

Analysis of optimize the route of network system in an operational research approach

Rudraman, Deepika and Ompal Singh

Principal of constituent Govt. College, Bhadpura Nawabganaj, Bareilly (U.P.)

Department of mathematics, Swami Sharddhanand College, University of Delhi.

Department of mathematics, SRM Institute of science and Technology Delhi- NCR Campus, Modinagar, (U.P)

Abstract:

This paper presented two major reasons that may decide to move service unit. Multiplication of the probability of each individual link length. The state number is random. There been dependence between the lengths of the various links, the totaling of the state probability would be a little more complicated but certainly possible. The major benefit gained from making the link length discrete is that the numbers of possible states of the whole network turn out to be a finite number, and consequently good solution methods are available. We talk about a location model on a network whose link length has possible. We expand the model to deal not only with location but also with the analysis problem.

Keyword: Relocation, State of a Network, Stochastic network, Probabilistic travel- time, Markovian decision, Transition, probability.

Introduction:

When a certain idle server is sent to a calling node, the server is specific because the busy home node is label as empty go, and are later to call as long as there are presented servers, served by other available servers located at diverse nodes. The vacant node will be ready to supply service only after the service is engaged by the available unit, whose work has been done. Since the ratio of require in each node is known, it may be a good idea to reenter the idle server in some home nodes to be able to react to future call moves. A similar strategy has been realized for the redevelopment of fire companies in New York City (Burman and MR Rahman, 1985).

Approaches in New York City based on an approximation with reasonable assumption where the approach took the analytic in an critical framework Hypercube model.

The general situation of being subdivided into many models according to some assumption obligatory on the model (Green, L., 2001). Several of the achievable influence that affect the assumption are:

(1) Servers may be similar in terms of the service they offer, for example, if the service time distribution is negative exponentially then the same rate (mean service rate) is use for all servers.

(2) Servers can also be famous as having a relationship with their domestic nodes. In other words, the server will return to a blocked home node upon end of a task. Or, the server may be allied with specific home nodes and must return to a individual node.

(3) Repositions may occur at such nodes that the candidate is a possible repositioning, not to repository units into isolated home nodes, but only to a node relatively close by region. May be by nodes acting as home nodes.

(4) The exchange of in rank and letter facilities available to the dispatch center may vary at one extreme. The dispatcher can divert the servers while they are in the process of moving to a different home node.

We decide between two major reasons that may lead to the decision to go units. The FIR is tied to possible change in arrangement topology. When we quantify link length in unit of go time, it can often occur (Green, L, & Kolesar, 1984) that the travel time between some nodes is not constant, but varies. This incident can be cause by a change in traffic load at diverse times of the day or a change in endure conditions. For example, travel time in the city from a uptown position in the morning is longer than in the evening. In some cities, the system has with intent changed the traffic way of various lanes during a few hours. This can force a server to select a diverse route (Hall and R.W., 2009) to reach the occupation node. Another example may be in military cases where several transportation are available after dark.

Consequently, what is specific as the shortest route to serve a call? However, we would not like to change the plan for the entire place; as such change is not a short term decision. Therefore we design an interim solution by temporarily resubmitting a server (ie, Jarvis, J.P., 1991). Settlement decisions are certainly not exempt from cost considerations. It takes time to move a unit from one place to another. This is a factor in itself. However, if there is no real-time communication capability between the moving unit and the dispatch center. The dispatcher will not be able to divert the moving unit to the call node while the unit is traveling, so the response time may incur additional costs in terms of response time. These costs should not exceed the benefits. Gain is measured here mainly in terms of development in expected response time, which is our key pointer of system routine. The subsequent sections cover several concerns above. We first discuss a setting representation on a network whose link length has likely. Later we expand the model to deal not only with place but also with healing problems.

