A probabilistic inventory model uncertain lead time with multi items using Weibull distribution

Ashish Negi[1], Ompal Singh[2]

[1,2]*Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus Modinagar, Ghaziabad − 201204 Uttar Pradesh*, ***India***

[1]*negi.ashish1995@gmail.com*, [2]ompalsit@srmist.edu.in

Abstract— For the current paper, we evolve a mathematical formulation for an inventory concern which is composed of various items. Demands and requirements for the following items are Weibull distribution. Our own goal is to establish the optimal quantity to order that curbs it to lowest the total cost for each item. it's known as individual replenishment norms. The overall cost is made up of the ordering cost, purchasing cost, shortage cost and handling cost. Utilizing the same formulation, we also enlarge a periodic review model that dictate the optimal time to order all the items. This is termed as a joint replenishment norms. Thereafter we juxtapose the overall cost of the individual replenishment norms and the joint replenishment norms and negotiate a rule that imposes a minimum overall cost. Our numerical studies depict that the joint replenishment norms is generally more suited than the individual policy. And now the joint replenishment policy provides less in handling and ordering costs, but higher shortage cost than the individual replenishment norms.

Keywords— Individual replenishment norms, joint replenishment norms, Weibull distribution.

I. INTRODUCTION

Inventory management has depicted one crucial thing that ought to be regarded by the management across the company's cash flow, specifically with regard to the funds accommodated in inventory. Through several of inventory has a repercussion of the lack of funding for the company's other divisions because the majority of the money are allocated to the inventory. On the contrary, The potential existence of an unmet demand will rise with fewer inventory on hand, and this will cost the credibility and esteem for the business going forward. Therefore we are compelled to build a mathematical formulation to determine the optimal inventory to support the business struggling with this problem. To construct a mathematical formulation, we must be aware of the existence of several stages where costs related to managing the inventory, including holding, ordering, shortage, and purchasing costs. For retailer, two ordinary When to place orders and how many (the optimal quantity) are among the inventory-related questions that need to be answered. These two questions are the most prevalent a component of the mathematical formulation to determine an optimal replenishment norms.

In the past few decades, there have been many pre-existing mathematical formulations for inventory problems have been established; the simplest and easiest to understand of the Economic Order Quantity (EOQ) which is widely used in this sector. It was developed originally by the engineer Ford Whitman Harris when he worked in the company Westinghouse Corporation. But it was introduced by R.H. Wilson (1934), after whom the model is named. The creation of more sophisticated and intricate inventory formulations has been started on this EOQ model. Some factors that have been embodies in the models in terms of demand distributions that involve quantity discount, multi-item, deterioration rate, and other formulations.

For a single item with an unpredictable rate of deterioration, a mathematical formulation for production system control has been created by Bukhari [1], if We analyze a perishable products model by taking into account the non-linear holding cost that dependent s with time it was developed by Ferguson et.al [2]. Zhang and Wang [3] have created a multi-item inventory problem with restrictions on storage capacity, and a collaborative inventory formulation to identify the optimal product assortment, display area, and shelf space for inventory replenishment is reviewed in Hariga et. al [4]. Kasthuri and others [5] created a fuzzy multi-item inventory model by consist of production costs and storage space. A fuzzy inventory model for the newsboy dilemma with multiple products that is limited by storage space. Among the common traits of inventory systems is unpredictable demand is thoroughly studied in Ding et.al [6]. Under a warehousing chance restriction by Ding [7], a multi-product newsboy dilemma with uncertain demand and unknown storage space is thoroughly examined.

