International Journal of Advanced Technology in Engineering and Science
Vol. No. 11, Issue No. 05, May 2023
www.ijates.com
ijates
ISSN 2348 - 7550

A Mathematical Study on Some Special Aspects of a Peculiar Ecological(AEC) Model -Local Stability & Stochastic Analysis

B.Pushpa Latha[1]
[1]II M.SC (Mathematics) Student
Department of Mathematics, Bapatla Engineering College
Bapatla-522102, India

K.V.L.N.Acharyulu[2]
[2]Associate Professor, Department of Mathematics
Bapatla Engineering College, Bapatla-522102,India
Email: dr.kvlna@gmail.com

&

M.Poojitha Santoshi [3]
[3]II M.SC (Mathematics) Student
Department of Mathematics, Bapatla Engineering College
Bapatla-522102, India

Abstract

In this study, we investigate the biodiversity of a peculiar three-species ecosystem in great detail using Ammensal- EnemyHost- Commensal (AEC) species. These species have varying rates that are proportionate to the sizes of their respective populations. The present study focuses primarily on an examination of the existence of local stability. During the course of this research, stochastic analysis is also looked into for this ecological model, and at the end, some useful outcomes have been established as a result of this investigation.

Key words: *Stability, Local Stability, Routh-Hurwitz Criterion and Stochastic analysis*

*Mathematics Subject Classification34D10,34D20,34D23,*35C10, 92D25,92D40.

1. Introduction:

Mathematical models evolved into practical biological instruments. Models are developed in an acceptable manner and put to use in order to shed light on the interactions that take place between the physical components and the mechanism that lies beneath the system that is being investigated. In the process of designing experiments and analyzing data, the interaction that develops between the empirical research and the theoretical model could prove to be an essential component. K.V.L.N.Acharyulu & N.Ch. Pattabhi Ramacharyulu et al [2-9]. investigated the local equilibrium and global stability of many conceptual models of ecology with regards to intercultural aspects. In the field of computational ecological science, the majority of researchers [1,18] and numerous mathematicians [10-17] have contributed significant insights, helpful concepts, and exciting applications to the study of the behavior of various ecological models.

A biological culture that remains in a state that is more or less constant for a period of time that is sufficiently lengthy should have the natural ability to tolerate hazards that arise from the atmosphere in abundance. The capacity of an ecosystem to maintain this state is typically referred to as system stability. If the number of species that make up a group stays the same over a sufficiently long period of time, then that group is thought to be stable. On the other hand, there is a sophisticated theory of mathematical stability that does not deal with actual items but rather with mathematical models of those objects (and it has a wide variety of applications in the fields of science and engineering). Therefore, if we have an accurate model of the environment that is expressed in terms of differential equations, then the conventional approaches to equilibrium theory will be able to determine the steadiness of the natural society based on our model. In a broad sense, equilibrium refers to particular solutions that a system of equations can provide. If the model trajectories in the process are stable, then it's possible to assume that the group or environment is stable as well.

2 Notations Adapted for use with three different species:

Within this ecosystem, the population strength of Ammesnal species is denoted by the value $N_1(t)$, which is calculated using the normal rate of growth a_1.With a normal rate of growth of a_2, $N_2(t)$ represents the population density of the EnemyHost at the given time. The Population density of the Commensal Species is Denoted by the Parameter $N_3(t)$, Which Corresponds to the normal rate of growth a_3. a_{ii} refers to the rate of decline of N_i resulting from insufficient N_i resources where i =1,2,3.a_{12} can be interpreted as the rate of decrease of the Ammesnal (N_1) that is caused by inhibition by the EnemyHost (N_2). The rate of expansion of the commensal due to its effective attacks on the Ammesnal species (N1) is shown by the symbol a_{31}.$K_i(=a_i/a_{ii})$ is a representation of the carrying capabilities of N_i, where i = 1, 2, 3, respectively. The expression $P(=a_{12}/a_{11})$ is the representation of the Prey-Ammesnal suffering rate's co-efficient. The commensal coefficient, denoted by Q $(=a_{31}/a_{33})$, is presented here.

