Comparative Study of Coded Modulation schemes using EXIT chart and BER characteristics

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Abstract

This paper , extensively studies on the topic Coded Modulation(CM) a scheme which combines both the coding as well as the modulation in together to get a bandwidth efficient scheme. This project contributes a genuine comparative study on different Coded Modulation schemes such as Trellis Coded Modulation (TCM), Bit interleaved Coded Modulation (BICM),Turbo Trellis Coded Modulation (TTCM) and Bit Interleaved Coded Modulation with Iterative Decoding (BICM-ID) in context of 8 level Phase Shift Keying(8PSK) over the Gaussian and uncorrelated Rayleigh channels. Here, the comparison are done in terms of decoding complexity, the bandwidth efficiency, the coding gain and the frame length by using the study tools such as EXtrinsic Information Transfer Charts(EXIT charts) and BER curve characteristics.

Keywords - Coded Modulation, BICM, BICM-ID, EXIT Charts, Iterative Decoding, Spectrum EfficiencyTCM, TTCM

1 Introduction

The radio spectrum is a scarce resource. Therefore, the main objective is to efficiently exploit the available spectrum. The error correction codes always add the redundancy by increasing the number of coded symbols. Since, the channel is band limited the transmitted symbol rate should be fixed which could result in lower rate information transmission rate. Now, the question is how can the channel be utilized in an efficient way still by adding the redundant bit. The key therefore is to combine the coding as well as modulation into a single unit and hence the name Coded Modulation.

Ungerboeck [1], in his paper fully describes how to employ the TCM schemes in redundant non-binary modulation(symbol based) with the combination of a finite state Forward Error Correction(FEC) encoder, which selects the coded signal sequence. The extra bits formed by corresponding convolution encoder will restrict the possible state transformation among the consecutive phasor to a certain legitimate constellation. The receiver tries to decode the incoming noisy signal by a trellis based soft-decision maximum-lilkelihod detector and tries to map it to the each of the legitimate phasor sequence by the restrictions imposed by the convolution encoder. The term "Trellis" is used to describe this scheme is because the overall operation can be described by a corresponding state transition diagram similar to that of binary convolution encoder. The only difference in TCM is that, here trellis branches are labelled with respect to the redundant non-binary modulated phasors. paper proposed an new approach know as "set partitioning" [1] which aims more directly at maximum free Euclidian Distance(ED).

Turbo codes [2] was a major milestone in the forward error correction codes which can even achieve an excellent bit error rates at low SNR. The original proposal was for the BPSK scheme but were soon successful with multilevel coded as well. Robertson [3] soon introduced the concept of the "Turbo Trellis Coded Modulation" (TTCM) employing two TCM codes as parallel concatenation of two recursive TCM encoder, and adapted puncturing mechanism to avoid the obvious disadvantage of the rate loss.

Bit-interleaved Coded Modulation (BICM) was a idea proposed by Zehavi [4] in order to improve the diversity of the code in Rayleigh channel. The design of the coded modulation schemes are affected by several factors such as High Free Euclidian Distance which is desired for the AWGN channel, while its was interested to note that a high Effective Code Length and a high minimum product distance were the main factors effecting the fading channel. The diversity of the code can be defined as "length" of the shortest error path [5] and one should be aware that the shortest error distance are not necessarily be the minimum distance error. Unfortunately there was no TCM codes available which can compensate the above said difficulties. In order to solve the problem Zehavi came with an idea to render the code's diversity equal to that smallest number of different bits, employing the bit-based interleaving.

Bit-Interleaved Modulation(BICM) was purposed to increase the diversity of the Ungerboeck TCM scheme under the Rayleigh channel. Li [6, 7] suggested a new iterative scheme of Bit-Interleaved Coded Modulation(BICM-ID) using Iterative Decoding which employed Set-Partitioning signal labelling system as that of Ungerboeck. Introduction of soft-decision feedback from the decoder's output to the demapper/demodulator input to iterate between them is advantageous of the fact that, it improves the reliability of the soft information passed to the demapper/demodulator.

EXtrinsic Information Transfer(EXIT) [8] chart analysis is powerful that is used to check the convergence of the iterated decoders. BER chart is one of most powerful tool to analysis the performance how good the decoder is, but it was not able to explain in detail how the decoder converges when an iterative decoding is done. EXIT chart measures the Mutual Information (MI) that is exchanged between the constituent decoder in a iterative process.

