International Journal of Advanced Technology in Engineering and Science
Vol. No.3, Special Issue No. 01, September 2015
www.ijates.com
ijates
ISSN 2348 - 7550

A NEW GUIDANCE STRATEGY FOR MISSILE INTERCEPTION

D. S. Chauhan[1] , **R. Sharma**[2]

[1-2]*Department of Ocean Engineering, IIT Madras, Chennai (TN), (India)*

ABSTRACT

This paper presents a new guidance strategy that is derived by a variation of the 'Proportional Navigation (PN)' guidance law by incorporating a term based on heading error in two dimensional planar engagement model. The planar engagement geometric model is prepared and the equations of motion are derived from basic principles. The analysis of model is based upon the evaluation of guidance strategy and our idea is to vary the navigation ratio that is dependent on the heading error and lead angle of the missile. This variation allows us to utilize both the guidance laws (i.e. PN guidance law and Retro-PN guidance law) depending upon parametric values. The new guidance law is conceptualized as the 'True Combined Proportional Navigation (TCPN)' guidance law and the velocity of interest in TCPN is the closing velocity and not the velocity of missile as velocity between target and missile is one which ultimately drives the LOS separation to zero. The TCPN is simulated in Matlab^{*TM} for some chosen parameters and our results show the efficiency and applicability of our proposed guidance strategy.

Keywords*: *PN Guidance, RPN Guidance, Navigation, Guidance, Missile Interception, Simulation.

I. INTRODUCTION

A navigation guidance law is an important part of missile interception and there exists various navigation guidance laws. It is well know that they have their ranges of application and suit a set of specific requirements, e.g. the 'Pure Proportional Navigation (PPN)' is applicable for head-on engagement and the 'Retro Proportional Navigation (RPN)' is applicable for tail-chase engagement, [1-3]. Our basic motivation is to develop a navigation guidance law that can incorporate technical strength and features of PN and RPN and thereby improves the range of applicability of the navigation guidance law. We explore an approach based on utilizing a varying 'Navigation Ratio (N)' that is dependent on 'Heading Error (HE)', [4-5]. The N can vary from negative or positive with respect to the changes in the HE and this change from negative to positive and vice versa is utilized to develop a navigation guidance strategy that switches between the PN and RPN. This development results into the 'True Combined Proportional Navigation (TCPN)' guidance law and our simulation results show that it performs better than other guidance laws for a range of applications. However, the TCPN guidance is also not applicable across the board and it is not applicable when for values of the HE where N approaches either zero or infinity.

The remaining of this paper is organized: Section 2 over views the existing guidance laws, Section 3 presents the PN, Section 4 discusses the numerical simulation results and Section 5 concludes the paper with identifying some critical future scope of research.

II. OVERVIEW OF GUIDANCE LAWS

The PN guidance law is an important and fundamental law in missile guidance and control. Because of its inherent strengths it is widely used and its variations that suit specific set of requirements are being investigated by various researchers. Following [1], the PN can be classified into: true PN law and pure PN law. The PN guidance law is:

$$\dot{\gamma} = \qquad (1)$$

where N is the navigation constant. The different variations of PN guidance laws are listed in Table 1.

Table 1: Different Variations of PN Guidance Laws.

S. No.	Variation of PN guidance Law	Description
1.	True Combined Proportional Navigation (TCPN)	Proposed in the present paper
2.	Pure Combined Proportional Navigation (PCPN)	Based on N and missile velocity, combined law for PN and RPN due to N varying from positive to negative
3.	Pure Proportional Navigation (PPN)	Based on N' and missile velocity
4.	True Proportional Navigation (TPN)	Based on N' and closing velocity

2.1 Pure Proportional Navigation (PPN)

From [4 and 6], we state:

$$\dot{\gamma} = \qquad (2)$$

$$a_m = N'V \qquad (3)$$

where is lateral acceleration command (latax) needed by guidance law. The equations 2-3 describe PPN law. The equation (2) is valid only when the lateral acceleration is perpendicular to the velocity of the missile. In PPN law, the latax is given by Equation (3) and is applied perpendicular to the velocity vector of the missile. If we ignore the angle-of-attack of the missile, then this direction of the latax is also the natural direction of the lift force which is generated by the airframe and the lifting surfaces whenever the missile maneuvers. This lift force is responsible for generating the actual lateral acceleration or latax. The angle-of-attack of a missile is never zero and for many highly maneuverable missiles it turns out to be quite high. This is where PPN departs from reality and its results are not applicable.