A multi-objective stochastic solid transportation problem (MOSSTP) with uncertainties in demand, conveyance capacity, and supply, consistent with Weibull distribution by Das et al.'s [8]. A two-tier supply chain formulation with single supplier and a single retailer that takes the merchant's inventory classification into account. Price-sensitive market demands and to create the mathematical model, two-parameter, time-varying Weibull distribution degradation has been assumed develop by Barman, Abhijit, et al [9]. A perishable item inventory control model that is dependent on the inventory rate and has a changeable demand rate over time. The framework predicted the possibility to define variables affected by Probability distribution functions and uncertainty by Patriarca, R., et al. [10]. A fuzzy supply chain model at a single point that incorporates Weibull distributed demand for milk commodities by Bhosale, M. R., and R. V. Latpate [11]. EOQ model with a quadratic demand rate, a two-parameter Weibull deterioration rate and under authorized shortages, a variable holding cost has been created. by Preety, et al [12]. Hadley, George, and Thomson M. Whitin. "Analysis of inventory systems." [13]. A probabilistic inventory model with multiple items [14].

For the current paper, we create a mathematical formulation to address an inventory problem with multiple products. These products' demand are following to the Weibull distribution. Finding an optimal ordering quantity to reduce each product's total price is our objective. It’s known as an individual replenishment norms. The total cost consists of the ordering cost, handling cost, purchasing cost and shortage costs. We also create a periodic review formulation with the same formulation, which establishes the optimal time to order each product for the research experiments. It’s known as a joint replenishment policy. After comparing the combined costs of the individual and collaborative replenishment policies, we arrive at a decision that provides minimum total cost. According to our numerical studies are the joint replenishment norms is often better than the individual replenishment norms. The joint replenishment norms gives less in ordering cost and handling cost, but higher shortage cost as compared to the individual replenishment norms. Here, comparison the study examines the differences between individual and joint replenishment norms, as well as the impact of joint ordering costs on the best course of action is studied.

II. THE MODEL

An inventory model that approximates the actual circumstances that retailers facing is the probabilistic EOQ model. In reality, Demand will vary over time. The demand fluctuation and uncertain lead time will be incorporated into this probabilistic inventory model. A shortage will result from variations in demand, especially during lead times when the store has a restricted supply of items to meet demand and the ordered goods have not yet arrived. Three different scenarios are possible for the probabilistic inventory formulation based on the scenario. The the start is when there is a constant demand during the lead time but the actual lead time varies. In the second case, lead time remains constant but demand is not during that lead time, and the last case is when both demand during lead time and lead time are distinct. We will discuss about the individual and joint replenishment norms model in this phase.

Individual replenishment norms

We derive a mathematical formulation for individual replenishment Norms in this phase, assuming that demand is fulfilled during the lead time under a group of two parameter Weibull distribution with a constant lead time. After that this model, we'll figure out each item's reorder point and optimal order quantity. The following assumption in developing this model.

(i) The Weibull distribution guides demand during the lead time.

(ii) The quantities of storage is limitless.

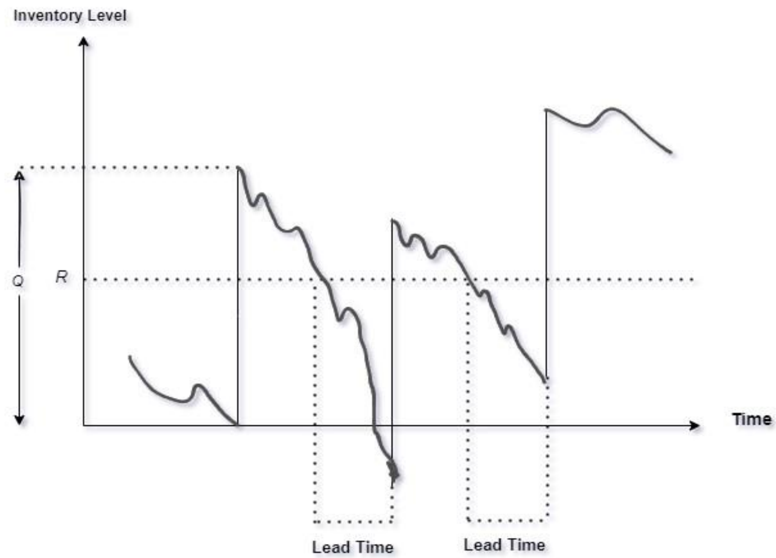
(iii) Lead time is constant and known.

(iv) When demand surpasses inventory volume during lead time than shortages arise..

