# A DISCRETE MODEL OF ROSSLER SYSTEM

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## ABSTRACT

*This paper considers a discrete version ofRosslersystem in 3-D state space. For suitable values of the parameters, the Jacobian is computed. Also eigenvalues and eigen vectors are evaluated. The corresponding dynamical properties are exhibited via time plots and phase diagrams in three dimensional space.*

***Keywords -- Difference Equations, Rossler System, Fixed Points, Eigen Values***

## I. INTRODUCTION

In 1963, meteorologist and mathematician Edward N. Lorenz published numerical studies of the solutions of a simplified model for atmospheric turbulence. Lorenz' equations model convective air currents and temperature changes in a planar vertical cell beneath a thunderhead. The Lorenz system consists of three differential equations [1,3,5]. The Rossler system plays an important role in the study of dynamical systems. In his work on continuous chaos, Rossler has been motivated by the search for chemical chaos, that is, chaotic behavior in far-from-equilibrium chemical kinetics. The Rossler system has only one quadratic nonlinearity. This model along with Lorenz model have attracted a large number of studies [7]. The defining equations of the Rossler system are

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + xz - cz \end{aligned} \qquad (1)$$

where$a, b, c \in R$ and they are positive and dimensionless.

## II. DISCRETE ROSSLER SYSTEM AND STABILITY ANALYSIS

A dynamical systemis a set of variables interacting over time. The changes in these variables, their time series, can exhibit various patterns of behavior. The science of nonlinear dynamics and chaos theory has sparked many researchers to develop mathematical models that simulate vector fields of nonlinear chaotic physical systems. Nonlinear phenomena arise in all fields of engineering, physics, chemistry, biology, economics, and sociology. The Rössler attractor exists in a three-dimensional state space. This paper considers the following discrete equation [2,6] of the Rossler system as

$$\begin{aligned} x(n+1) &= x(n) - h[y(n) + z(n)] \\ y(n+1) &= y(n) + h[x + ay] \\ z(n+1) &= z(n) + h[b + z(x - c)] \end{aligned} \qquad (2)$$

In order to find the fixed points, the three Rossler equations are set to zero and the $(x, y, z)$ coordinates of each fixed point were determined by solving the resulting equations. Thisyields the fixed point coordinates

$$E_1 = \frac{c + \sqrt{c^2 - 4ab}}{2}, -\left(\frac{c + \sqrt{c^2 - 4ab}}{2a}\right), \frac{c + \sqrt{c^2 - 4ab}}{2a}$$

$$E_2 = \frac{c - \sqrt{c^2 - 4ab}}{2}, -\left(\frac{c - \sqrt{c^2 - 4ab}}{2a}\right), \frac{c - \sqrt{c^2 - 4ab}}{2a}$$

The Jacobian matrix J for the system is given by

$$J(x, y, z) = \begin{pmatrix} 1 & -h & -h \\ h & 1 + ah & 0 \\ hz & 0 & 1 + h(x - c) \end{pmatrix}$$

The eigenvalues can be determined by solving the characteristic equation.

## III. NUMERICAL STUDY

Some properties of the Rössler system can be deduced via linear methods such as eigenvectors. The stability of each of these fixed points can be analyzed by determining their respective eigenvalues and eigen vectors. In this section for different set of parameters, the eigen values and corresponding eigen vectors are evaluated. Also time plots and phase portraits are presented [4]. The eigenvectors have several interesting implications. The magnitude of a negative eigen value characterizes the level of attraction along the corresponding eigenvector. Similarly the magnitude of a positive eigen value characterizes the level of repulsion along the corresponding eigenvector.

### EXAMPLE:1

For the equilibrium point $E_1$ consider the Rossler parameter values of $h = 0.001, a = 0.1, b = 0.1$, and $c = 4.7$ yield the Eigenvalues are $\lambda_{1,2} = 0.9990 \pm i0.0069$ and $\lambda_3 = 1.0001$ and also the Eigenvectors are

$$v_1 = \begin{bmatrix} 0.7071 \\ -0.0354 - i0.7062 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0.7071 \\ -0.0354 + i0.7062 \\ 0 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.1995 \\ -0.0416 \\ 0.9790 \end{bmatrix}.$$