2 System Model

The schematic for the coded modulation is illustrated in the Figure 1. The source here will be producing some random information bits, which is then encoded by the one of the respective encoders and consecutively interleaved by random-interleavers . The interleaved bits/symbols are then modulated according to symbol rule for each of corresponding modulation schemes.The channel discussed here for the coded modulation schemes are that of the

AWGN and Rayleigh distributed flat fading.

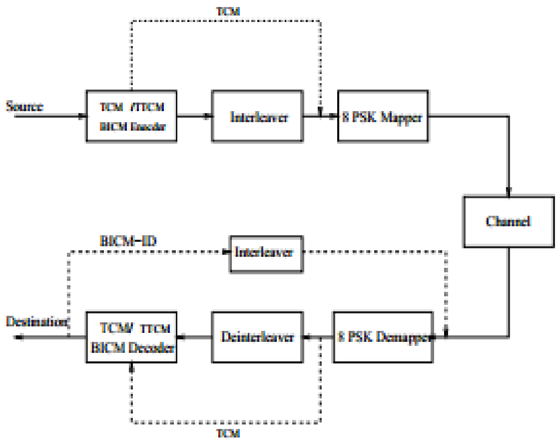

Figure 1: System Overview for different coded Modulation Schemes [9]

The relationship between the AWGN and Rayleigh fading channel can be expressed as:

$$y_t = \alpha_t x_t + n_t \quad (1)$$

where x_t is the transmitted discrete signal and y_t is received signal. α_t is the Rayleigh-distrubuted fading having an expected squared value of $E(\alpha_t^2)$=1 and n_t is the complex AWGN having a noise variance of $N_0/2$ per dimension. For an AWGN channel $\alpha_t = 1$. The receiver side consist of demodulator/demaper followed by a de-interleaver and will be having one of the TCM, TTCM or BICM decoders which explained in the previous chapter. When considering the BICM-ID schemes the decoder output is is interleaved and then feedback to the demmaper input as illustrated in the Figure 1.

The study of coded modulation schemes are with Ungerbock's TCM, Roberson's TTCM, Zehavi's BICM and Li's BICM-ID. To make this little more elaborate the following generator polynomials are used to study and stimulate the performance of the TCM/ TTCM schemes are given in the upper part Table 1. These are RSC codes which will be adding one parity bits to information bits. Hence, the corresponding coding rate for a 2^{m+1} ary PSK is $R = \frac{m}{m+1}$. The number of decoding staged associated with the constraint length k is 2^{K-1}.

Lower part of the Table 1 shows the respective generator polynomial for the BICM/BICM-ID schemes. As noted previously they are non-systematic codes having maximum hamming distance. Here, as TCM/TTCM only one parity bit is added to the information bits and hence, the coding rate and bandwidth efficiency remains the same as that of the TCM/TTCM.

Decoders utilises the Log based- Maximum A Posteriori (Log-MAP) [11] algorithm for its soft decision. Log-MAP algorithm was chooses since, they are the numerical stable version and to reduce the complexity as well as

Scheme	State	g^1	g^2	g^3
TCM/TTCM	8	11	02	04
	16	23	04	16
	32	45	16	34
	64	103	30	66
BICM/BICM-ID	8	04	02	06
		01	04	0 7
	16	07	01	04
		02	05	07
	32	14	06	16
		03	10	17
	64	15	06	15
		06	15	17

Table 1: Generator polynomial for Ungerbock TCM coded with best maximum distance [1] as well as the Paaske's [10] generator polynomials for BICM codes. Both the codes are expressed in octal representation form.

the numerical problem.

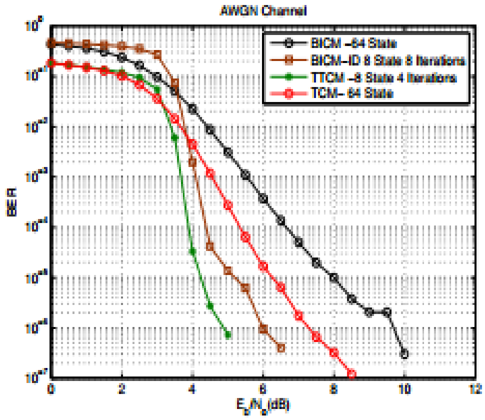

Figure 2: BER performance of Coded modulation employing 8 PSK and using a Frame length 2000 information bits