2.2 True Proportional Navigation (TPN)

In TPN the closing velocity between missile and target is considered as it drives the LOS separation to zero. Though, the is not directly available unless the missile carries an inertial navigation unit, but is easily available from Doppler data of the seeker. From [4 and 6], we state:

$$a_m = N' \qquad (4)$$

where is the closing velocity given by $-dR$ and is the rate of change of line of sight. For PN, N' is used and it is effective Navigation constant (N' > 2). In the equation (4) if the N turns out to be negative and used to find lateral acceleration using expression for classical PN, then this guidance law becomes the 'Retro-Proportional Navigation (RPN)' guidance law. This strategy has been used for defining the 'Combined Proportional Navigation (CPN)' guidance law using time varying navigation ratio (N) as given by the equation (4).

Now, if the N is used to generate latax with , then it is considered as the 'True Combined Proportional Navigation (TCPN)' guidance law. Furthermore, if the N is used with , then it is considered as the 'Pure Combined Proportional Navigation (PCPN)' guidance law.

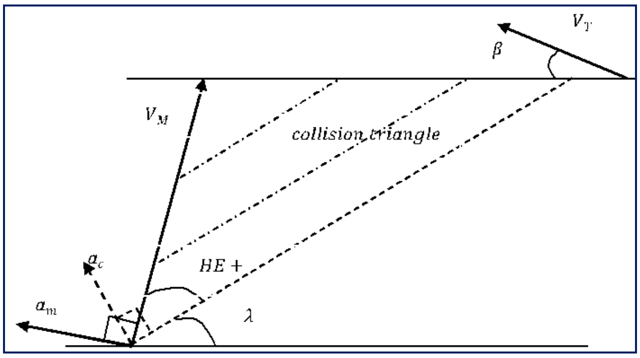

Figure 1: Collision Triangle and the Heading Error.

Fig. 1 shows the collision triangle and heading error. For successful interception, missile velocity vector and target velocity vector needs to be inside the collision triangle, and they are inside collision triangle if they satisfy collision conditions which are: (a) rate of change of LOS is zero and (b) closing velocity is negative or the LOS separation is reducing in between the missile and target. The lead angle is theoretically correct angle for missile to be on collision course because missile is not always fired in the collision course. The missile is fired in the direction of targets and that results into an error in the lead angle. This error is the 'Heading Error (HE)'. From [4 and 5], we state the N as time varying navigation ratio:

$$N = \frac{N'.V_c}{V_m.\cos(\varepsilon +} \qquad (5)$$

and this has been used in [3]. This is used in the present work as shown in Fig. 1. In the equation (5), the N' is effective navigation ratio (N' ≥ 2), and N is the varying navigation Ratio. The N' is set as constant. The advantages of time-varying navigation ratio is that if the N'>2, Vc >0, Vm>0, then the sign of navigation ratio can be determined by $\cos(\varepsilon + H$ for $\cos(\varepsilon + H>=0$, N>0 (PN), and for $\cos(\varepsilon + H<0$, N<0 (RPN).

III. PROPORTIONAL NAVIGATION

Following [6-8] the guidance law can be stated:

$$a_c = N V_c. \tag{6}$$

and as shown in Fig. 1 the target can maneuver evasively with acceleration magnitude . Furthermore,

$$\beta = \tag{7}$$

where is the target acceleration, is the angular velocity of target, is the magnitude of target velocity. The is missile velocity vector. The components of the target velocity vector in the earth or inertial coordinate system can be found by integrating above and we observe:

$$V_{TX} = -V_T \cos\beta ; V_{TY} = -V_T \,\mathrm{si} \tag{8}$$

The target position components in the earth fixed coordinate system are found by directly integrating the target velocity components. Therefore, the differentials for the components of the target position are:

$$\dot{R}_{TX} = V_{TX}; \dot{R}_{TY} = \tag{9}$$

Similarly, the missile velocity and position differential equations are:

$$\dot{V}_{MX} = a_{MX}; \dot{V}_{MY} = a \tag{10}$$

$$\dot{R}_{MX} = V_{MX}; \dot{R}_{MY} = V \tag{11}$$

where a and a are the missile acceleration components in the earth coordinate system. In order to find the missile acceleration components, the components of the relative missile-target separation need to be computed. For this the components of the relative missile-target separation are computed by:

$$R_{TMX} = R_{TX} - R_{MX}; R_{TMY} = R_{TY} - R \tag{12}$$

and the LOS angle can be found, using trigonometry, in terms of relative separation components, e.g.