It is presumed that none of these variables or parameters under consideration have a negative value.

3 .Basic Equations:

The basic equations of a specific ecosystem for multi-evolving three species with Ammensal, Enemy host, and Commensal species are described as follows in non-linear ordinary differential equations:

(i) The equations of growth rate for Ammensal species (N_1):

$$\frac{dN_1}{dt} = a_{11}N_1\left((1-m_1)K_1 - N_1 - PN_2\right) \quad (3.1)$$

(ii) The equations of growth rate for EnemyHost species (N_2):

$$\frac{dN_2}{dt} = a_{22}N_2\left((1-m_2)K_2 - N_2\right) \quad (3.2)$$

(iii) The equations of growth rate for Commensal species (N_3):

$$\frac{dN_3}{dt} = a_{33}N_3\left((1-m_3)K_3 - N_3 + QN_2\right) \quad (3.3)$$

All possible equilibrium points $E^*\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$ are

(i) $\overline{N_1}=0;\ \overline{N_2}=0;\ \overline{N_3}=0$

(ii) $\overline{N_1}=(1-m_1)K_1;\ \overline{N_2}=0;\ \overline{N_3}=0$

(iii) $\overline{N_1}=0\ ;\ \overline{N_2}=(1-m_2)K_2\ ;\ \overline{N_3}=0$

(iv) $\overline{N_1}=0;\ \overline{N_2}=0;\ \overline{N_3}=(1-m_3)K_3$

(v) $\overline{N_1}=(1-m_1)K_1-(1-m_2)PK_2;\ \overline{N_2}=(1-m_2)K_2;\ \overline{N_3}=0$

where $(1-m_1)K_1 > (1-m_2)PK_2$

(vi) $\overline{N_1}=(1-m_1)K_1\ ;\ \overline{N_2}=0\ \ \overline{N_3}=(1-m_3)K_3$

(vii) $\overline{N_1}=0;\ \overline{N_2}=(1-m_2)K_2;\ \overline{N_3}=(1-m_3)K_3+(1-m_2)QK_2$

where $(1-m_3)K_3 > (1-m_2)QK_2$

(viii) $\overline{N_1}=(1-m_1)K_1-(1-m_2)PK_2\ ;\ \overline{N_2}=(1-m_2)K_2\ ;\ \overline{N_3}=(1-m_3)K_3+(1-m_2)QK_2$

where $(1-m_1)K_1 > (1-m_2)PK_2 \ \& \ (1-m_3)K_3 > (1-m_2)QK_2$

4. Study of Local Stability in Ecological (AEC) Model:

Local stability and Stochastic analysis are some of the aspects covered in this article as we attempt to determine the peculiar nature three-species mathematical model. Theorems that are pertinent to the study are developed.

Theorem (4.1):

The ecosystem which consists of Ecological Model with AEC species is locally stable at positive equlibrium point $E^*\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$

Only if $X_1 X_2 > X_3$

where $X_1 = a_{11}\overline{N_1} + a_{22}\overline{N_2} + a_{33}\overline{N_3}$, $X_2 = a_{11}a_{22}\overline{N_1}\,\overline{N_2} + a_{11}a_{33}\overline{N_1}\,\overline{N_3} + a_{22}a_{33}\overline{N_2}\,\overline{N_3}$

$X_3 = a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3}$ where $\Delta = \begin{bmatrix} -a_{11}\overline{N_1} & -Pa_{11}\overline{N_1} & 0 \\ 0 & -a_{22}\overline{N_2} & 0 \\ 0 & Qa_{33}\overline{N_3} & -a_{33}\overline{N_3} \end{bmatrix}$ with the internal existed conditions

$(1-m_1)K_1 - \overline{N_1} - P\overline{N}_2 = 0,\ (1-m_2)K_2 - \overline{N}_2 = 0,\ (1-m_3)K_3 - \overline{N_3} + Q\overline{N_2} = 0.$

Proof:

The characteristicequation of A is $\left|\Delta - \lambda I\right| = 0$

$$\begin{vmatrix} -a_{11}\overline{N_1} - \lambda & -Pa_{11}\overline{N_1} & 0 \\ 0 & -a_{22}\overline{N_2} - \lambda & 0 \\ 0 & Qa_{33}\overline{N_3} & -a_{33}\overline{N_3} - \lambda \end{vmatrix} = 0$$

$$\lambda^3 + \lambda^2\left(a_{11}\overline{N_1} + a_{22}\overline{N_2} + a_{33}\overline{N_3}\right) + \lambda\left(a_{11}a_{22}\overline{N_1}\,\overline{N_2} + a_{11}a_{33}\overline{N_1}\,\overline{N_3} + a_{22}a_{33}\overline{N_2}\,\overline{N_3}\right)$$

$$+ a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3} = 0$$

$$let\left(a_{11}\overline{N_1} + a_{22}\overline{N_2} + a_{33}\overline{N_3}\right) = X_1 > 0;$$

$$\left(a_{11}a_{22}\overline{N_1}\,\overline{N_2} + a_{11}a_{33}\overline{N_1}\,\overline{N_3} + a_{22}a_{33}\overline{N_2}\,\overline{N_3}\right) = X_2 > 0$$

$$a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3} = X_3 > 0$$

$$\Rightarrow \lambda^3 + X_1\lambda^2 + X_2\lambda + X_3 = 0$$

By arranging them in a Routh Array, the first column contains only positive numbers.

$\textit{Those are}\, 1>0,\ X_1>0,\ X_1X_2-X_3>0\ \therefore\ X_1X_2>X_3\ \ \&\, X_3>0$

Where $X_1=a_{11}\overline{N_1}+a_{22}\overline{N_2}+a_{33}\overline{N_3}$

$X_2=a_{11}a_{22}\overline{N_1}\,\overline{N_2}+a_{11}a_{33}\overline{N_1}\,\overline{N_3}+a_{22}a_{33}\overline{N_2}\,\overline{N_3}$

$X_3=a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3}$

$$\frac{X_1X_2-1.X_3}{X_1}=\frac{1}{a_{11}\overline{N_1}+a_{22}\ \overline{N_2}+a_{33}\ \overline{N_3}}\Big\{\Big[a_{11}{}^2a_{22}\ \ \overline{N_1}+a_{11}a_{22}{}^2\overline{N_2}\Big]\overline{N_1}\,\overline{N_2}$$

$$+\Big[a_{11}{}^2a_{33}\overline{N_1}+a_{11}a_{33}{}^2\overline{N_3}\Big]\overline{N_1}\,\overline{N_3}+\Big[a_{22}{}^2a_{33}\overline{N_2}+a_{22}a_{33}{}^2\overline{N_3}\Big]\overline{N_2}\,\overline{N_3}+2a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3}\Big\}\ >0$$

Hence**,** By Routh-Hurwitz criterion, the system is locally stableat $\mathrm{E}^*\left(\overline{\mathrm{N}_1},\overline{\mathrm{N}_2},\overline{\mathrm{N}_3}\right)$.

5. Stochastic Analysis of Ecological (AEC) Model:

The noise effect is shown by the following system of nonlinear ordinary differential equations on Ammensal- EnemyHost – Commensal species.

(i) The equation for the growth rate of Ammensal species (N_1):

$$\frac{\mathrm{dN}_1}{\mathrm{dt}}=(1-m_1)a_1N_1-a_{11}N_1^2-a_{12}N_1N_2+\alpha_1\Omega_1(t) \tag{5.1}$$

(ii) The equation for the growth rate of EnemyHost species (N_2):

$$\frac{\mathrm{dN}_2}{\mathrm{dt}}=(1-m_2)a_2N_2-a_{22}N_2^2+\alpha_2\Omega_2(t) \tag{5.2}$$

(iii) The equation for the growth rate of Commensal species (N_3):

$$\frac{\mathrm{dN}_3}{\mathrm{dt}}=(1-m_3)a_3N_3-a_{33}N_3^2+a_{32}N_3N_2+\alpha_3\Omega_3(t) \tag{5.3}$$

$\textit{Here}\ \Omega(t)=\big[\Omega_1(t),\Omega_2(t),\Omega_3(t)\big]$ represents a 3D Gaussian white noise process which satisfies $E\big[\Omega_i(t)\big]=0\ \textit{where}\ i=1,2,3\ \&\ \alpha_1,\alpha_2,\alpha_3\ \textit{are real constants},$

Where $\delta_{i,j}$ is the Kronecker symbol ; δ is the $\delta-$dirac function.