1. Location on a stochastic network:

The notion of a stochastic network with some link lengths suggests how such a network is located for these service units. In fact, the section could have been placed. However, we found it more convenient to limit the discussion to the periodic travel time, and present the major topic here, presenting the probability problem as an introductory section (Jordan and WC, 1986.) has gone. A sample network, consisting of five nodes and six links. As you can easily see, link length is not a market in figure 1. This would mean that the length be not stable, that is, the travel time is not deterministic. We would like to emphasize that figure 1 does not represent the geographic topology of the deal - for example, there is a link connecting nodes 2 and 3, while there is no direct link between nodes 2 and 5. However, in relation to travel time, their values are random variables. The probability distribution functions that provide values for each entity link. The most universal approach would

be to assume that (Larsen & R.C., 1999) the travel time for each link is given by a stable probability distribution function. This would, however, create a very complex problem. We therefore assume that each link can have a finite number of possible lengths, and a distinct probability distribution of length is given.

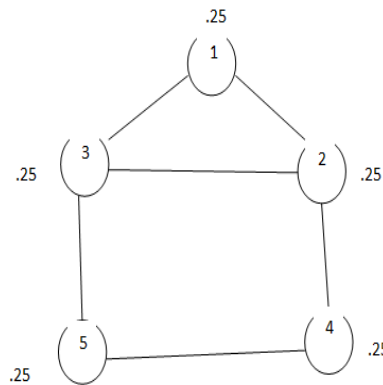


Figure 1. Sample stochastic network G.

We suppose that the top 2 and 3 are associated with the link values, say 1 and 9 units with probability of 0.4 and 0.6, respectively. Assume the link (4, 5) can also have two values, say 4 and 6, respectively, with probabilities of 0.5 and 0.5, and these probability do not depend on the people of link (2, 3), it Also that the rest of the links maintain a stable length, then the entire network can be in any of the four states as shown in Table 1.

State No.	Link(2,3)	Link(4,5)	State probability
1	1	4	0.2
2	1	6	0.2
3	9	4	0.3
4	9	6	0.3

Table 1. Four Possible States of a Sample Network.

The state probability in table 1 is simply the multiplication of the probability of each individual link length. The state number is random. There been dependence between the length of the various links, the totaling of the state probability would be a little more complicated but certainly possible. The major benefit gained from making (Halpern and J, 1999) the link length discrete is that the numbers of possible states of the whole network turn out to be a finite number, and consequently good solution methods are available. It is important to note that the outcomes of this model are likely to be more accurate than just using the expected length of a link as a substitute for the real travel time, which is quite common in perform. Consider turn back to the network G of figure 1. Assume that only the length of link (2, 3) may vary and value and have the values 1 and 9 with probability 0.4 and 0.6, respectively. The other links maintain deterministic lengths. The network can be in either one of two states, arbitrarily designated as state 1 and 2, with probabilities $\pi_1 = 0.4$ and $\pi_2 = 0.6$, respectively. From the

figure 2, where we have also set arbitrary values for the deterministic lengths. Choose $l_1(2,3) = 1$, that is, the length of link (2, 3) under state 1 is 1, similarly $l_2(2,3) = 9$. In a similar manner, we distinguish between shortest distance on the network when the network is in state 1, d_1 and in state 2, d_2 . In the current section, we assume that repositioning is not allowed, thus we would like to find permanent home nodes for two service units. However, we would like to take into account the stochastic nature of the network, and find the optimal location subject to possible change of link (2, 3).