$$\lambda = \tan^{-1}\frac{R_T}{R_T} \tag{13}$$

The relative velocity components in earth coordinates are:

$$V_{TMX} = V_{TX} - V_{MX}; V_{TMY} = V_{TY} - V \tag{14}$$

and the LOS rate is computed:

$$\frac{d}{dt}\left[\tan^{-1}\frac{R_{TMY}}{R_{TMX}}\right] = \left[\frac{1}{1+\left(\frac{R_{TMY}}{R_{TMX}}\right)^2}\right]\frac{d}{dt}\left(\frac{R_{TM}}{R_{TM}}\right.$$

By using the quotient rule and simplifying we get:

$$\dot{\lambda} = \left[\frac{R_{TMX}V_{TMY} - R_{TMY}V_{T}}{R_{TM}^2}\right. \tag{15}$$

The relative separation between missile and target is R, is expressed in terms of its inertial components by application of the distance formula:

$$R_{TM} = \sqrt{R_{TMX}^2 + R_{T}} \tag{16}$$

Since the closing velocity is defined as the negative rate of change of the missile target separation.

It can be obtained by differentiating above equation, i.e.,

$$V_c = -\dot{R}_{TM} = \left[\frac{-(R_{TMX}V_{TMX} + R_{TMY}V_{TM}}{R_{TM}}\right. \tag{17}$$

and the magnitude of the missile guidance command is computed:

$$a_c = N\left[\frac{R_{TMX}R_{TMY}(V_{TMX}^2 - V_{TMY}^2) + V_{TMX}V_{TMY}(R_{TMY}^2 - R_{TMX}}{R_{TM}^3}\right. \quad (18)$$

The missile acceleration components in earth coordinates are computed by trigonometry using angular definition:

$$a_{MX} = -a_c \sin\lambda;\ a_{MY} = a_c\,\mathrm{co} \quad (19)$$

The initial angle of missile velocity vector with respect to LOS is the missile lead angle and it is:

$$\varepsilon = \sin^{-1}\left(\frac{V_T \sin(\beta +}{V_M}\right. \quad (20)$$

The initial angular deviation of missile from collision triangle is known as a ‘Heading Error (HE)’.

IV. NUMERICAL SIMULATION RESULTS

The numerical equations listed above are solved using the second-order Runge – Kutta method and the step sizes have been input by the user. Our numerical simulation results show that the optimum results are obtained for step size of 0.01 till range is less than 1000 m, and further lowering of step size to 0.0002 can accurately capture the miss-distance.

4.1 Results w.r.t. for different Scenarios

Fig. 2 presents trajectories of missile which are considered to be launched for a scenario in which missile is being fired from a war-ship trying to intercept an anti-ship missile being fired from airborne platform. Here the target is accelerating toward the platform which need to be defended with a maneuvering acceleration of ‘-0.1 g’. For the trajectory of interceptor the initial position coordinates are (0 m, 10 m) and initial position of target is at (10000 m, 1000 m).

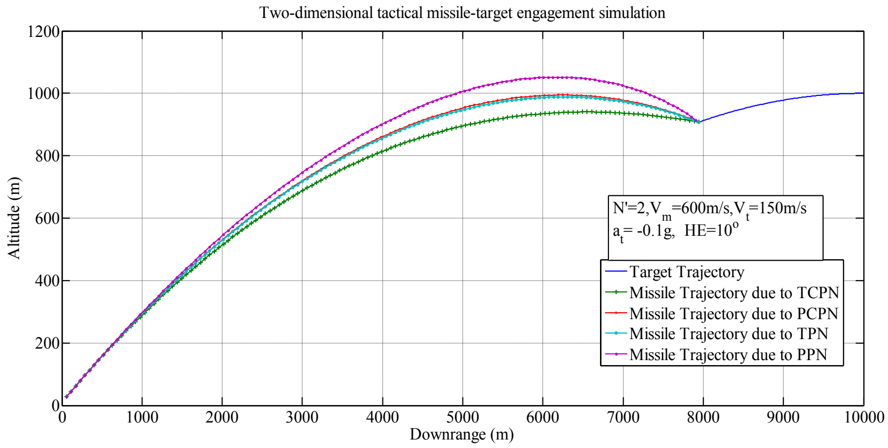

Figure 2: Trajectories of Missile and Interceptor for Ship Launched Scenario.