For the equilibrium point $E_2$ consider the Rossler parameter values of $h = 0.001, a = 0.1, b = 0.1$, and $c = 4.7$ yield the Eigenvalues are $\lambda_{1,2} = 1.0000 \pm i0.0069$ and $\lambda_3 = 1.0001$ and also the Eigenvectors are

$$v_1 = \begin{bmatrix} 0.0215 + i0.1442 \\ 0.0201 - i0.0063 \\ 0.9891 \end{bmatrix}, v_2 = \begin{bmatrix} 0.0215 - i0.1442 \\ 0.0201 + i0.0063 \\ 0.9891 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.0309 \\ -0.7076 \\ 0.7059 \end{bmatrix}.$$

### EXAMPLE:2

For the equilibrium point $E_1$ consider the Rossler parameter values of $h = 0.001, a = 0.1, b = 0.1$ and $c = 9.7$ yield the Eigenvalues are $\lambda_{1,2} = 1.0005 \pm i0.0100$ and $\lambda_3 = 0.9030$ and also the Eigenvectors

$$v_1 = \begin{bmatrix} 0.7071 \\ -0.0357 - i0.7062 \\ 0.0007 - i0.0001 \end{bmatrix},\ v_2 = \begin{bmatrix} 0.7071 \\ -0.0357 + i0.7062 \\ 0.0007 + i0.0001 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.1015 \\ -0.0104 \\ 0.9948 \end{bmatrix}.$$

For the equilibrium point $E_2$ consider the Rossler parameter values of $h = 0.001, a = 0.1, b = 0.1$ and $c = 9.7$ yield the Eigenvalues are $\lambda_{1,2} = 1.0000 \pm i0.0990$ and $\lambda_3 = 1.0010$ and also Eigenvectors are $v_1 = \begin{bmatrix} 0.0001 + i0.1015 \\ 0.0103 - i0.0001 \\ 0.9948 \end{bmatrix}, v_2 = \begin{bmatrix} 0.0001 - i0.1015 \\ 0.0103 + i0.0001 \\ 0.9948 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 0.0008 \\ -0.7071 \\ 0.7071 \end{bmatrix}$