3 Results and Discussion

This section deals with the BER characteristic of TCM,TTCM,BICM and BICM-ID under AWGN and uncorrelated Rayleigh Channel. In this section complexity of the coded modulation is compared with overall coding gain obtained from the uncoded 4 -PSK modulation scheme. The complexity is compared in terms of the number of decoding stages as well as the number of decoding iteration. For TCM or BICM code the complexity is equal to the number of decoding stages namely to $S = 2^{K-1}$ where K is the **constrain length**.The TTCM's overall complexity is proportional to $2 \cdot S \cdot t$ where t is the number of iterations, here the overall complexity is doubled since, TTCM invoke two TCM codes. As in the case of the BICM-ID the decoder uses only one decoder but during every iterations demodulator /de-mapper is invoked .However, the overall complexity of de mapper is insignificant when compared to overall complexity of decoder used. Hence, the complexity of BICM-ID with t iteration and using S state component is proportional to $t \cdot S$.

3.1 Performance Over AWGN Channel

Figure 2 demonstrates the BER characteristic of coded modulation under AWGN channel while trying to maintain the overall decoding complexity of all the coded modulation the same in terms of the number of trellis states. The simulation uses 64-State TCM, 64 State BICM, 8 State TTCM with four iterations and 8 State BICM-ID along with 8 Iteration with a overall frame length of 2000 information bits. At a BER 10^{-4} TTCM outperforms all other schemes while maintaing same complexity, TTCM achieves 1 dB better performance than BICM-ID, 1.5 dB better than TCM and BICM performance degrades with almost 1.8 dB lower E_b/N_0.

3.2 Performance over Uncorrelated Narrowband Rayleigh Fading Channels

The uncorrelated Rayleigh fading channels means that, using an infinite-length interleaver over narrowband Rayleigh Fading channels. Figure 3 shows the performance of 8 State TTCM using four iterations,16-state BICM-ID employing four iterations 64-state TCM and 64 state BICM,. The states where selected in a such a way that all the coded modulation did have similar complexity, As it can be seen from the Figure 3 TTCM performs best, followed by BICM-ID, BICM and TCM. At BER of 10^{-4} TTCM performs about 1.8 dB better in terms of the required E_b/N_o value than BICM-ID, 2.7 dB better than BICM and 5 db better than TCM. As it is evident from the Figure 3, the Error flow of TTCM was lower than the associated Error Free Feedback(EFF) bound of the BICM-ID. One the other hand, the BERs of TTCM and BICM-ID was almost identical at E_b/N_o =7 dB

3.3 Effect of Block Length on Coded Modulation

This section compares the interleaving effect of block length on the coded Modulation. It is much evident form the Figure4 that high interleaving block is appropriate for the iterative TTCM and BICM-ID schemes. Here BICM-ID used for the simulations are 16 state with 4 iteration while TTCM uses 8 state with 4 iterations. At a BER of 10^{-4} the 300 bits/Frame performed worst with almost 1 dB degradation when compared over the 1500 bits/Frame. When 3000 frame length was used there was only slight improvement in its SNR performance. in short, BICM-ID

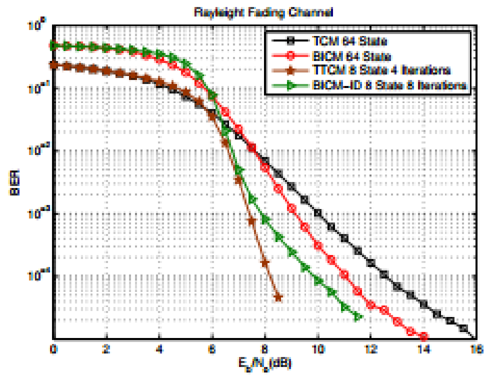

Figure 3: BER performance of Coded modulation employing 8 PSK and using a Frame length 2000 information bits

scheme has more advantage when use it is used over a larger frame length. The TTCM performance improves when the frame length is increased and on the whole, here TTCM exhibit best performance.