$N_1(t)=u_1(t)+s^*,\ N_2(t)=u_2(t)+p^*\ \&\ N_3(t)=u_3(t)+q^*$

$$\frac{dN_1}{dt}=\frac{du_1}{dt},\ \frac{dN_2}{dt}=\frac{du_2}{dt},\ \frac{dN_3}{dt}=\frac{du_3}{dt}$$

$$\frac{du_1}{dt}=-A_{11}u_1s^*-a_{12}u_2s^*+\alpha_1\,\Omega_1(t)\ where\ A_{11}=2a_{11} \quad (5.4)$$

$$\frac{du_2}{dt}=-A_{22}u_2p^*+\alpha_2\,\Omega_2(t)\ where\ A_{22}=2a_{22} \quad (5.5)$$

$$\frac{du_3}{dt}=-A_{33}u_3q^*+a_{32}u_2q^*+\alpha_3\,\Omega_3(t)\ \ where\ A_{33}=2a_{33} \quad (5.6)$$

Taking Fourier technique of (4.4),(4.5)&(4.6),we get

$$\text{Eq (5.4)}\ \Rightarrow\alpha_1\,\overline{\Omega_1}\,(\omega)=\left(i\omega+A_{11}s^*\right)\overline{u_1}(\omega)+a_{12}\,s^*\overline{u_2}(\omega) \quad (5.7)$$

$$\text{Eq (5.5)}\ \Rightarrow\alpha_2\,\overline{\Omega_2}\,(\omega)=\left(i\omega+A_{22}p^*\right)\overline{u_2}(\omega) \quad (5.8)$$

$$\text{Eq(5.6)}\ \Rightarrow\alpha_3\,\overline{\Omega_3}\,(\omega)=\left(i\omega+A_{33}q^*\right)\overline{u_3}(\omega)-a_{32}q^*\overline{u_2}(\omega) \quad (5.9)$$

The matrix representtaion of (5.7),(5.8) & (5.9) is $\Rightarrow M(\omega)\overline{\mathrm{u}(\omega)}=\Omega(\omega)$ (5.10)

$$\text{Here } \mathrm{M}(\omega)=\begin{bmatrix}\mathrm{M}_{11} & \mathrm{M}_{12} & \mathrm{M}_{13}\\ \mathrm{M}_{21} & \mathrm{M}_{22} & \mathrm{M}_{23}\\ \mathrm{M}_{31} & \mathrm{M}_{32} & \mathrm{M}_{33}\end{bmatrix};\ \overline{u}(\omega)=\begin{bmatrix}\overline{u_1}(\omega)\\ \overline{u_2}(\omega)\\ \overline{u_3}(\omega)\end{bmatrix};\ \overline{\Omega}(\omega)=\begin{bmatrix}\psi_1\overline{\Omega}_1(\omega)\\ \psi_2\overline{\Omega}_2(\omega)\\ \psi_3\overline{\Omega}_3(\omega)\end{bmatrix}$$

$$\text{where } \mathrm{M}(\omega)=\begin{bmatrix}-i\omega+A_{11}s^* & a_{12}s^* & 0\\ 0 & i\omega+A_{22}p^* & 0\\ 0 & -a_{32}q^* & i\omega+A_{33}q^*\end{bmatrix}$$

$$\Rightarrow\left|\det M(\omega)\right|=R(\omega)^2+I(\omega)^2$$

$$\Rightarrow\left|\det M(\omega)\right|^2=(-\omega A_{33}q^*-\omega^2A_{22}p^*-\omega^2A_{11}s^*+A_{11}A_{22}A_{33}s^*p^*q^*)^2$$