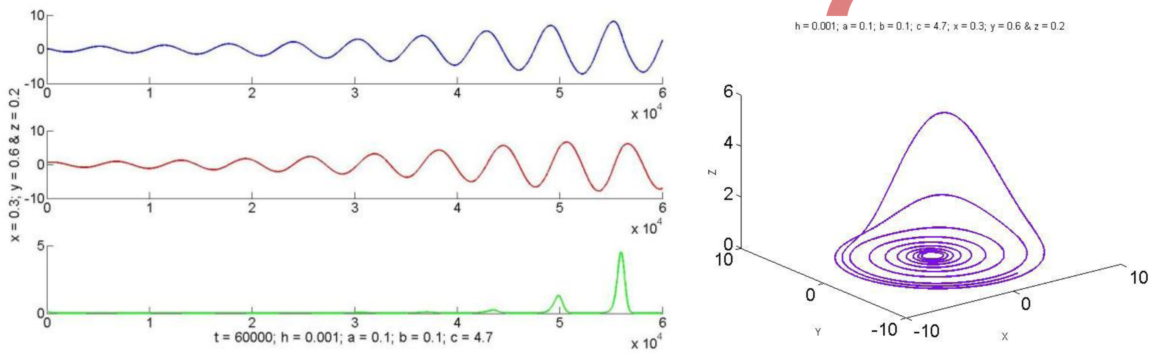



Figure-1.Time Series and Phase space trajectories values of c = 4. 7

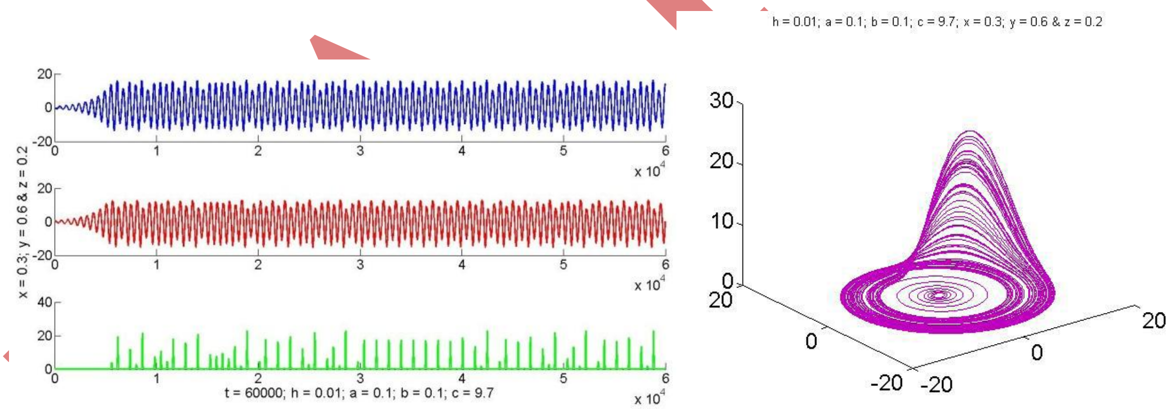



Figure- 2Time Series and Phase space trajectories values of c = 9.7

**EXAMPLE: 3**

For the equilibrium point $E_1$ consider the Rossler parameter values of $h = 0.01, a = 0.1, b = 0.1$ and $c = 14$ yield the Eigenvalues are $\lambda_{1,2} = 1.0000 \pm i0.0100$ and

$\lambda_3 = 0.9860$ and also the Eigenvectors

are $v_1 = \begin{bmatrix} 0.7071 \\ -0.0357 - i0.7062 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0.7071 \\ -0.0357 + i0.7062 \\ 0 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 0.0709 \\ -0.0050 \\ 0.9975 \end{bmatrix}$

For the equilibrium point $E_2$ consider the Rossler parameter values of $h = 0.001, a = 0.1, b = 0.1$ and $c = 14$ yield the Eigenvalues are $\lambda_{1,2} = 1.0000 \pm i0.0119$ and $\lambda_3 = 1.0001$ and the Eigen values

are $v_1 = \begin{bmatrix} 0.0004 + i0.0845 \\ 0.0071 - i0.0001 \\ 0.9964 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0.0004 - i0.0845 \\ 0.0071 + i0.0001 \\ 0.9964 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 0.0010 \\ -0.7072 \\ 0.7071 \end{bmatrix}$

