

# A FINITE AUTOMATA MODEL FOR LAKE EUTROPHICATION

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## **ABSTRACT**

*A lake's trophic state is a measure of its "biological productivity", which is further a measure of the mass of plants and animals in a lake. Hence, it is important to look at systematic, efficient and automated approaches to study the various trophic states and find out the causes that lead to the trophic variations. The purpose of this paper is to study the computational aspects of the changing trophic levels of the lakes and to find a transition rule then generate grammar that will describe the changing trophic states of modelled lake system. A model to the changing trophic levels of the lakes by NDFA, using a Prisoner's Dilemma game is then generated and then a DFA from the NDFA is produced.*

**Keywords:** *DFA, Grammar for the Lake Eutrophication, NDFA, Prisoner's Dilemma Game, Trophic States of Lakes.*

## **I. INTRODUCTION**

The hydrologic status of water of a lake in terms of quantity and quality is the result of complex processes of physical, chemical and biological inputs. The different trophic levels of lakes and the causes is used to study the discrete game between human and environment. We intend to study the computational aspects of the changing trophic levels of the lakes. Here we are discussing "To find a transition rule and thus a grammar that will describe the changing trophic states of modelled lake system".

The word 'eutrophic' is a descendent of the Greek word "eutrophos" which means well-fed. Rast and Thornton's (1996) defines "eutrophication is basically the natural ageing process of lakes". Other famous definitions includes Chorus and Bartram's (1999), which states that "eutrophication is the enhancement of the natural process of biological production caused by nutrient enrichment" and very popular definition of South Africa stating that "eutrophication is the process of nutrient enrichment of waters which results in the stimulation of an array of symptomatic changes, amongst which increased production of algae and aquatic macrophytes, deterioration of water quality and other symptomatic changes are found to be undesirable and interfere with water uses [OECD, 1982]". The above definition is a little complex. In short, eutrophication is nutrient enrichment that causes problems. Therefore, a simplified definition for the word is adopted. Eutrophication is the process of excessive nutrient enrichment of waters that typically results in problems associated with macrophyte, algal or cyanobacterial growth. The causes and effects of eutrophication are complex. Eutrophication has occurred naturally over centuries as lakes aged and were filled in with sediments (Carpenter 1981). However, human activities have always proved to be an accelerator for the rate and to extent

eutrophication through both point-source discharges and non-point loadings of limiting nutrients, such as nitrogen and phosphorus, causing cultural eutrophication, with drastic consequences for water sources, fisheries, and water bodies (Carpenter et al. 1998). The major consequences of cultural eutrophication includes cyanobacteria (blue green algae), contaminated drinking water supplies, demotion of recreational opportunities, and deficiency of oxygen.

## II. PROBLEM STATEMENT

To study the various trophic states and find out the causes that lead to the trophic variations and to model the changing trophic levels of lakes by NDFA, using a Prisoner's Dilemma game and then find out a DFA from the NDFA. Finally a grammar is to be generated that will describe the changing trophic level of lakes.

## III. RELATED WORK

### 3.1 Trophic States of Lakes

Depending on the pH, seed values, Nitrogen, Phosphorous, etc. the lakes are generally classified into four major states: Oligotrophic (O), Mesotrophic (M), Eutrophic (E) and Hypereutrophic (HE).

### 3.2 For Our Modelling of the Lake System we Consider Seven States as

Oligotrophic (O), Fuzzy mesotrophic (FM), Mesotrophic (M), Fuzzy eutrophic (FE), Eutrophic (E), Fuzzy hypereutrophic (FH) and Hypereutrophic (H)

## IV. PROBLEM SOLVING APPROACH

### 4.1 Agent

Agents are autonomous decision making entities. Each agent individually assess its situation and make decision on the basis of set of rules.

### 4.2 Game

A game is a three tuple  $G (P, \Sigma, \Pi)$  where:

P: set of players.

$\Sigma$ : strategy space.

$\Pi$ : the set of payoffs associated with the strategies.

### 4.3 Two Person Prisoner's Dilemma

The Prisoner's Dilemma (PD) game epitomizes the evolution of cooperation among rational, unrelated agents, in the absence of a central enforcing authority. It is a symmetric game, two player and two pure strategies.

#### 4.4 Strategic Form of Prisoner's Dilemma Game

		PLAYER II	
		C	D
PLAYER I	C	(3,3)	(0,5)
	D	(5,0)	(1,1)

#### 4.5 Equivalent Strategic Form

		PLAYER II	
		C	D
PLAYER I	C	(R,R)	(S,T)
	D	(T,S)	(P,P)

*R*: reward for mutual cooperation

*S*: sucker's payoff

*T*: temptation to defect

*P*: punishment for mutual defection

Constrained through the payoff ordering:  $T > R > P > S$

#### 4.6 The Lake Eutrophication Model

We assume the discrete interactions between human and lake environment to follow the Two Prisoner's Dilemma Game. We model the game using an N DFA. We obtain the DFA from the N DFA. We obtain the Grammar from the DFA.

## V. SOLUTION

Q Comprises the following states, which represent the various trophic levels of a lake:

1. Oligotrophic(*O*)
2. Fuzzy mesotrophic(*FM*)
3. Mesotrophic(*M*)
4. Fuzzy eutrophic(*FE*)

5. Eutrophic(*E*)
6. Fuzzy hypereutrophic(*FH*)
7. Hypereutrophic(*H*)

$\Sigma$ :The alphabet comprises the inputs, which are the strategies used by the players in playing G, given by the table below:

<u>STRATEGY</u>	<u>SYMBOL</u>	<u>CODED AS:</u>
Cooperate, Cooperate	( <i>C,C</i> )	1
Cooperate, Defect	( <i>C,D</i> )	2
Defect, Cooperate	( <i>D,C</i> )	3
Defect, Defect	( <i>D,D</i> )	4

*q* is the Oligotrophic state, representing the initial state of the trophic levels of the lakes.

