2,2}$  is the channel from 2nd transmit antenna to 2nd receive antenna,

$x_1, x_2$  are the transmitted symbols and  $n_1, n_2$  is the noise on 1, 2 receive antennas.

The equation can be represented in matrix notation as follows:

$$[y_1 \ y_2] = [h_{1,1} \ h_{1,2} \ h_{2,1} \ h_{2,2}][x_1 \ x_2] + [n_1 \ n_2] \quad (3)$$

Equivalently,

$$Y = HX + N \quad (4)$$

Where,

$Y$  = Received Symbol Matrix,  $H$  = Channel Matrix,

$X$  = Transmitted Symbol Matrix,  $N$  = Noise Matrix.

To solve for  $x$ , we need to find a matrix  $W$  which satisfies  $WH = I$ . The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (HH^H)^{-1} H^H \quad (5)$$

Where,

$W$  - Equalization Matrix, and  $H$  - Channel Matrix.

The pseudo inverse for a general  $m \times n$  matrix is this matrix, where

$$H^H H = [h_{1,1} \ h_{2,1}]^* [h_{1,1} \ h_{2,1} \ h_{1,2} \ h_{2,2}] [h_{1,1} \ h_{1,2} \ h_{2,1} \ h_{2,2}] \\ = [ |h_{1,1}|^2 + |h_{2,1}|^2 \ h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} \ |h_{1,2}|^2 + |h_{2,2}|^2 ] \quad (6)$$

**B. BER with ZF Equalizer with 2x2 and 4x4 MIMO**

Notably, the  $HH^H$  matrix's off-diagonal terms are not zero. Since off-diagonal terms are not zero, the Zero Forcing Equalizer tries to null out the interfering terms when performing the equalization; for example, when solving for  $x_1$ , the interference from  $x_2$  is tried to be nulled, and vice versa. Amplification of noise may occur while doing this. The Zero Forcing equalizer is therefore not the ideal equalizer for the task.

The implementation is straightforward and rather simple. Additionally, it can be seen that the channel for symbols transmitted from each spatial dimension (space is an antenna) is similar to a 1x1 Rayleigh fading channel after zero-forcing equalization.

In Rayleigh fading with Zero Forcing equalization, the BER for 2x2 and 4x4 MIMO channels is therefore comparable to the BER computed for a 1x1 channel in Rayleigh fading.

The bit error rate is calculated for BPSK modulation in a Rayleigh fading channel as,

$$P_b = \frac{1}{2} (1 - \sqrt{E_b / N_0}) \quad (7) \text{Where}$$

$P_b$  - Bit Error Rate

$\frac{E_b}{N_0}$  - Signal to Noise Ratio

$N_0$

#### IV MINIMUM MEAN SQUARE ERROR EQUALIZER

In a traditional method, the Mean Square Error (MSE) is calculated and an attempt is made to reduce the mistake by using a Minimal Mean Square Error (MMSE). Because of this, it alludes to the most accurate common indicator of estimate quality. The primary characteristic of MMSE is that it partially removes ISI but reduces the overall power of the noise and ISI components in the output. Assume that  $y$  is a known random variable and that  $x$  is an unknown random variable. Each function of the measurement  $y$  constitutes an estimator  $\hat{x}(y)$ , and its mean square error is defined as,

$$MSE = E \{ (X - \hat{X})^2 \} \quad (8)$$

where  $x$  and  $y$  are taken into account along with the expectation. The term  $AY + b$  is used to obtain a minimum MSE overall estimate, also referred to as a linear MMSE estimator.  $A$  and  $b$  are matrices and vectors, respectively, if the measurement  $Y$  is a random vector.

##### A. Minimum Mean Square Error Equalizer for 2x2 MIMO Channel

If we take a look at a 2x2 MIMO channel, the signal received on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}][x_1 \ x_2] + n_1 \quad (9)$$

On the second receive antenna, the signal is received as follows:

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}][x_1 \ x_2] + n_2 \quad (10)$$

$y_1, y_2$  are the received symbol on the first and second antenna respectively.

$h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to 1<sup>st</sup> receive antenna,

$h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 1<sup>st</sup> receive antenna,

$h_{2,1}$  is the channel from 1<sup>st</sup> transmit antenna to 2<sup>nd</sup> receive antenna,

$h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 2<sup>nd</sup> receive antenna,

$x_1, x_2$  are the transmitted symbols and

$n_1, n_2$  is the noise on 1<sup>st</sup>, 2<sup>nd</sup> receive antennas.