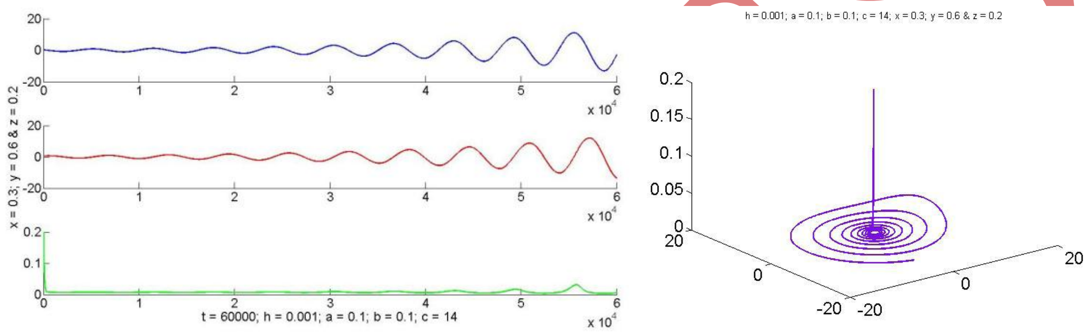


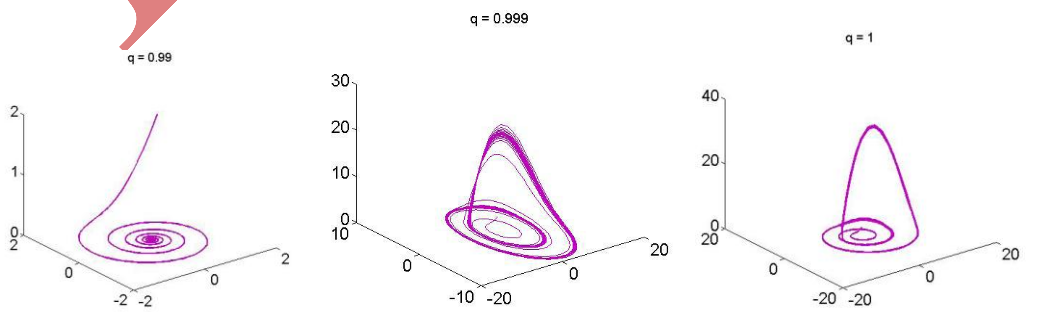

Figure-3Time Series and Phase space trajectories values of c = 14

## IV. MAGNIFICATION OF DISCRETE ROSSLER'S SYSTEM

The discrete version of the Rossler's Equation is given by

$$x(n+1) = (x(n) - h[y(n) + z(n)])q$$
$$y(n+1) = (y(n) + h[x + ay])q \qquad (3)$$
$$z(n+1) = (z(n) + h[b + z(x - c)])q$$

Following diagrams in 3-Dimensions are presented for different values of q from 0.99 to 1.0035. Let $h = 0.05$ ; $a = 0.2$ ; $b = 0.2$, $c = 4.7$ and $x = 1.2$; $y = 1.3$; $z = 1.6$ with the iterative times increasing $q = 0.99 - 1.0035$ .



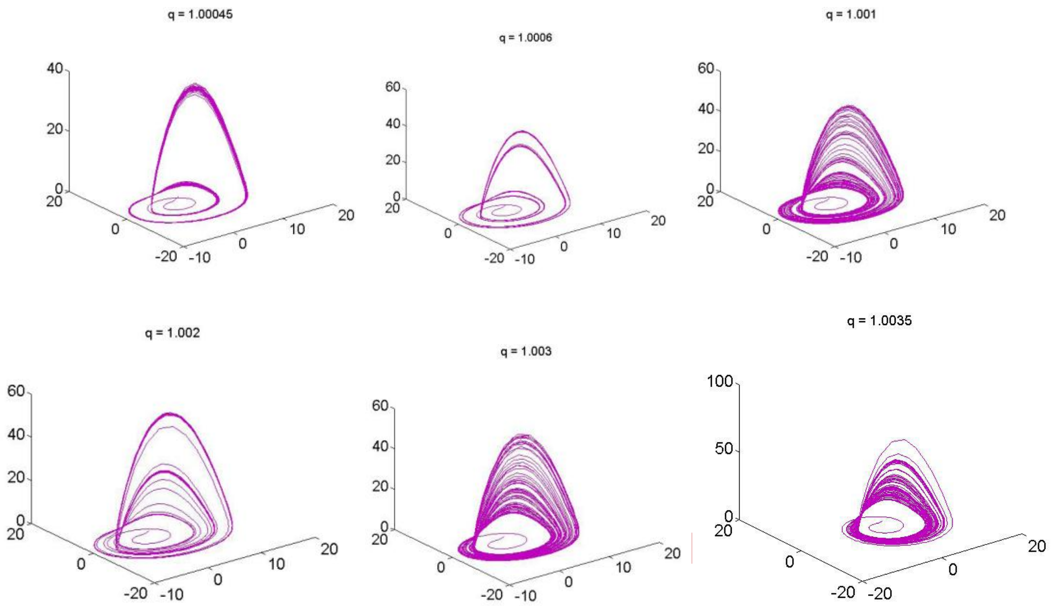



Figure-4 Phase space trajectories of 6000 point

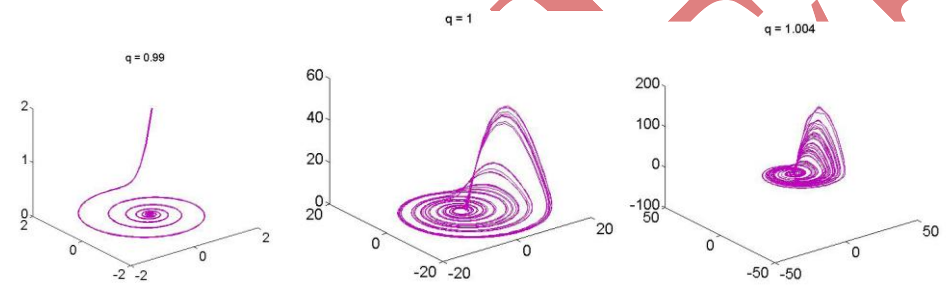



Figure-5 Phase space trajectories of 6000 points at h = 0.05; a = 0.2; b = 2; c = 10.7

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