(v) Unmet demand during the lead time will be fulfilled in the next duration.

(vi) Every refill always utilizes the same order quantity.

(vii) Holding costs are based on the average quantity of commodities preserved.



Graph 1. Probabilistic EOQ Model

In developing our model, we employ the following notations.

- D = Average demand during a single planning cycle

- C = Purchase unit/ cost.
- Q = the optimum quantity for an order.
- K = Ordering unit/cost for an order.
- h = Holding unit/cost proportion during the planning period.
- S = shortage unit/ cost.
- 1 − α = service level, retailer's guarantee in delivering customers' demand. (0 < α < 1).
- R = Reorder point.
- $f(x)=\begin{cases}\frac{\gamma}{\lambda}\left(\frac{x}{\lambda}\right)^{\gamma-1} e^{-\left(\frac{x}{\lambda}\right)^{\gamma}}, & x\geq 0\\ 0, & x<0\end{cases}$ is Weibull distribution (2 Parameter) with density function that includes demand in the lead time. Where $\gamma>0$ is the shape parameter and $\lambda>0$ is the Scale parameter of the distribution.

- T AC = Total average cost.
- U = Limit on quantity when there is pricing difference.

Graph 1 illustrates the problem scenario by showing the inventory level periodically throughout the arranging period. From that graph, It is obvious that shortages may arise during lead times when demand above the reorder point R. The costs associated with ordering, holding, shortage, and purchase make up the total inventory cost taken into account when developing the model.

- The amount that a merchant must pay to purchase items from a supplier is known as the purchasing cost. If the yearly demand is D units, then the yearly purchasing cost is CD.
- The costs incurred each time an order is set up known as the ordering cost. If the ordering cost is K, then yearly ordering cost is $\frac{KD}{Q}$.
- Rent or the cost of goods insurance are examples of holding costs that are typically incurred when storing and maintaining items. In general, it is shown as a proportion of the unit cost of purchase, viz. Ch In our formulation, the yearly holding cost is provided by $Ch\left[\frac{Q}{2}+R-E(X)\right]$, where X indicates a random variable for demand for lead time.
- In our formulation, the shortage cost arises when demand surpasses the reorder point during the lead time. A procedure to calculate the yearly shortage cost is as follows, $\frac{sD}{Q}\left[\int_R^{\infty}(x-R)f(x)dx\right]$.

So, the yearly total average cost for our formulation is:

$$TAC(Q,R)=CD+\frac{KD}{Q}+Ch\left[\frac{Q}{2}+R-E(X)\right]+\frac{sD}{Q}\left[\int_R^{\infty}(x-R)f(x)dx\right] \qquad (1)$$

Now find the minimum total yearly inventory cost, conditions $\frac{\partial TAC}{\partial Q} = 0$ and $\frac{\partial TAC}{\partial R} = 0$ must be hold. The condition $\frac{\partial TAC}{\partial Q} = 0$ will give:

$$Q = \sqrt{\frac{2D\left(K + s\left[\int_R^\infty (x-R) f(x)dx\right]\right)}{Ch}} \tag{2}$$

Now condition $\frac{\partial TAC}{\partial R} = 0$, we hav $\int_R^\infty f(x)dx = \frac{ChQ}{sK}$ (3)

The retailer's incapacity of accommodating customer demand is indicated by the right side of equation (3). So equation (3) as

$$\int_R^\infty f(x)dx = \alpha \tag{4}$$

The following Hadley-Whit in algorithm (Hadley and. whitin [13]) is the process to determine our model's reorder point and optimal order quantity as follows.

- To begin, we apply the EOQ, i.e. $Q = \sqrt{\frac{2DK}{Ch}}$
- Find the value of R from equation (3).
- Find Q using equation (2) and the value of R found in step (ii).
- Continue steps (ii) and (iii) until we figure out the optimal values for Q *and* R, which are those at which the difference between each iteration's values of Q *and* R is small than 1.
- Find the optimum TAC using equation (1).