State 1, $\pi_1 = 0.4$ State 2, $\pi_2 = 0.6$

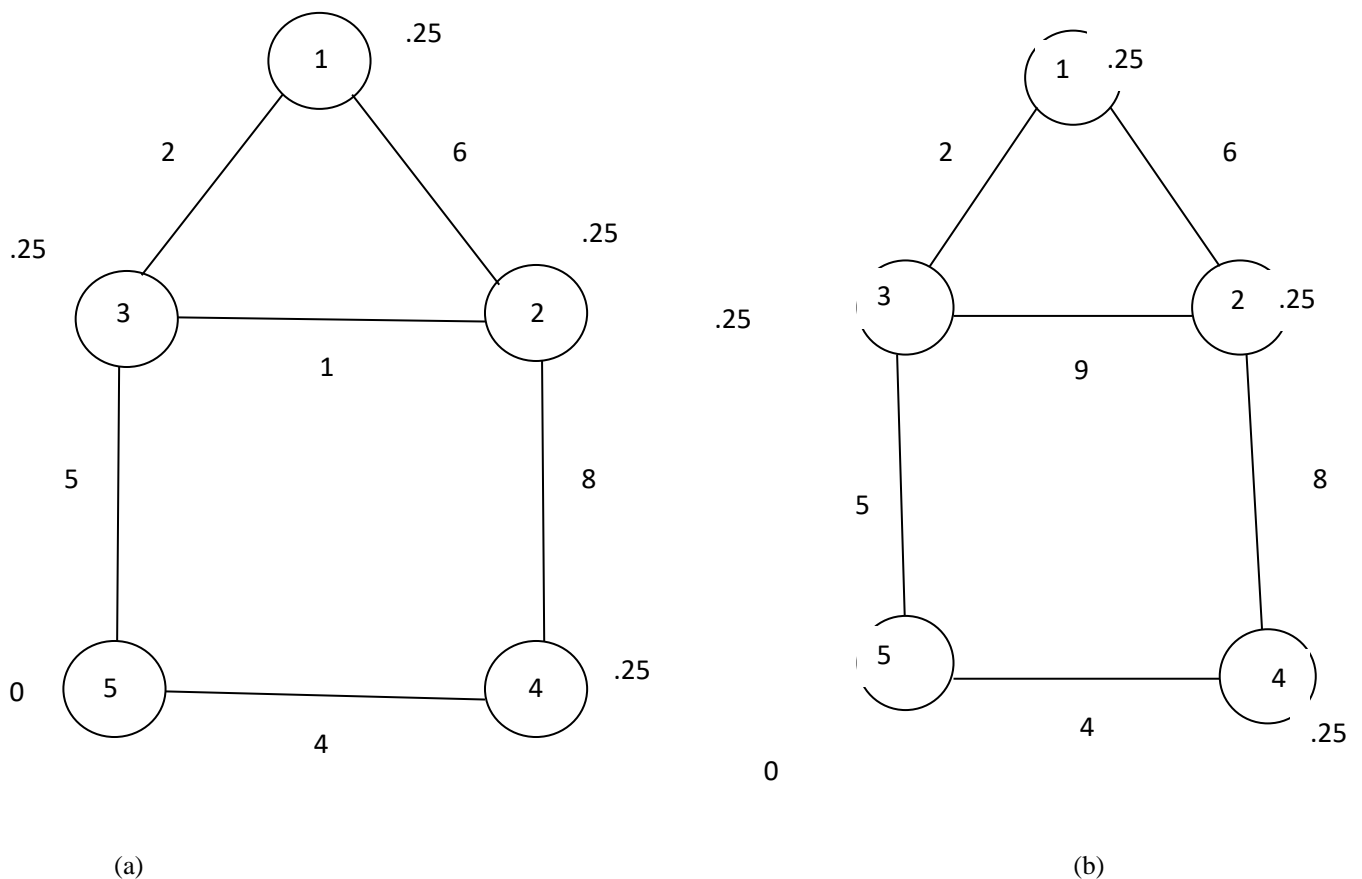


Figure 2. Two states of network G.

Let us ignore for the time being state number 2 (Figure 2(b)). We are looking for a pair of nodes that will minimize the expected response time to a call when the network is in state 1. Denote this pair K^* , so we have to find the set K^* such that

$$\sum_{j=1}^5 h_j d_1(K^*, j) \leq \sum_{j=1}^5 h_j d_1(K, j) \tag{1}$$



Where K is any subject of two nodes, h_j is the proportion of the demand generated at node j, $d_1(K, j)$ is the shortest distance between the closest home node in K and any node j, when the network is in state 1, $d_1(K, j)$ is the shortest distance between the closest home node in K^* and any node j when the system is in state 1.