Matrix notation can be used to display the equation as follows:

$$[y_1 \ y_2] = [h_{1,1} \ h_{1,2} \ h_{2,1} \ h_{2,2}][x_1 \ x_2] + [n_1 \ n_2] \quad (11) \text{Equivalently,}$$

$$Y = HX + N \quad (12)$$

Where,

$Y$  = Received Symbol Matrix,  $H$  = Channel Matrix,



$X$  = Transmitted Symbol Matrix,  $N$  = Noise Matrix.

Finding a coefficient  $W$  that minimizes the criterion is the goal of the Minimal Mean Square Error (MMSE) technique.

$$E \{ [W_{y-x}] [W_{y-x}]^H \} \quad (13)$$

$W$ -Equalization Matrix,  $H$ -Channel Matrix,  
 $n$  - Channel Noise and  $y$  - Received signal.

Finding a matrix  $W$  that satisfies  $WH = I$  is necessary to solve for  $x$ . Given by, is the Minimal Mean Square Error (MMSE) detector for satisfying this condition.

$$W = [H^H H + N_o I]^{-1} H^H \quad (14)$$

The pseudo inverse for a general  $m \times n$  matrix is this matrix, where

$$H^H H = [ \begin{matrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{matrix} ]^* [ \begin{matrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{matrix} ] \\ = [ |h_{1,1}|^2 + |h_{2,1}|^2 \quad h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} \quad |h_{1,2}|^2 + |h_{2,2}|^2 ] \quad (15)$$

In the Zero Forcing Equalizer, two equations, namely equations (5) and (14) are compared. Both equations are similar, with the exception of  $N_o I$ .

Indeed, the channel from the second broadcast antenna to the first receive antenna is where the MMSE equalizer minimizes to the Zero Forcing equalizer when the noise term's value is zero.

## I. SIMULATION MODEL

- The system was designed using the steps listed below.
- Create an arbitrary binary sequence of +1s and -1s.
- These should be paired up into two symbol groups, and two symbols should be sent at once.
- Following the multiplication of the symbols by the channel, white Gaussian noise is introduced.
- Equilibrate the symbols you received.
- To plot the theoretical and simulation findings by iterating for various values.
- To plot the simulation and theoretical findings for various values.

## II. SIMULATION RESULT AND DISCUSSION

### A. Simulation Analysis for ZF

Let's now talk about the performance characteristics of the ZF equalizer receiver as simulated by BER. According to expectations, the simulated outcomes for a 22 MIMO system employing BPSK modulation in the Rayleigh channel are displaying findings that are identical to those produced for the 11 system for BPSK modulation in the Rayleigh channel depicted in Fig. 3. Our data rate boost is made possible by the ZF equalizer. In some channel situations, we might not be able to increase data rates by two. It is possible for channels to be

correlated (the coefficients are almost the same).

BER for 2x2 MIMO with BPSK modulation and ZF equalizer (Rayleigh channel) (13)

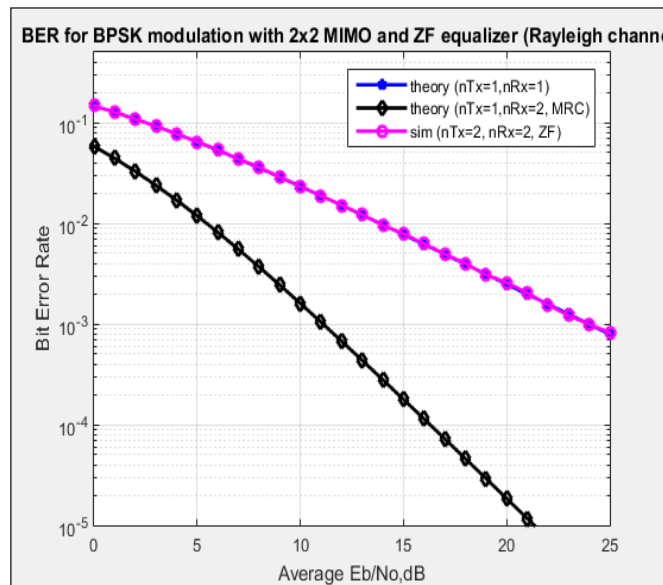


Fig. 3: 2x2 MIMO Channel BER Figure with ZF Equalizer (BPSK Modulation in Rayleigh Channel)

BER for 4x4 MIMO with BPSK modulation and ZF equalizer (Rayleigh channel).