Joint replenishment norms

The optimal time for replenishment is the decision variable in our formulation for the joint replenishment norms. Another name for this is the periodic review formulation. In this formulation we consider that all product is ordered concurrently (joint order). The ordering amount for each item varies but there is a set ordering time interval. Certain costs, like ordering cost, should be less when things are ordered jointly compared to order separately. We display the joint replenishment norms ordering cost.

	Item 1	**Item 2**	**Item 3**
Yearly Demand	**540**	**380**	**750**
Ordering Cost ($)	**6**	**8**	**5**
Holding Cost	**0.02**	**0.017**	**0.02**

Shortage Cost ($)	**1.6**	**2.5**	**1.4**
Purchase Cost ($)	**13**	**16**	**9**
γ	**5**	**5**	**3**
λ	**2**	**1**	**2**

Table 1. Data

By K^* where K^* is lesser than the total of any ordering cost individually. Given that the variable used to figure out the optimal timing for replenishing, T, we use the relation $Q_i = D_i T$ for each item i in individual replenishment norms formulation, and then figure out the optimal T. Our formulation becomes:

$$TAC(T) = \frac{K^*}{T} + \sum_{i=1}^{n} \left\{ \begin{array}{l} C_i D_i + C_i h_i \left(\frac{TD_i}{2} + R_i - E\left(X_i\right) \right) \\ + \frac{S_i}{T} \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i \end{array} \right\} \tag{5}$$

Where n indicate the number of items is jointly ordered. Take 1st derivative of $TAC(T)$ and equivalent it to 0 will provide an optimal replenishing time as:

$$T = \sqrt{\frac{2\left(K^* + \sum_{i=1}^{n} s \int_{R_i}^{\infty} (x_i - R_i) f(x_i) dx_i \right)}{\sum_{i=1}^{n} C_i D_i h_i}} \tag{6}$$

Numerical Studies

This section we take into consideration a merchant who purchases three distinct products from a single supplier and offers them to customers. Any product's demand is Weibull distributed with numerous of parameter values (γ and λ), and all data is shown in Table 1.

Table 2. Result for Individual Replenishment Norms

	Item 1	Item 2	Item 3
Q	200	150	800
R	25	18	22
Purchase Unit/Cost	7000	6500	6700
Ordering Unit/Cost	12.50	17.67	11.25
Holding Unit/Cost	12.2	16.89	11.33
Shortage Unit/Cost	0.39	0.35	0.22
Inventory Unit/Cost	6824.25	6248.67	6523.56
TAC	19596.48		

Using the Hadley-Whitin algorithm, we obtain the following outcomes for the individual replenishment norms, which are shownin Table 2. Along with the total average cost is 19596.48, for each product's optimal ordering quantity and reorder point are shown.

For joint replenishment norms, using the joint ordering unit/cost K^* = 9 the results are shown in Table 3.

Table 3. Result for Joint Replenishment Norms

	Item 1	**Item 2**	**Item 3**
T(year)	**0.26**		
Q	**156**	**114**	**228**
Purchase Unit/Cost	**7000**	**6500**	**6700**
Ordering Unit/Cost	**30.56**		
Holding Unit/Cost	**11.78**	**14.83**	**11.12**
Shortage Unit/Cost	**0.43**	**0.46**	**0.24**
Inventory Unit/Cost	**6802.20**	**6196.64**	**6483.36**
TAC	**19482.20**		

So In Table 3, the optimal ordering time T is close to 0.26 years, The optimal amount to order for each item are 156,114 and 228 respectively which are lesser than the optimal order quantities for individual replenishment norms. This information has an impact on shortage cost and holding cost. The holding unit/cost for the individual joint norms is more than the holding unit/cost in the joint replenishment norms but shortage cost is less. These outcomes make sense because our holding costs increase with the quantity of items we own, while our shortage costs decrease with that quantity. Overall, the overall cost of inventory for joint replenishment norms in these numerical tests is lesser than the overall cost of inventory for individual replenishment norm.