K											
J	(1,2)	(1,3)	(1,4)	(1,5)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	h_j
1	0	0	0	0	2	3	3	2	2	7	0.25
2	0	1	3	3	0	0	0	1	1	6	0.25
3	1	0	2	2	0	1	1	0	0	5	0.25
4	8	9	0	4	8	0	4	0	4	0	0.25
5	6	5	4	0	5	4	0	4	0	0	0.25
$\sum_{j=1}^5 h_j d_1(K, j)$	2.25	2.5	1.25	2.25	2.5	1	2	0.75	1.75	4.5	

Table 2. Expected Response Time (ERT_1^i) for possible two home nodes in G under State 1.

The aforementioned (table 2) shows the briefest distance between under all potential blend of two home hubs. The last segment gives the interest share. The main concerns bring up the normal reaction time for (Hill and D.M., 1998) every possible pair. You can see efficiently, the ideal scraper area for this issue would be in pinnacle 3 and 4. This will yield an ideal reaction period of 0.75.

K											
J	(1,2)	(1,3)	(1,4)	(1,5)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	h_j
1	0	0	0	0	2	6	6	2	2	7	0.25
2	0	6	6	6	0	0	0	8	8	8	0.25
3	2	0	2	2	0	8	8	0	0	5	0.25
4	8	9	0	4	8	0	4	0	4	0	0.25
5	7	5	4	0	5	4	0	4	0	0	0.25
$\sum_{j=1}^5 h_j d_1(K, j)$	2.5	3.75	2	3	2.5	3.5	4.5	2.5	3.5	5	

Table 3. Expected Response Time (ERT_2^i) for possible two home nodes in G under State 2.

Table 3 present the same calculation perform for the network under the state. 2. The optimal home location in this state would be at nodes 1 and 4, yielding the desired reaction time of 2. We should locate two servers where it is observed that 40% of the time the network is in state 1 and 60% of the time it is in state 2, please remember that we do not allow a change of location in the current discussion.

Let us first examine a numerical solution. It is informed by ERT_r^i . When the network state is in r , I give the expected response time, and the home nodes are situated in one of the ten possible pairs represent in the headings of Table 2 and Table 3. Since we are looking for a permanent location for the server, we would, in fact, want to reduce the expected response time under two states, a weighted average of by ERT_r^i and ERT_2^i . Equation (2) describes what the objective function should be.

$$\min_{i=1,\dots,10} (\pi_1 ERT_1^i + \pi_2 ERT_2^i) \tag{2}$$

K	(1,2)	(1,3)	(1,4)	(1,5)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	π
ERT_1^i	2.25	2.5	1.25	2.25	2.5	1	2	0.75	1.75	4.5	0.4
ERT_2^i	2.5	3.75	2	3	2.5	3.5	4.5	2.5	3.5	5	0.6
$\pi_1 ERT_1^i + \pi_2 ERT_2^i$	2.4	3.25	1.7	2.7	2.5	2.5	3.5	1.8	2.8	4.8	

Table 4. Weighted ERT for two States for the network G.

Incidentally, the pair (1, 4); it has the negligible ERT for the stochastic organization, 1.7. Nonetheless, expect that the preeminent areas are as yet on hubs, instead of on connection. That supposition that was valid for the aloofness organization. There is no reason for stress; notwithstanding, it is practical to demonstrate that under a sensible series of expectations additionally here there exists at any rate one bunch of ideal areas that completely dwell on the hubs of the organization.

1. Repositioning on a stochastic network.

It is the dispatching focus is completely educated on the condition of the organization and the area of the units. At whatever point the organization changes from state 1 to state 2 or the other way around, the dispatcher can respond quickly and adjust (Goldman and P.R., 1999) the area of home hubs if so wanted. We expect here that the organization is clogged, that is, the limit of administration units is endless, and subsequently lines can't be produced. We need to utilize a few presumptions.

