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FITTING OF A PARTIALLY REPARAMETERIZED GOMPERTZ MODEL TO BROILER DATA

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ABSTRACT

Different non-linear growth models were fitted to the growth data of broiler and Gompertz model was found appropriate to the data under intensive system. One of the estimated parameters of the fitted Gompertz model showed a non-linear behavior. Thus, the concept of partial reparameterization by expected-value parameters was used to mitigate nonlinear behavior of the estimated parameter. The growth data of broiler refitted to an explicit form of the partially reparameterized version of Gompertz model showed superior results.

***Keywords*: Curvature Effects, Growth, Nonlinear Model, Parameter and Reparameterization.**

I. INTRODUCTION

In the North-Eastern Region (NEH) of India, there is a high demand of meat (animal proteins). Broiler farming will play an important role to meet the demand of food and nutrition in this particular region. Growth parameters are important not only as selection criteria but also in terms of feed management techniques. Therefore, it is preferable to model the growth trend that defines periodic changes in the underlying characteristic. There have been quite a few studies undertaken toward the determination of growth trend in broilers in the NEH region of India. Further, the nonlinear models fitted to maximum of the data usually resulted in highly nonlinear behavior of the estimated parameters. Many authors highlighted the importance of reparameterization in nonlinear model fitting especially to tackle the issue of nonlinear behavior of the estimated parameters ([1-5]). In fact, a little attention is given to the various reparameterizations and consequently, the parameter estimates hardly satisfy any of the optimum properties. The present study aims to estimate growth rate curves and their parameters using different nonlinear growth models to determine the age-live weight relationship of broiler under intensive system. The suitability of reparameterized model to mitigate the nonlinear behavior of estimated parameter is also demonstrated.

II. MATERIALS AND METHODS

The following nonlinear models will provide a reasonable representation of average weight W_t at time t whose model function is of the form $W_t = f(t,\beta) + \varepsilon_t$:

Logistic:

$$W_t = \frac{\beta_1}{1+\beta_2 \exp(-\beta_3 t)} \quad (1)$$

Gompertz:

$$W_t = \beta_1 \exp[-\beta_2 \exp(-\beta_3 t)] \quad (2)$$

Von-Bertalanffy:

$$W_t = \frac{\beta_1}{[1-\beta_2 \exp(-\beta_3 t)]^3} \quad (3)$$

Where β_1, β_2 and β_3 are the parameters to be estimated. The parameter β_1 represents the limiting growth value or asymptotic size, β_2 the scaling parameter and β_3, the rate of maturity.

If 'β_1' is likely to be an offensive parameter say, in equation (2), then it can be partially reparameterized by expected-value parameter. To obtain an expected-value parameter from above equation (2), we need to choose value t_1 of the regressor variable t, within the observed range of t. Then, we get the expected value from equation (2) as follows:

$$W_1 = \beta_1 \exp[-\beta_2 \exp(-\beta_3 t_1)]$$

Solving this equation for the parameter 'β_1' only, we get

$$\beta_1 = \frac{W_1}{\exp[-\beta_2 \exp(-\beta_3 t_1)]}$$

Substituting back into the original equation (2), we get

$$W_t = W_1 \frac{\exp[-\beta_2 \exp(-\beta_3 t)]}{\exp[-\beta_2 \exp(-\beta_3 t_1)]} \quad (4)$$

The above model is proposed to eliminate the nonlinear behaviour of the estimated parameter. Here, the likely offensive parameter 'β_1' is reparameterized by expected-value parameter while the other parameters are not changed.

2. 1Criteria for Model Selection

To examine model performance, summary statistics like root mean square error (RMSE) and mean absolute error (MAE) are generally used:

$$RMSE = \left[\sum_{t=1}^{n} \left(W_t - \hat{W}_t\right)^2 / n\right]^{1/2};$$

$$MAE = \sum_{t=1}^{n} \left|\left(W_t - \hat{W}_t\right)\right| / n \text{, and}$$

where

$\hat{W}_t$ Predicted weight of t^{th} observation;

$\overline{W}$ Average weight;

n Number of observations, $t = 1,2,...,n$.