The transition function is described by the following matrix:

Inputs	1	2	3	4
States				
<b>O</b>	O	O	FM,M,FE,E,FH,H	O,H
FM	O	O	M,FE,E,FH,H	O,H
M	O,FM	O	FE,E,FH,H	O,H
FE	O,FM,M	O	E,FH,H	O,H
E	O,FM,M,FE	O	FH,H	O,H
FH	O,FM,M,FE,E	O	H	O,H
<b>H</b>	FH	FH	H	H

*h*, the set of final states, includes the terminals O & H.

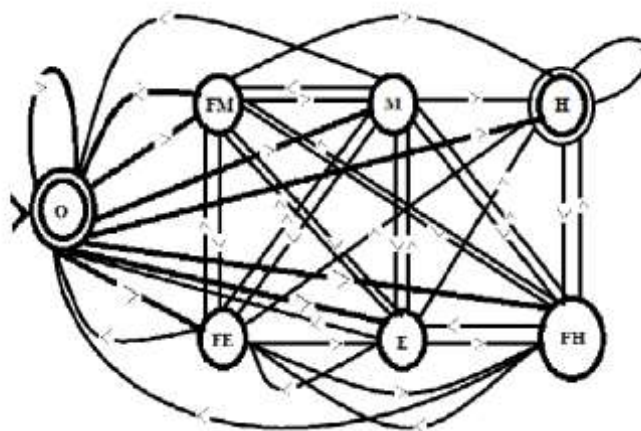


Fig.1 NDFA

### 5.1 Dfa For Lake Eutrophication

On the basis of the obtained NDFA for the lake eutrophication the following DFA is obtained by the rule of:

1. State multiplicity and
2. State minimization.

Q: Comprises the following states, which represent the various trophic levels of a lake and their suitable combinations:

<u>STATES</u>	<u>CODED AS</u>
FM,M,FE,E,FH,H	a
O,H	b
O,FM,M,FE,E,FH	c
O,FH	d
M,FE,E,FH,H	e
FE,E,FH,H	f
E,F,H	g
FH,H	h
O,FM,M,FE,E	i
O,FM,M,FE	j
O,FM,M	k
O,FM	l
<b>O</b>	<b>O</b>
<b>H</b>	<b>H</b>
<b>FH</b>	<b>FH</b>

$\Sigma$ : The alphabet comprises the inputs, which are the strategies used by the players in playing  $G$ , given by the table below:

<u>STRATEGY</u>	<u>SYMBOL</u>	<u>CODED AS:</u>
Cooperate, Cooperate	$(C,C)$	1
Cooperate, Defect	$(C,D)$	2
Defect, Cooperate	$(D,C)$	3
Defect, Defect	$(D,D)$	4

q is the Oligotrophic state, representing the initial state of the trophic levels of the lakes.

The transition function is described by the following matrix:

<u>INPUTS--&gt;</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
<u>STATES</u>				
<b>O</b>	<b>O</b>	<b>O</b>	a	b
a	c	d	e	b
b	d	d	a	b
c	c	d	a	b
d	i	<b>O</b>	a	b
e	c	d	f	b
f	c	d	g	b
g	c	d	h	b
h	c	d	<b>H</b>	b
<b>H</b>	<b>FH</b>	<b>FH</b>	<b>H</b>	<b>H</b>
<b>FH</b>	i	<b>O</b>	<b>H</b>	b
i	j	<b>O</b>	a	b
j	k	<b>O</b>	a	b
k	l	<b>O</b>	a	b
l	<b>O</b>	<b>O</b>	a	b

h , the set of final states, includes the terminals O, a, b, h & H.

## 5.2 Grammar for the Lake Eutrophication

Based on the Deterministic Finite Automata (DFA) the following grammar is given for the lake eutrophication and the changing trophic levels of lakes.

<u>STATES</u>	<u>CODED AS</u>	<u>CODING FOR GRAMMAR</u>
FM,M,FE,E,FH,H	a	J
O,H	b	K
O,FM,M,FE,E,FH	c	L
O,FH	d	M
M,FE,E,FH,H	e	N
FE,E,FH,H	f	O
E,F,H	g	P
FH,H	h	Q
O,FM,M,FE,E	i	R
O,FM,M,FE	j	S
O,FM,M	k	T

O,FM	l	U
<b>O</b>	<b>O</b>	V
<b>H</b>	<b>H</b>	W
<b>FH</b>	<b>FH</b>	X

Corresponding CFG is:

→ *V	→	V/J/K/ε
*J	→	L/M/N/K/ε
*K	→	M/J/K/ε
L	→	L/M/J/K
M	→	R/V/J/K
N	→	L/M/O/K
O	→	L/M/P/K
P	→	L/M/Q/K
*Q	→	L/M/W/K/ε
R	→	S/V/J/K
S	→	T/V/J/K
U	→	V/J/K
*W	→	X/W/ε
X	→	R/V/W/K

## VI. CONCLUSION

The changing trophic levels of lakes and Lake Eutrophication can be very well understood with appropriate computational logics over a discrete time using the interactions between human and Lake Environment, NDFA, DFA and the grammar.

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