The computed miss-distance obtained with respect to different guidance laws is listed in Table 2 and it can be seen from the values of miss-distance that proposed new strategy is efficient than other variations of the PN guidance law.

Table 2: Miss-Distance w.r.t Different Guidance Strategies

Guidance law	Miss-distance (m)
TCPN	1.1637
PCPN	19.93
TPN	17.79
PPN	42.94

Fig. 3 shows the engagement senerio for high speed incoming target threat with target velocity as 1500 m/s. Again we can see from Table 3 that even though the miss-distance is larger than in case of low speed targets (Table 2), still the TCPN is effeicient that other variations of the PN guidance law.

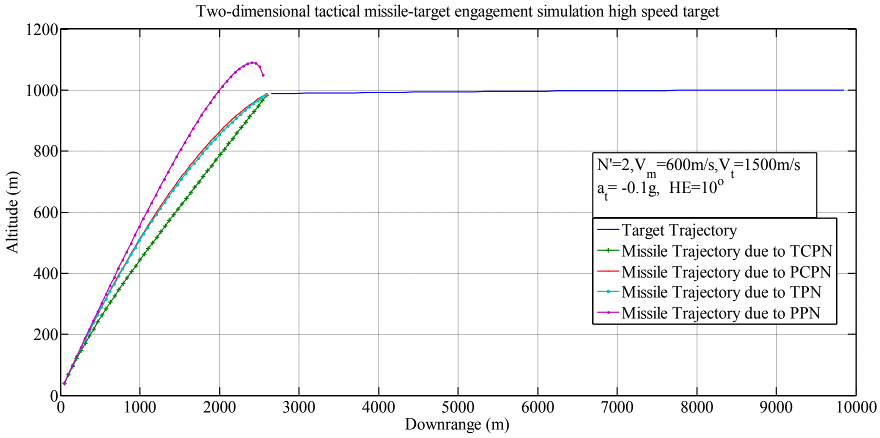

Figure 3: Trajectories of Missile and Interceptor for Ship Launched Scenario for High Speed Targets.

Table 3: Miss-Distance w.r.t. Different Guidance Strategies

Guidance law	Miss-distance (m)
TCPN	55.23
PCPN	62.404
TPN	61.3
PPN	118.34

Fig. 4 shows the trajectories of missile and target in the scenario of the missile being launched from the airborne platform. Here, the heading error is considered at -20º for a non-maneuvering target. We observe from Fig. 7 that as the target velocity goes on increasing intercepting range goes on reducing, e.g. when target velocity is 300 m/s, intercept occurs at around 6475 m and so on till the downrange is at 2195 m for the target velocity at 2000 m/s. The trajectory of interceptor with initial position coordinates at (0 m, 10,000 m) and initial position of target is at (10000 m, 10000 m). Here, the N'=2 and the target velocity , is tested for 300, 600, 900, 1500, and 2000 m/s. It can be noted here that the initial missile is travelling in the wrong direction due to heading error but latter on-course it is corrected with effect of guidance law. From Table 5, we can observe that as the target velocity goes on increasing, miss-distance due to the varying navigation ratio goes lower in magnitude compare to miss-distance due to constant navigation ratio.