The better model will have the least values of these statistics. It is, further, recommended for residual analysis to check the model assumptions such as independence or the randomness assumption of the residuals and the normality assumption. To test the independence assumption of residuals, run test procedure is available in the literature ([6]). Further, Shapiro-Wilk's test was applied to check the normality assumption but, it is not so stringent for selecting nonlinear models because their residuals may not follow normal distribution.

Moreover, the curvature in a nonlinear model consists of two components: the intrinsic (IN) curvature and parameter effects (PE) curvature. Details of the root mean square (RMS) IN and PE measures of curvature and curvature critical value are given in Bates and Watts ([7-8]). According to Ratkowsky ([9]), the IN curvature is typically smaller than the PE curvature, which can be affected by altering the parameterization of the model. Severe curvature effects are indicated by values of IN and PE exceeding the critical value i.e., $1/\sqrt{F_{p,(n-p)}(0.05)}$, p is the number of parameters involved in the model. In usual, PE is computed when IN is within permissible limits and a lower value of PE suggests that the model exhibits close-to-linear behavior ([6]).

Hougaard's measure of skewness, g_t, can also be employed to assess whether a parameter is close to linear or whether it contains considerable nonlinearity. Hougaard's measure is computed as follows:

$$E\left[\hat{\beta}_t - E\left(\hat{\beta}_t\right)\right]^3 = -(MSE)^2 \sum_{jkl}^{p} L_{jk} L_{kl} L_{jl} \left(W_{jkl} + W_{kjl} + W_{lkj}\right)$$

where the sum is a triple sum over the number of parameters,

$$\mathbf{L} = [\mathbf{J'J}]^{-1},$$

$$W_{jkl} = \sum_{m=1}^{n} J_{mj} H_{mkl},$$

J is the Jacobian matrix, J_m is the Jacobian vector, H is the Hessian matrix, H_m is its component evaluated at observation m and β_t is the t^{th} parameter. This third moment is normalized using the standard error to give Hougaard's measure of skewness as:

$$g_t = \frac{E\left[\hat{\beta}_t - E\left(\hat{\beta}_t\right)\right]^3}{\left(MSE * L_{tt}\right)^{3/2}}.$$

According to Ratkowsky ([6]), if $|g_t| < 0.1$, the estimator $\hat{\beta}_t$ of parameter β_t is very close-to-linear in behavior and, if $0.1 < |g_t| < 0.25$, the estimator is reasonably close-to-linear. If $0.25 < |g_t| < 1$, the skewness is very apparent. For $|g_t| > 1$, the nonlinear behavior is considerable.

2.2 Description of Data

The weekly average growth dataset of body weight/ broiler in gm under intensive system observed by Fanai ([10]) was considered. The experiment was conducted to compare the growth performance of broiler under intensive system and backyard system. A total of 600 commercial broilers were divided into two groups, 300 chicks were reared in the College farm under intensive system following standard management practices and remaining 300 were equally distributed to 10 farmers to rear under backyard system. All the production parameters like growth rate, feed conversion rate, mortality rate and economics under backyard system as well as intensive system of management from birth till 13 weeks of age were recorded and analyzed. They observed that overall performance of broiler was comparatively better under intensive system than backyard system. Thus the growth data of broiler under intensive system is further considered for the present study.

III. FIGURES AND TABLES

Table 1: Summary Statistics for Fitting of Various Growth Models on Broiler Data Under Intensive System

Parameter Estimates	Growth Models		
	Logistic	Gompertz	Reparameterized Gompertz
β_1 (W_1)	5205.60 (132.50)	6329.30 (170.60)	2304.80 (21.60)
β_2	28.43 (3.03)	4.76 (0.15)	4.76 (0.15)
β_3	0.44 (0.02)	0.22 (0.01)	0.22 (0.01)
Curvature			
RMS IN Curvature	0.03	0.02	0.02
RMS PE Curvature	0.34	0.34	0.21
Critical Value	0.52	0.52	0.52
Hougaard's Skewness			
β_1 (W_1)	0.29	0.26	0.01
β_2	0.50	0.24	0.24
β_3	0.11	0.05	0.05
Goodness of fit			
RMSE	85.48	47.73	47.73
MAE	62.44	36.67	36.67
Residual Analysis			
Run test $(\|Z\|)$	1.73	0.00	0.00
Shapiro-Wilk test *p*-value	0.79	0.21	0.21

Note: Figures in parentheses are the corresponding asymptotic standard errors.