It is possible the overall inventory cost for joint replenishment norm gives more than the overall inventory cost for individual replenishment norm in this formulation. This may occur when the savings from joint replenishment policy's ordering cost are not as much as the total of the individual replenishment norms ordering costs. Our numerical studies, the joint ordering cost is 30.56 in Table 3 compared to the summation of the individual ordering cost of 41.42 in Table 2, a saving is 10.86.

Conclusion

For the current paper, describes a mathematical formulation for a probabilistic multi-item inventory problem in which the demand is Weibull distributed for each item with distinct values of γ and λ. For the individual replenishment norms, we have determined the reorder point and optimal ordering quantity for the joint replenishment norms, we have determined the ideal ordering time. In our numerical based experiments, thereafter to determine the most effective course of action, we examine the individual and joint replenishment norms. From our numerical experiment with three products we achieve in our experiment and In contrast to the individual replenishment norms, we find that the joint replenishment norms results in a lower overall yearly inventory cost due to a different distribution of demand in a period of lead time.

Future Scope

There are some limitations also for our model since it fails to take into account an assortment of factors, including the rate at which the items deteriorate, quantity discounts, and possibly non-linear holding costs. It might be fascinating to explore these elements further and examine how they reflect the best course of action, as well as the optimal ordering quantity, timing, and reorder point. It will pave the way for future development.

References

[1] Bukhari, Fawzy. "Adaptive control of a production-inventory model with uncertain deterioration rate." *Applied Mathematics* 2.9 (2011): 1170-1174.

[2] Ferguson, Mark, Vaidy Jayaraman, and Gilvan C. Souza. "Note: An application of the EOQ model with nonlinear holding cost to inventory management of perishables." *European journal of operational research* 180.1 (2007): 485-490.

[3] Zhang, Bin, and Xiayang Wang. "Optimal policy and simple algorithm for a deteriorated multi-item EOQ problem." *American Journal of Operations Research* 1.2 (2011): 46.

[4] Hariga, Moncer A., Abdulrahman Al-Ahmari, and Abdel-Rahman A. Mohamed. "A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions." *European Journal of Operational Research* 181.1 (2007): 239-251.R. Nicole, "Title of paper with only first word capitalized," J. Name Stand. Abbrev., in press.

[5] Kasthuri, R., et al. "Multi-item fuzzy inventory model involving three constraints: A Karush-Kuhn-Tucker conditions approach." *American Journal of Operations Research* 1.3 (2011): 155.

[6] Ding, Sibo, and Yuan Gao. "The (σ, S) policy for uncertain multi-product newsboy problem." *Expert Systems with Applications* 41.8 (2014): 3769-3776.

[7] Ding, Sibo. "Uncertain multi-product newsboy problem with chance constraint." *Applied mathematics and computation* 223 (2013): 139-146.

[8] Das, Amrit, and Gyu M. Lee. "A multi-objective stochastic solid transportation problem with the supply, demand, and conveyance capacity following the Weibull distribution." *Mathematics* 9.15 (2021): 1757.

[9] Barman, Abhijit, et al. "Pricing and inventory decision in a two-layer supply chain under the weibull distribution product deterioration: An application of NSGA-II." *RAIRO-Operations Research* 57.4 (2023): 2279-2300.

[10] Patriarca, R., et al. "EOQ inventory model for perishable products under uncertainty." *Production Engineering* 14.5-6 (2020): 601-612.

[11] Bhosale, M. R., and R. V. Latpate. "Single stage fuzzy supply chain model with Weibull distributed demand for milk commodities." *Granular Computing* 6 (2021): 255-266.

[12] Poswal, Preety, et al. "Fuzzy optimization model of two parameter weibull deteriorating rate with quadratic demand and variable holding cost under allowable shortages." *Yugoslav Journal of Operations Research* 32.4 (2022): 453-470.

[13] Hadley, George, and Thomson M. Whitin. "Analysis of inventory systems." (1962).

[14] Lesmono, Dharma, and Taufik Limansyah. "A multi item probabilistic inventory model." *Journal of Physics: Conference Series*. Vol. 893. No. 1. IOP Publishing, 2017.