- (1) The organization G totally associated, one can go from any hub of the organization to some other hub through the connections of the organization.
- (2) The travel rate along the connections is steady, for example, it take half of the assign effort to go along half of a connection.
- (3) Time range between the places of states are apparently perpetual than movement times on the organization, when a choice is made to migrate a unit, a converse activity won't be expected to occur very soon.
- (4) The condition of the organization is the novel boundary to recognize the home peak. It isn't significant what the historical backdrop of the organization had been before it reached; similar arrangement of areas should be utilized each time the organization is in that states, paying little heed to the historical backdrop of the organization.

(5) There is a cost work related with the migration of inert help unit. In any case, the cost capacity can be either straight to the distance or a no decreasing curved capacity.

Add to the over five presumptions the two introductory ones, complete data and endless limit, (Green and L, 1998) and we are currently completely prepared to measure. This methodology, nonetheless, isn't exceptionally exact, for two significant reasons. To begin with, we need to consider the expense of movement. Significantly under the natural oversimplified approach that we applied above; migrating would not be advantageous when the normal movement cost is more noteworthy than 0.2, which is the contrast somewhere in the range of 1.7 and 1.5. The subsequent explanation is more key. It isn't really steady with accept that when repositioning is expected the incomparable arrangement of home peak under each practical state adjust with the preeminent arrangement of home hubs related with that specific state had this state been the solitary condition of the organization. The line of idea shows the requirement for a broad model that takes into account both ERT and cost. To do that, we need to acquire two extra information things.

- (1) An explicit cost functions for relocation.
- (2) The conversionprobability between the diversestate of the system.

The change probability can be given is a square network. Each line of the lattice represent one of the potential situation of the organization; every segment is additionally related with a potential state. The components of the framework are the restrictive probability for the change starting with one state then onto the next. The amount of the components in each line should approach 1. Such a framework is normally named a Markovian progress system or a stochastic lattice. A immediately forward Markovian system for our succeeding.

To state From state	1	2
1	0.25	0.75
2	0.5	0.5

Figure 3. A sample Markovian transition matrix.

The above information establish that on the off chance that it is realize that the organization is presently at state 1, it has a 25% possibility of being in state 1 in whenever range, and a 75% difference in changing to state 2, it is realize that the organization is currently in state 2, it has a fifty-fifty possibility of being in both of the achievable states during whenever stretch. Dismiss the migration costs; the ERT can be communicate as follows.

$$ERT = \pi_1 \sum_{j=1}^5 h_j d_1(K(1), j) + \pi_2 \sum_{j=1}^5 h_j d_2(K(2), j) \tag{3}$$



Where $K(1)$ and $K(2)$ are the sets of designated home nodes for states 1 and 2, respectively, and π_i is the steady-states probability of state i , $i = 1, 2$. The probability π_i can be calculated using Markov analysis. The method is base on solving a set of linear equations. Let P denote the transition matrix and π the steady-state probability vector. The vector π is solving the system of equation.

$$\pi P = \pi \quad (4)$$

$$\sum_i \pi_i = 1$$

The main difference is the interpretation of π_i , which is the stable state probability for the non-periodic case. Here π_i is the fraction of time the system is in state i . The purpose is to minimize the sum of the preferred response time and the preferred relocate expenses.

$$\text{Min } (\lambda \text{ ERT} + B) \quad (5)$$

Where λ is the number of post per one change epoch and B is the preferred relocation expenses.