$$+(-\omega^3+\omega A_{22}A_{33}p^*q^*+A_{11}A_{33}s^*q^*+\omega A_{11}A_{22}p^*s^*)^2$$

$$From\ \left(4.10\right)\Rightarrow\overline{u}(\omega)=\left[M(\omega)\right]^{-1}+\overline{\Omega}(\omega)$$

$$\text{where } \left[\mathrm{M}(\omega)\right]^{-1}=k(\omega)=\frac{1}{\mathrm{M}(\omega)}\begin{bmatrix}\mathrm{M}_{11}{}^{CF(1,1)^T}(\omega) & \mathrm{M}_{21}{}^{CF(1,2)^T}(\omega) & \mathrm{M}_{31}{}^{CF(1,3)^T}(\omega)\\ \mathrm{M}_{12}{}^{CF(2,1)^T}(\omega) & \mathrm{M}_{22}{}^{CF(2,2)^T}(\omega) & \mathrm{M}_{32}{}^{CF(2,3)^T}(\omega)\\ \mathrm{M}_{13}{}^{CF(3,1)^T}(\omega) & \mathrm{M}_{23}{}^{CF(3,2)^T}(\omega) & \mathrm{M}_{33}{}^{CF(3,3)^T}(\omega)\end{bmatrix}$$

$$Now\ \sigma_{u_1}{}^2 = \frac{1}{2\pi}\sum_{i=1}^{3}\int_{-\infty}^{\infty}\alpha_i\left|\frac{M_{i1}{}^{CF(2,i)^T}}{|M(\omega)|}\right|^2 d\omega, \sigma_{u_2}{}^2 = \frac{1}{2\pi}\sum_{i=1}^{3}\int_{-\infty}^{\infty}\alpha_i\left|\frac{M_{i2}{}^{CF(2,i)^T}}{|M(\omega)|}\right|^2 d\omega,$$

$$\sigma_{u_3}{}^2 = \frac{1}{2\pi}\sum_{i=1}^{3}\int_{-\infty}^{\infty}\alpha_i\left|\frac{M_{i3}{}^{CF(3,i)^T}}{|M(\omega)|}\right|^2 d\omega \quad \left|M_{11}{}^{CF(1,1)^T}(\omega)\right|^2 = (a_{22}a_{33}\,p^*q^* - \omega^2)^2 + \omega^2(a_{22}\,p^* + a_{33}\,q^*)^2;$$

$$\left|M_{12}{}^{CF(2,1)^T}(\omega)\right|^2 = 0, \quad \left|M_{13}{}^{CF(3,1)^T}(\omega)\right|^2 = 0;$$

$$\left|M_{21}{}^{CF(1,2)^T}(\omega)\right|^2 = (A_{12}A_{33}s^*q^*)^2 + (A_{12}\omega s^*)^2;$$

$$\left|M_{23}{}^{CF(2,2)^T}(\omega)\right|^2 = (A_{11}A_{32}s^*q^*)^2 + \omega^2(A_{32}q^*)^2;$$

$$\left|M_{22}{}^{CF(3,2)^T}(\omega)\right|^2 = (A_{11}A_{33}s^*q^* - \omega)^2 + \omega^2(A_{33}q^* + A_{11}s^*)^2;$$

$$\left|M_{31}{}^{CF(1,3)^T}(\omega)\right|^2 = 0; \quad \left|M_{32}{}^{CF(2,3)^T}(\omega)\right|^2 = 0;$$

$$\left|M_{33}{}^{CF(3,3)^T}(\omega)\right|^2 = (A_{11}A_{22}s^*p^* - \omega^2)^2 + \omega^2(A_{11}s^* + A_{22}p^*)^2$$

$Case(1):\ If\ \alpha_1 = 0\ \&\ \alpha_2 = 0\ then\ \sigma_{u_1}{}^2 = 0,\quad \sigma_{u_2}{}^2 = 0$

$$\sigma_{u_3}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha_3\left[(A_{11}A_{22}s^*\,\mathrm{p}^* - \omega^2)^2 + \omega^2(A_{11}s^* + A_{22}p^*)^2\right]}{|M(\omega)|^2}d\omega$$