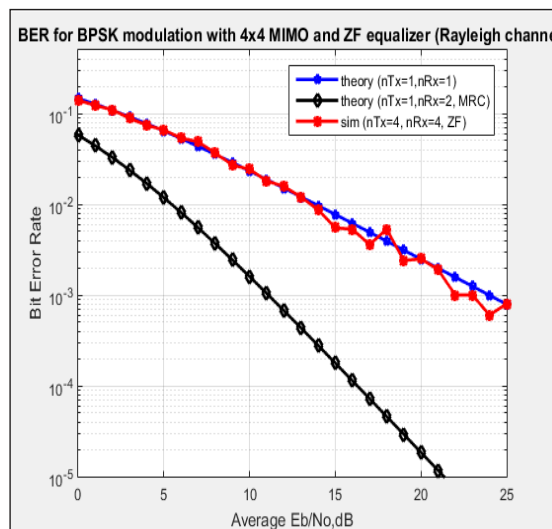


Fig. 4: 4x4 MIMO Channel BER Figure with ZF Equalizer (BPSK Modulation in Rayleigh Channel)

According to Fig. 4, as SNR rises, BER drops and the availability of receiving antennas improves. Fig. 4 depicts a 4x4 MIMO scenario with some discontinuity brought on by interference effects.

It is noted the following things. Only when the channel is noiseless does the Zero Forcing Equalizer, which eliminates all ISI, remain perfect. When the channel is noisy, the Zero Forcing Equalizer performs poorly,

greatly amplifies the noise, and has a tiny magnitude (i.e. near to zero). For wireless communication links, ZF Equalizer is insufficient since it fully disregards the noise effect. On static channels with high SNR, though, it works nicely.

*B. Simulation Analysis for MMSE*

BER for 2x2 MIMO with BPSK modulation and MMSE equalization (Rayleigh channel).

Fig. 5 shows that for MMSE equalizer the number of receiver and transmitter is 2x2.

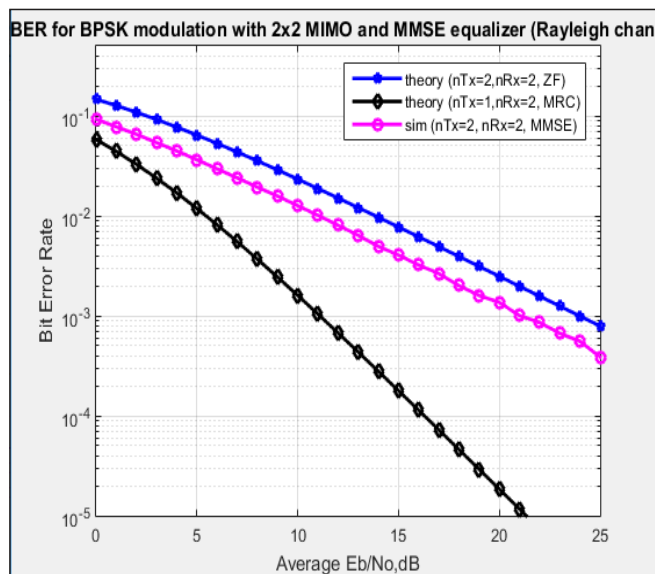


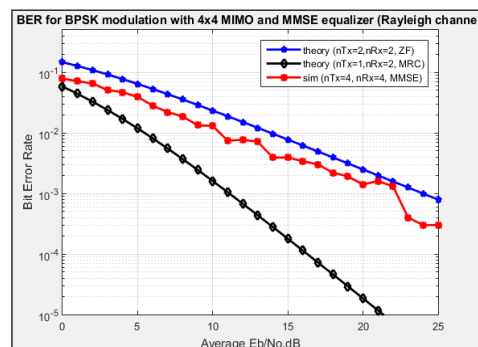
Fig. 5: 2x2 MIMO channel BER plot with MMSE Equalizer (BPSK Modulation in Rayleigh Channel)

The two equalizers, ZF and MMSE, are demonstrated in Fig. 5 from the above. It has been found that the Bit Error Rate (BER) for MMSE reduces more quickly than ZF with an increase in SNR, and MMSE performs better than the ZF equalizer. Also, MMSE performs better than ZF.