3. Mathematical Modeling:

Let $G(N, L)$ be the system with N the set of node and L the set of relations. In contrast the lengths of the links are assumed to be random variables to reflect that in practice travel times between any two points of the network can vary markedly even during the same day. The main difficulty of commerce with probabilistic link length is the incredible in the computational difficulty of almost all usual network harms. When the random variables are permanent these computational difficulty make even (Geoffrion and A.M.,1999) simple network harms almost impossible to solve. Let us denote by $l_r(i, j)$ the length of link (i, j) when the state of the system is r . According to the definition of states above, if r and s are two distinct states of the network then $l_r(i, j) \neq l_s(i, j)$ for at least one link $(i, j) \in L$. We denote by m the number of states and by π_r the probability that the network is in state r , $r = 1, \dots, m$. In general, the number of states is a function of the degree of the dependence among the random variables $l(i, j)$. For example, when there is complete statistical independence,

$$m = \prod_{(i,j) \in L} n_{ij} \quad (6)$$

Where n_{ij} is the number of various that link (i, j) . The probability π_r are assumed to be given directly when the length of links are not statistically independent. Let us indicate by K a set of p point on the system $G(N, L)$. The p median problem can now be modified as follow. Find a set of point K^* such that

$$\sum_{r=1}^m \pi_r \sum_{j=1}^n h_j d_r(K^*, j) \leq \sum_{r=1}^m \pi_r \sum_{j=1}^n h_j d_r(K, j) \quad (7)$$

For any $K \in G$, where $d_r(K, j)$ is the shortest distance (travel time) between the closet point in the set K and node j when the system is in state r . We take into account that the network can be in any state r with probability

π_r , and then with probability h_j , a call can arrive from any node j , and consequently a distance between the closest ability and node j is incur.

3.1 Major results:

Even though the random variables are assumed to be discrete, the problem is still very hard. There are two main reason for that.

- (1) The shortest travel time as well as the shortest path between any two point may change with the state of the state of the system, and when m is large many such change may occur.
- (2) When there is more than one facility in the system, the choice on assigning a particular facility to a certain calling node depends on the state of the system.

The median problem, there exists an analogous result, (Ompal Singh, 2019) that an optimal solution exist on the nodes of G . Actually, this result is also accurate when we replace $d_r(K, j)$ in equation (7) by $U(d_r(K, j))$, where U is a convex utility function of travel time and the sign of the inequality in equation (7) is reversed. An important assumption that is essential for proving equation (7) is that the time required to travel a fraction θ of the link (i, j) is equal to $\theta l_r(i, j)$ for all $r = 1, 2, \dots, m$. This assumption is both straightforward and reasonable since the system can be easily reconstructed for this statement to hold. The problem equation (7) can be formulated as an integer programming problem. We define two type of binary decision variables.

$$Y_{ijr} = \begin{cases} 1, & \text{if the facility located at node } i \text{ is server node } \\ & j \text{ when the network is in state } r \\ 0, & \text{otherwise} \end{cases}$$

$$Y_i = \begin{cases} 1, & \text{if the facility located at node } i \\ 0, & \text{otherwise} \end{cases}$$

The problem is

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{r=1}^m \pi_r h_j d_r(i, j) Y_{ijr} \quad (8)$$

Subject to

$$Y_j + \sum_{\substack{i=1 \\ i \neq j}}^n Y_{ijr} = 1, \quad j = 1, \dots, n, \quad r = 1, \dots, m$$

$$Y_i \geq Y_{ijr}, \quad j = 1, \dots, n, \quad j \neq i, \quad r = 1, \dots, m$$

$$\sum_{i=1}^n Y_i = p$$

The first set of constraint assures that each node j is assign to a facility. The second set of constraint limits the assignment of nodes of those nodes where facilities are situated, and the last constraint restricts the number of facilities to p .

