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Investigating Algorithms and Computing Methods for Determining the Generalized Commute Probability of Complete Sets

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ABSTRACT

The calculation of the generalized commuting probability, which quantifies the likelihood of subsets of elements commuting within a finite group, is a fundamental problem in computational group theory. This abstract presents an overview of methods and algorithms developed for efficiently computing the generalized commuting probability of finite groups. The presented approaches contribute to the advancement of computational group theory, enabling researchers and practitioners to explore and understand the structure and properties of finite groups in various mathematical and scientific domains.

Keywords: Probability, Group, Elements, Finite ring, Algorithms

I. INTRODUCTION

The concept of commuting elements in a group lies at the heart of group theory, providing insights into the structure and properties of the group. When two elements commute, their order of multiplication does not affect the result, and they can be rearranged without altering the outcome. The commuting probability, defined as the probability that two randomly chosen elements commute, has been extensively studied and utilized in various branches of mathematics and physics.

In recent years, there has been growing interest in a generalization of the commuting probability known as the generalized commuting probability. Unlike the traditional commuting probability, which focuses on the commutativity of pairs of elements, the generalized commuting probability extends this notion to consider larger subsets of elements within a finite group. Specifically, it quantifies the likelihood that a randomly chosen subset of elements, of a given size, commute with each other.

The study of the generalized commuting probability of finite groups is a rich and challenging area of research, with applications spanning from algebraic number theory to quantum information theory. Understanding the behavior of the generalized commuting probability provides valuable insights into the structure and symmetry of finite groups, shedding light on their algebraic properties and applications.

The calculation of the generalized commuting probability is a computationally demanding task, as it requires considering all possible subsets of a given size within the group and determining whether they commute. Naively computing the commuting probability for each subset is infeasible for large groups due to the exponential growth in the number of subsets. Consequently, the development of efficient computational methods and algorithms becomes crucial in order to tackle this problem and explore the generalized commuting probability for a wide range of groups.

Previous research in this field has focused on various aspects of computing the generalized commuting probability, including exact computation methods, approximate estimation techniques, and algorithms tailored for specific classes of groups. Enumeration-based methods have been employed to exhaustively compute the commuting probability for small groups, but their applicability is limited due to their exponential time complexity. Approximate methods, such as sampling-based approaches and Monte Carlo simulations, offer a trade-off between computational efficiency and accuracy but require careful analysis and design to ensure reliable results.

Despite the progress made in the field, challenges remain in efficiently calculating the generalized commuting probability for larger groups. One challenge arises from the inherent combinatorial nature of the problem, as the number of possible subsets grows exponentially with the group size. Additionally, the presence of non-commuting elements and the dependence of the generalized commuting probability on the chosen subset size introduce additional complexity.

II. COMMUTING PROBABILITY OF FINITE RINGS

In the context of finite rings, the concept of commuting probability does not have a direct interpretation. However, properties such as commutativity and the existence of zero divisors can have important implications for the structure and properties of finite rings.

For any two elements s and r of a ring R, we write [s,r] to denote the additive commentator of s and r. That is, [s, r] = sr−rs. By K(S, R) we denote the set {[sr-rs]: s∈S, r∈R} and [S,R] denotes the subgroup of (R,+) generated by K(S,R). Note that [R, R] is the commutator subgroup of (R, +). Also, for any x∈R, we write [x,R] to denote the subgroup of (R,+) consisting of all elements of the form [x,y] where y∈R.

The commuting probability of R, denoted by Pr(R), is the probability that a randomly chosen pair of elements of R commute. That is Pr(R)=|{(s,r)∈R×R:sr=rs}/|R×R|. The study of commuting probability of a finite ring was initiated by MacHale in the year 1976. Many papers have been written on commuting probability of finite groups in the last few decades. However, the study of the commuting probability of a finite ring was neglected. After many years, in the year 2013, MacHale resumes the study of commuting probability of finite rings together with Buckley and Ní Shé. In this paper, we obtain several bounds for Pr(R) through a generalization of Pr(R). We generalize Pr(R) as the following ratio

$$Pr(S,R) = |\{(s,r) \in S \times R: sr = rs\}/|S \times R|$$

if and only if R is a finite ring and S is a subring of it. Keep in mind that Pr(S,R) is the chance that any given pair of components, one from subring S and the other from subring R, commute. The subring S's commuting probability in the ring R is denoted by Pr(S,R). By definition, if Z(S,R)=S, then Pr(S,R)=1 and Pr(R,R)=Pr(R).

The study of commuting probability of a finite ring R, given by the ratio,

$$Pr(R) := \frac{|\{(r,s) \in R \times R : rs = sr\}|}{|R \times R|}$$

originated with MacHale back in 1976. In this subsection, we review the results on Pr(R) that will be used in the subsequent chapters and throughout the thesis.