$Case(2):\ If\ \alpha_1 = 0\ \&\ \alpha_3 = 0$

$$then\ \sigma_{u_1}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha_2\left[(A_{12}A_{33}s^*\,\mathrm{q}^*)^2 + (A_{12}\omega s^*)^2\right]}{|M(\omega)|^2}d\omega$$

$$\sigma_{u_2}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha_2\left[(A_{11}A_{33}s^*\,\mathrm{q}^* - \omega)^2 + \omega^2(A_{33}q^* + A_{11}s^*)^2\right]}{|M(\omega)|^2}d\omega$$

$$\sigma_{u_3}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha_2\left[(A_{11}A_{32}s^*\,\mathrm{q}^*)^2 + (A_{32}\omega q^*)^2\right]}{|M(\omega)|^2}d\omega$$

$Case(3):\ If\ \alpha_2 = 0\ \&\ \alpha_3 = 0$

$$then\ \sigma_{u_1}{}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha_1\left[(A_{22}A_{33}p^*\,\mathrm{q}^* - \omega^2)^2 + \omega^2(A_{22}p^* + A_{33}\,\mathrm{q}^*)^2\right]}{|M(\omega)|^2}d\omega$$

$$\sigma_{u_2}{}^2 = 0;\quad \sigma_{u_3}{}^2 = 0$$

In order to accomplish this, stochastic analysis is carried out.

6.Conclusions:

The following observations and inferences can be made based on the findings of this research study, which is a thorough examination of an AEC ecosystem with three species:

(i).The Routh-Hurwitz criterion demonstrates that this model is stable on a local level.

(iii). The consistency of the system is successfully discussed through the use of stochastic analysis.

Acknowledgments:

The authors are grateful to '**Dr. Nazeer Shaik garu**, Principal, Bapatla Engineering College, Bapatla, **Our Magnanimous Management, Bapatla Education Society** and **HOD & Faculty Members**, Dept. of Mathematics, Bapatla Engineering College for their constant encouragement and valuable support for successfully completing our PG (M.SC-Maths) project **'A Mathematical Study on Some Special Aspects of A Peculiar Ecological Model'**

7. Reference:

[1].Kapur J.N.,Mathematical Modelling, Wiley Eser,1985.
[2].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Ammensal-Enemy Specie Pair With Limited And Unlimited Resources Respectively-A Numerical Approach, Int. J. Open Problems Compt. Math (IJOPCM), Vol. 3, No. 1, March 2010,73-91.
[3].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Enemy- Ammensal Species Pair With Limited Resources –A Numerical Study, Int. J. Open Problems Compt. Math (IJOPCM), Vol. 3, No. 3, September 2010,339-356,
[4].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; Mortal Ammensal and an Enemy Ecological Model with Immigration for Ammensal Species at a Constant Rate, International Journal of Bio-Science and Bio-Technology, Vol. 3, No.1, Marc 2011,39-48,
[5].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Immigrated Ecological Ammensalism with Limited Resources"- International Journal of Advanced Science and Technology, Vol. 27 ,2011, 87-92.
[6].K.V.L.N.Acharyulu and N.Ch.Pattabhi Ramacharyulu; A Numerical Study on an Ammensal - Enemy Species Pair with Unlimited Resources and Mortality Rate for Enemy Species"- International Journal of Advanced Science & Technology,Vol.30, May 2011,13-24.
[7].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Ecological Ammensalism with Multifarious restraints- A Numerical Study" International Journal of Bio-Science and Bio-Technology, Vol. 3, No. 2, June 2011,1-12.
[8].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; Multiple Constraints in Ecological Ammensalism- A Numerical Approach , Int. J. Advance. Soft Comput. Appl., Vol. 3, No. 2, July 2011,1-15.
[9].K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; On the Carrying capacity of Enemy Species, Inhibition coefficient of Ammensal Species and Dominance reversal time in An Ecological Ammensalism - A Special case study with Numerical approach, International Journal of Advanced Science and Technology, Vol. 43, June,2012,49-58.
[10].Lotka A.J.(1925). Elements of Physical Biology, Williams and willians, Baltimore,1925.