3.2 Repositioning-Relocated on probabilities Networks.

We talked about the issue of finding office on organizations with probabilistic connection length. It was expected that the organization can be at some random moment in any of a limited number of states, with each state varying from the wide range of various by a change in any event one connection travel time. At continually discrete time spans, the organization can make a progress from one state each other strongly, and a Markovian change grid depict these advance. Allow P to indicate the progress framework with $p \in P$ being the likelihood of a change from a state r to a state s . We further expect that for the framework P there exists a consistent state likelihood vector, which we signify by π subject to the constraint $\sum_{r=1}^m \pi_r = 1$. Administration requirements are produced only at the hubs $G(N, L)$. Given any of the ages, h_i is the likelihood that specific interests emerge at hub I . The probability $\{h_i\}$, which are indistinguishable in all age, reflect only the qualities of hub $\{i\}$ and don't rely upon workers area and on the age's history. The p office are to be found once and all, here we permit the alternative of migrate, at a cost, at least one of (Chruuch and R.L, 2010) the p workers in light of changes in the condition of the framework. The Markovian reliance among state is because of a specific level of consistency and relationship that exists concerning changes in the condition of the organization. The framework works as follows, at whatever point there is an interest for administration, a worker from the adjacent office is selected by the framework administrator and goes to serve concern. At whatever point there is a difference in condition of the society, the administrator has the choice of migrate at least one of the people.

Conclusion:

This finishes up the conversation on repositioning on stochastic organizations. We presently go to the below average of conditions where repositioning is a suitable alternative, that is where the organization can turn into a blocked organization, and repositioning choices rely upon the status of the different help units. Each time repositioning model is utilized the mean help time is overhauled by utilizing the yield aftereffects of the model, and in this way the repositioning model is rehashed until there are no more changes in the estimation of the mean assistance time. The best conditions, as far as data, would be the situation of wonderful data, where the dispatcher knows the specific area of every unit and may build up moment correspondences with it. The most pessimistic scenario would be the point at which the dispatching focus can contact workers just when they are inactive at their home hub.

References:

- [1] Adar, Z., N. Ahituv, and O. Berman, "The value of an Information system for service Network," Computers and Operations Research 12(1), pp 72-85, 1985.
- [2] Berman and M.R. Rahman, "Optimal location-relocation decision on stochastic network," International Journal of Transportation of science, 19(3), pp203-221, 1985.



- [3] Berman and M.R. Rahman, “ Optimal path of single service unit on a network to a Nonemergency destination,” International Journal of Transportation of science, 17(2), pp 218-232, 1983.
- [4] Green, L., “A multiple dispatch queuing model of policy patrol Operation” Management science,30 (6), pp 653- 664, June 2001.
- [5] Green, L., and Kolesar, “A comparison of the multiple dispatch and MIMIC priority queuing models of police patrols” Management science, 30(6), pp 665-670,June 1984.
- [6] Hall and R.W. “Direct versus terminal freight routing on a network with concave costs” Operation Research, 21, pp 287-298, 2009.
- [7] Jarvis, J.P., “Optimal assignment in a Markovian queuing system” Computers and Operations Research 8, pp 17-23, 1991.
- [8] Jordan and W.C. “Scale economics on multi-commodity distribution network” Computers and Operations Research, pp 27-35, 1986.
- [9] Larson and R.C. “A hypercube queuing model for facility location and redistricting in Urban emergency services “Computers and Operations Research 1, pp 11-23, 1999.
- [10] Halpern and J, “The accuracy of estimates for the performance criteria on certain emergency service queuing system” transportation Science 11, pp 223-242, 1999.
- [11] Hill and D.M. “Growth allocation model for the Boston” Journal of the American Institute of Planners 31, pp 11-120 1998.
- [12] Goldman and P.R. “A Domination theorem for optimal location” Journal of Operation Research 2, pp 213-220, 1999.
- [13] Green and L. “A Queuing system in which customers require a random number of services” Journal of Operation Research 1, pp 325-335, 1998
- [14] Geoffrion and A.M. “Proper efficiency and the theory of vector maximization” Journal of Mathematics analysis and applications 22, pp 618-623, 1999.
- [15] Chruch and R.L. “Locating on obnoxious facility on a network” Journal of Transportation of Sciences 12, pp 107-118, 2010.
- [16] Ompal Singh “Characterization of Model on Location Facilities” Pertinika Journal of science and technology 27(3),pp 1451-1458,2019.
- [17] Ompal Singh “Characterization of Model on Zoning Process”Pertinika Journal of science and technology 27(4), pp 2373-2379, 2019.