By above equation, we have

$$\Pr(R) = \frac{1}{|R|^2} \sum_{r \in R} |C_R(r)|$$

and hence

$$\Pr(R) = \frac{|Z(R)|}{|R|} + \frac{1}{|R|^2} \sum_{r \in R/Z(R)} |C_R(r)|$$

III. APPROACHES FOR COMPUTING THE GENERALIZED COMMUTING PROBABILITY

Approaches for computing the generalized commuting probability of finite groups can be classified into several categories, each with its own advantages and limitations. In this section, we will review some of the commonly employed methods and discuss their characteristics.

Enumeration-Based Methods

Enumeration-based methods involve exhaustively considering all possible subsets of a given size within the group and checking whether they commute. These methods provide an exact computation of the generalized commuting probability but are limited to small groups due to their exponential time complexity.

One approach is to generate all possible subsets using combinatorial techniques such as generating all combinations or permutations. For each subset, the commutativity of its elements is verified. This approach guarantees accuracy but becomes computationally infeasible for large groups due to the exponential growth in the number of subsets.

Group Presentation and Relators

Another approach is to use the group presentation and relators to calculate the generalized commuting probability. Group presentation represents the group in terms of generators and relators, where the relators capture the defining relations of the group. By analyzing the relators, it is possible to determine whether a given subset commutes or not.

This method can be particularly useful when dealing with groups with known presentations, such as certain classes of finite groups or specific mathematical structures. However, determining the commuting probability using group presentation and relators can still be computationally challenging for larger groups with complex presentations.

Sampling-Based Methods

Sampling-based methods provide an approximate estimation of the generalized commuting probability by randomly sampling subsets from the group and checking their commutativity. These methods are computationally more efficient than enumeration-based methods but introduce a certain degree of error. One approach is to randomly sample a large number of subsets and calculate the proportion of commutative subsets among them. By increasing the number of samples, the estimation can converge to the true value of the generalized commuting probability. Monte Carlo simulations are often employed to generate random samples and obtain statistically reliable estimates.

Statistical Estimation Techniques

Statistical estimation techniques utilize statistical methods to infer the generalized commuting probability based on a limited set of observed data. These methods can be employed when only partial information about the group is available or when direct computation is infeasible.

One such technique is maximum likelihood estimation (MLE), where a statistical model is formulated based on the observed data, and the likelihood of the data given a certain parameter (the generalized commuting probability) is maximized. MLE provides an estimate of the parameter that maximizes the likelihood of the observed data.

Heuristic and Optimization-Based Approaches

Heuristic and optimization-based approaches aim to find approximate solutions to the generalized commuting probability problem by leveraging techniques from optimization and computational intelligence. Metaheuristic algorithms, such as genetic algorithms or simulated annealing, can be applied to explore the solution space and search for subsets with high commutativity. These methods often trade off exactness for computational efficiency and may provide good approximations for large groups. It is worth noting that the choice of the most suitable approach depends on factors such as the size of the group, available computational resources, desired accuracy, and the specific properties of the group under consideration.

IV. PERFORMANCE METRICS

Several performance metrics can be employed to evaluate the performance of algorithms for calculating the generalized commuting probability. The choice of metrics depends on the specific goals and requirements of the study. Some commonly used metrics include:

Computation Time

The time required to compute the generalized commuting probability for a given group. This metric provides insights into the efficiency and scalability of the algorithms.

Space Complexity

The amount of memory or storage required by the algorithms. This metric is important when dealing with large groups or limited computational resources.

Accuracy

The accuracy of the computed generalized commuting probability compared to the exact value (if available). This metric measures the reliability of the algorithms and their ability to produce accurate results.

Approximation Error

For approximate methods, the approximation error quantifies the deviation of the estimated probability from the true value. It can be measured using metrics such as mean squared error or relative error.

Scalability

The ability of the algorithms to handle larger groups efficiently. This metric examines how the computation time and resources required by the algorithms grow as the group size increases.

Robustness

The stability and consistency of the algorithms across different groups and datasets. Robust algorithms should perform well across a variety of group structures without significant fluctuations in their performance.

V. CONCLUSION

The study of computational methods and algorithms for calculating the generalized commuting probability of finite groups has yielded significant advancements in the field of computational group theory. These methods have provided valuable tools for analyzing the structure and properties of finite groups and have found applications in diverse areas of mathematics and beyond. By combining theoretical foundations, algorithmic design, and empirical evaluations, researchers and practitioners can continue to advance the field of computational group theory. Further research can focus on refining existing algorithms, developing novel approaches, and exploring applications of the generalized commuting probability in various fields.

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