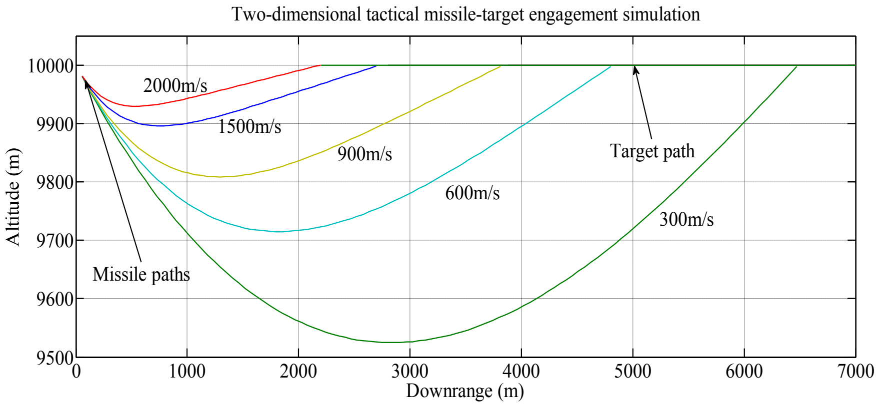

Figure 4: Trajectories of missile and interceptor for launch from airborne platform.

Table 4: Range of interception w.r.t. different .

Target Velocity (m./s)	Intercepting downrange (m.)
300	6475
600	4803
900	3812
1500	2699
2000	2195

Table 5: Comparison of miss-distance due constant navigation ratio (N') and varying navigation ratio (N’).

Target velocity (m./s)	300	600	900	1500	2000
Miss distance (m) with N. (*10^3)	0.1310	0.0841	0.0664	0.4036	0.2847
Miss distance (m) with N’. (10^3)	0.0801	0.0893	0.1178	0.3212	0.0279

We can observe from Fig. 5 that with an increase in effective navigation ratio, the HE error is removed more rapidly. The interceptor has a shorter trajectory to intercept but it also results in larger missile acceleration at the beginning of the flight. Fig. 5 shows the varying navigation constant with respect to different values of target velocity. The navigation constant is time varying whose value depends on interceptor’s flight path angle and LOS angle.

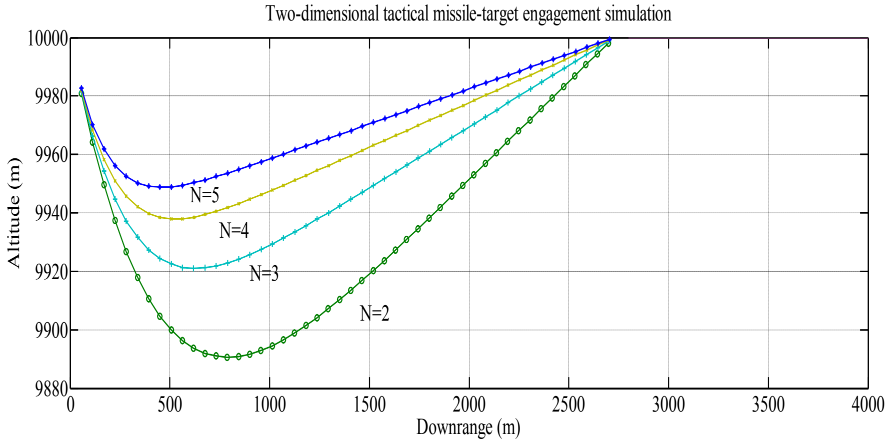

Figure 5: Effective Navigation Ratio vs Heading Error.

We can observe from Fig. 6 that value of varying navigation ratio has positive values up till heading error is < 90°, as heading error goes beyond 90°, the varying navigation ratio becomes negative, and this negative varying navigation ratio represent RPN guidance law [2]. At the cost of a higher intercept time, the RPN guidance law demands lower terminal lateral acceleration than proportional navigation and can intercept high-velocity targets from many initial conditions that the classical proportional navigation cannot.

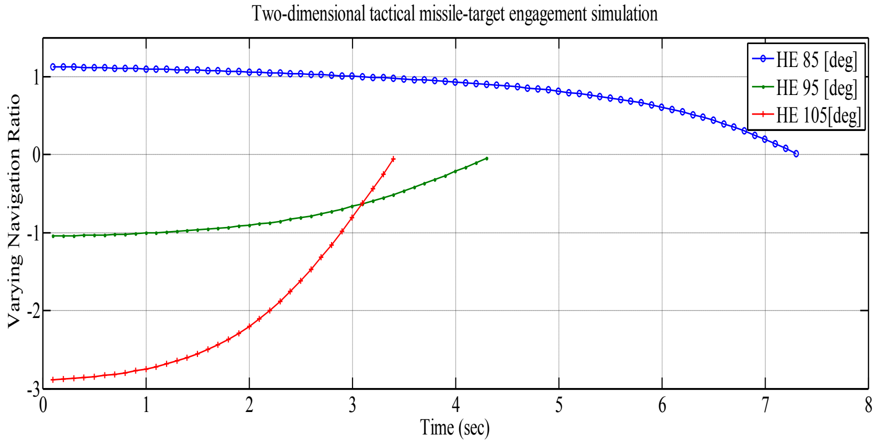

Figure 6: Change in varying navigation ratio with time.