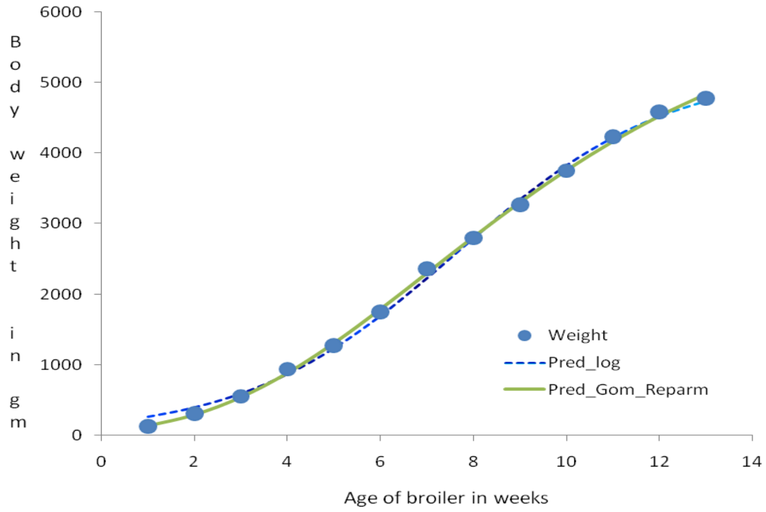

Fig. 1: Graphical Display of Observed and Predicted Growth of Broiler Under Intensive System (Reparameterized Gompertz Model is Adjudged to be the Best fit)

IV. RESULTS AND DISCUSSION

The above dataset was fitted to different nonlinear models using SAS 9.3 version available at College of Agriculture, CAU, Imphal. Different sets of initial parameter values have been tried so that a global convergence criterion is met for fitting of nonlinear models. The global convergence criteria have been met for logistic and Gompertz models. The estimates of parameters, RMSE, MAE, curvature effects, Hougaard's skewness coefficients, run test statistic (|*Z*|) value and Shapiro-Wilk test p-value for the above two models under intensive system are presented in Table 1. Gompertz model shows better performance than other model when the criteria of RMSE and MAE are used to identify the best-fit model. Further, independence assumption about residuals is satisfied since the run test |*Z*| values (lies between 0.00 – 1.73, given in Table 1) are well below the critical value of 1.96 of normal distribution at 5% level of significance. Also, the significance values of Shapiro-Wilk test for residuals clearly indicate (p>0.05) that residuals are normally distributed. The asymptotic weight of broiler estimated by the Gompertz model is approximately 6329.30 gm under intensive system. Moreover, RMS IN curvature (0.02) and RMS PE curvature of Bates and Watts (0.34) are less than the corresponding critical value 0.52 and they are acceptable. However, Hougaard's skewness value of the estimated parameter of Gompertz model say, $\hat{\beta}_1$ is greater than 0.25 which shows that the nonlinear behavior is very apparent. To rectify the above problem, a partially reparameterized Gompertz model is proposed considering the parameter $\hat{\beta}_1$ as an offensive parameter, given in equation (4). The parameter β_1 is replaced by W_1 in the process of reparameterization as β_1 is considered to be the offensive parameter. A value of t_1=7 was chosen and the corresponding value of W_1= 2362.5 is taken as an initial value for computation of the final estimate of the parameter W_1, which gives the best result in terms of least nonlinear behavior. The reparameterized model was refitted to the data and the results are again presented in Table 1. Further improvements in Hougaard's skewness and curvature effects are also seen in this refitted model. The graph of fitted model along with observed growth data is also depicted in Fig. 1 which shows the appropriateness of the proposed model.

V. CONCLUSION

It is summarized that Gompertz model is adjudged to be the best fit for the present data set. We also conclude that under the intensive system, we can expect the maximum size of approximately 6329.30 gm in weight of broiler. As such one of the estimated parameter of Gompertz model was showing non-linear behavior, it was partially reparameterized by expected value parameter. The partially reparameterized model was refitted to the same data set and the results showed appropriateness of the proposed model. Thus, reparameterization will help to mitigate the nonlinear behavior of the estimated parameters.

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