BER for BPSK modulation using an MMSE equalizer and 4x4 MIMO (Rayleigh channel).

Fig. 6: 4x4 MIMO channel BER plot with MMSE Equalizer (BPSK Modulation in Rayleigh Channel)

According to Fig. 6, it exhibits some discontinuity brought on by the interference-effect. According to the aforementioned graphs, BER will decrease if the number of receiving and transmitting antennas in the





MMSE equalizer is increased.

The ZF and MMSE equalizers' BER values in the (2x2) and (4x4) MIMO systems are compared in the table below. The Table shows that the BER values steadily decline. Also, this table shows that the simulated bit error rate (simBer) of the MMSE in both the (2x2) and (4x4) MIMO systems is significantly lower than the ZF.

TABLE I: BER Values Comparison between The ZF AND MMSE Equalizer IN (2x2) AND (4x4) MIMO System

$E_b$ $N_0$	BER Values for ZF and MMSE Equalizer in(2x2) and (4x4) MIMO System			
	simBerZF 2x2	simBer MMSE 2x2	simBerZF 4x4	simBer MMS E 4x4
0	0.1450	0.0945	0.1363	0.0884
1	0.1307	0.0804	0.1313	0.0681
2	0.1057	0.0673	0.1054	0.0596
3	0.0935	0.0531	0.0834	0.0495
4	0.0740	0.0441	0.0772	0.0477
5	0.0603	0.0352	0.0679	0.0386
6	0.0537	0.0285	0.0493	0.0279
7	0.0437	0.0243	0.0407	0.0268
8	0.0340	0.0182	0.0380	0.0197
9	0.0329	0.0152	0.0311	0.0170
10	0.0222	0.0123	0.0234	0.0133
11	0.0171	0.0099	0.0196	0.0089
12	0.0162	0.0080	0.0168	0.0086
13	0.0138	0.0057	0.0123	0.0058
14	0.0107	0.0065	0.0115	0.0053
15	0.0074	0.0047	0.0077	0.0030
16	0.0059	0.0031	0.0064	0.0040
17	0.0063	0.0026	0.0035	0.0025
18	0.0043	0.0027	0.0032	0.0018
19	0.0031	0.0018	0.0034	0.0018
20	0.0020	0.0013	0.0018	0.0017
21	0.0019	0.0012	0.0019	0.0009
22	0.0011	0.0007	0.0016	0.0005



23	0.0009	0.0004	0.0014	0.0003
24	0.0014	0.0003	0.0007	0.0010
25	0.0005	0.0003	0.0007	0.0002

It is concluded that MMSE is more accurate and works well for a variety of channel state conditions stated in SNR ratio based on the simulation results examined thus far. An approximation method with a higher signal-to-noise ratio is the zero forcing method. Although it requires significantly more computational labor, the MMSE performs effectively for all signal to noise ratios.

### III. CONCLUSION

This work attempts to present specific details on the Minimal Mean Square Error (MMSE) and Zero Forcing (ZF) Equalizers, as well as to demonstrate a performance comparison between them based on the MIMO receiver. With all types of equalizers, all simulation results pertaining to the Bit Error Rate (BER) aspects will be displayed. For the analysis of simulation, this research makes use of the RF signal processing lab framework. It will be determined by reviewing all the simulation data that the Zero forcing equalization does not meet a certain requirement. First of all, it will perform poorly when the signal to noise ratio increases; hence, it is only thought to be a competent receiver in noise-free environments. The frequency response of the channel can be all zeros, but that cannot be changed. Thirdly, although having a finite length, the channel impulse response does not meet all requirements. ZF is not more effective than MMSE given these restrictions.

The MMSE equalizer-based receiver is more accurate and suitable for a wide variety of channel state conditions indicated in SNR ratio, according to the simulation findings for MMSE. The MMSE equalizer offers superior noise protection and eliminates a marginal noise, whereas the ZF equalizer increases the noise in the channel. Also, the outcomes of Zero Forcing and MMSE equalizers were simulated and compared. According to the outcomes, MMSE equalizer outperforms ZF equalizer.

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