A new guidance law is designed to intercept targets that are of higher speeds than the interceptor. The RPN Guidance law is a modification of the classical PN guidance law. It uses a negative navigation constant, and it consequently attempts to intercept the target at the interception point that is farther from the initial target position by rotating the interceptor flight-path angle opposite to the direction of rotation of the LOS. The time for interception is more for the RPN than for the PN. However, the RPN performs significantly better than the classical PN law in terms of missile capturability, the interceptor lateral acceleration or “latax” demand, and closing velocity when used against high-speed targets. The miss distances at various heading errors are listed in Table 6. Table 7 shows the varying navigation ratio with respect to different target velocities. We observe that there is not much change in the navigation ratio gradient due to the change in target velocities, as is observed during changes in heading errors. From the results it can be stated that the HE can used to control N for more efficient guidance law performance irrespective of target velocity. This feature is favorable in interception of high velocity incoming threats.

Table 6: Miss distance at various heading errors.

HE (°)	20	40	60	80
Miss distance (m.)	58.85	77.46	27.69	3828

Table 7: Variation of N values at different Target Velocities with constant HE.

Target Velocity (m/s)	Varying Navigation Constant range
300	2.878—2.811
600	3.878—3.851
900	4.878—4.864
1500	6.878—6.873
2000	8.545—8.542

V. CONCLUSION

This paper has presented a variant of PN – TCPN – guidance law and our results show that it is efficient for both the low and high speed targets. The higher values of effective navigation ratios results into a tighter interception trajectory which demands a higher initial acceleration but also lead to fast removal of the heading error. We have observed that using the heading error as an control variable, it is possible to change from the PN guidance law to the RPN guidance law. The use of varying N to generate latax ensures better capturability, than the use of effective navigation constant (N'). The simulation results showed that the TCPN performs better than other guidance strategies as it combines the advantages of both the PN and RPN. Also, it incorporates the influence of heading error which is a crucial parameter in guidance law. In the TCPN heading errors are removed faster against other laws.

However, in this paper the theoretical aspects of the TCPN have not been investigated in detail. This needs an exploration and our future work will go in this direction. Currently, this is under investigation.

VI. ACKNOWLEDGEMENTS

This research was supported by the internal research grants of IIT Madras through research scheme: OE12S018 though HTRA support scheme of the MHRD, GoI, India.

VII. TRADEMARKS AND COPYRIGHTS

*Trademark and copyright with MathWorks, Inc., USA.

REFERENCES

[1] D. Ghose, Guidance of Missiles: An Introduction to Proportional Navigation, NPTEL (2012).

[2] Prasanna, H. M., Ghose, D.: Retro-Proportional-Navigation: A New Guidance Law for Interception of High Speed Targets. J. Guidance. Control. Dyn. 35, 377–386 (2012).

[3] Li, Y., Yan, L., Zhao, J., Liu, F., Wang, T.: Combined proportional navigation law for interception of high-speed targets. Defense Technolgy. 10, 298–303 (2014).

[4] Siouris, G. M.: Missile Guidance and Control Systems. Springer, (2004).

[5] Babu, K. R., Sarma, I. G., Swamy, K. N.: Switched Bias Proportional Navigation for Homing Guidance Against Highly Maneuvering Targets. 17, (1994).

[6] Zarchan, P.: Tactical and Strategic Missile Guidance Sixth Edition. American Institute of Aeronautics and Astronautics, Inc. (2012).

[7] Lin, C.F.: Modern Navigation Guidance and Control Processing. Prentice Hall (1991).

[8] Swee, J.C.S.: Missile Terminal Guidance And Control Against Evasive Targets, (2000).