[11].Lakshmi Narayan K.(2005).A Mathematical study of a prey – predator Ecological Model with a partial cover for the prey and alternative food for the predator, Ph.D. Thesis, JNTU.
[12].Meyer WJ. Concepts of mathematical modelling. McGraw-Hill; 1985.
[13].Mesterton-Gibbons Michael. A technique for finding optimal two species harvesting policies. Ecol Modell 1996;92:235–44.
[14]. Paul Colinvaux A. Ecology. New York: John Wiley; 1986.
[15].Phani kumar N. Seshagiri Rao. N & Pattabhi Ramacharyulu N.Ch.,On the stability of a host -A flourishing commensal species pair with limited resources". International journal of logic based intelligent systems,3(1),2009,pp. 45-54.
[16].PhanikumarN.,Pattabhi ramacharyulu N.Ch.,A three species eco-system consisting of a prey predator and host commensal to the prey" International journal of open problems compt.math, 3(1),2010,pp.92-113.
[17].Srinivas NC. Some mathematical aspects of modelling in biomedical sciences. Ph.D. thesis. Kakatiya University; 1991.
[18].Volterra V. Leconssen La Theorie Mathematique De La Leitte Pou Lavie. Paris: Gouthier-Villars; 1931.

Brief Bio-data of the Authors:

B.Pushpa Latha: She is studying M.Sc(Maths),Second year, Department of Mathematics, Bapatla Engineering College. She presented a paper entitled 'A Mathematical Study on Some Special Aspects of a Peculiar Ecological(AEC) Model -Local Stability & Stochastic Analysis ' in 3rd International Conference on Emerging Trends and Researches in Professional Development of Faculties in Multi Disciplinary Domains conducted by Jan Nayak Ch. Devi Lal Vidyapeeth, Sirsa under the research work which was done in her PG project She completed her under graduation in Government degree college for women , Bapatla and secured good marks. She achieved a certificate of appreciation from DIGITAL LITERACY event. She got second price in quiz on science day celebrations in her degree college. Two research articles were published in Conference Proceedings and International peer reviewed Journals.

Dr.K.V.L.N.Acharyulu: He is working as Associate Professor in the Department of Mathematics, Bapatla Engineering College, Bapatla which is a prestigious institution of Andhra Pradesh. He took his M.Phil. Degree in Mathematics from the University of Madras and stood in first Rank ,R.K.M. Vivekananda College, Chennai. Nearly for the last twenty two years he is rendering his services to the students and he is applauded by one and all for his best way of teaching. He has participated in many seminars and presented his plenty of papers on various topics. More than 130 research articles were published in various International reputed and peer reviewed Journals. He obtained his Ph.D from ANU under the able guidance of Prof. N.Ch.Pattabhi Ramacharyulu, NIT, Warangal. He edited more than 100 books under his Editorship. He wrote more than 10 chapters and authored five books. He is a Member of Various Professional Bodies and created three world

International Journal of Advanced Technology in Engineering and Science
Vol. No. 11, Issue No. 05, May 2023
www.ijates.com
ijates
ISSN 2348 - 7550

M.Poojitha Santoshi : She is studying M.Sc(Maths),Second year, Department of Mathematics, Bapatla Engineering College. She presented a paper entitled 'A Mathematical Study on Some Special Aspects of a Peculiar Ecological Model -Global Stability & Diffusion Analysis ' in 3rd International Conference on Emerging Trends and Researches in Professional Development of Faculties in Multi Disciplinary Domains conducted by Jan Nayak Ch. Devi Lal Vidyapeeth, Sirsa under the research work which was done in her PG project. She completed her graduation in Chaitanya Bharathi Degree College, Andhra Pradesh. She got appreciated with many prizes and rewards in her graduation for getting good marks in Mathematics. She has quite interest in Maths from her childhood. She has a zeal to do research in Mathematics and publish valuable research Papers. Two research articles were published in Conference Proceedings and International peer reviewed Journals.