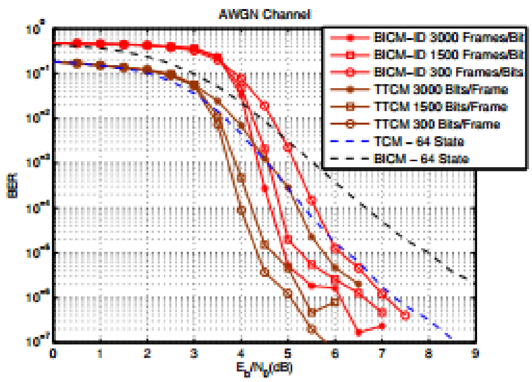

Figure 4: Effects of block length on Coded Modulation

3.4 Study using EXIT chart anaylsis

Firstly, we will be studying the EXIT band chart characteristic [12] of BICM-ID schemes. Since, BICM-ID is a serial concatenated code having the outer code will be of a 2/3 rate convolutional encoder of code memory equal to three and the inner code is of a De-mapper.

Figure 5 shows the effect of the Frame length on the performance of the code over both AWGN and Rayleigh Channel. It can be seen from the figure that performance of the BICM-ID code is very remarkable for a system

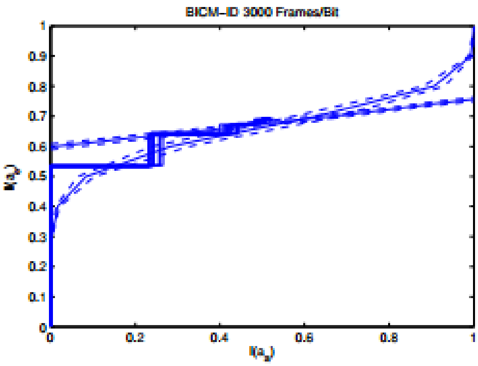

Figure 5: EXIT band Chart trajectories for BICM-ID at a SNR =4.5 dB

with the longer frame length. However, even for short frame length it can give good results but suffers a overall degradation of about 2 db at a BER of 10^{-4} for both AWGN and narrow band flat fading channels. Note that larger block length can lead to the earlier convergence, in term of low SNR and number of iterations used but, at higher SNR values the effect of the frame length is insignificant.

Now, we can discusses in detailed about the TTCM EXIT band chart characteristics, the TTCM is nothing but a parallel concatenated code of two TCM decoder. The coding rate is of 2/3 and coding memory of three is used here.

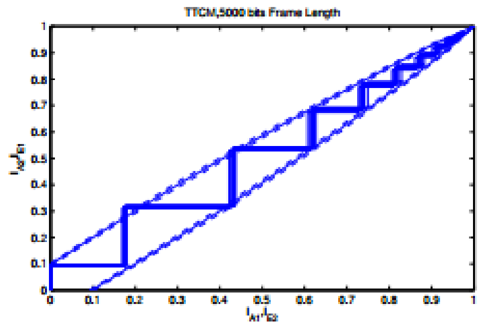

Figure 6: EXIT band chart trajectories for TTCM at SNR=2.5 dB

EXIT band chart trajectories [12] of TTCM decoders for the frame length of 5000 bits are used here for simulations. This actually shows how the mutual information are exchanged between each other until they converges. From these figures we can see band graph are different for different frame lengths and also trajectories show the different channel realisation at the same SNR value its self. The different channel realisation trajectories will fit the EXIT band chart quite well as seen from the Figures. The *threshold* E_b/N_0 for the TTCM is 2.5 db and

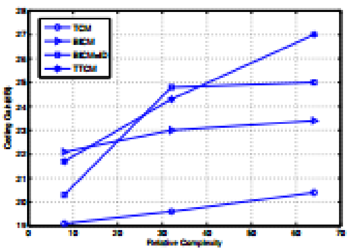

Figure 7: Coding gain at a BER of 10^{-4} decoding complexity when compared to the uncoded QPSK under Rayleigh channel

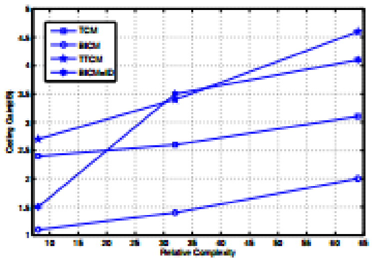

Figure 8: Coding gain at a BER of 10^{-4} against the decoding complexity when compared to the uncoded QPSK under AWGN channel

from the EXIT chart given in the Figure 6 the EXIT chart tunnel is opened and the decoder can exchange mutual information smoothly. This can gives the confidence that the simulations are correct.

3.5 Coding Gain Vs Decoding Complexity

This section investigate the coding gain of the coded modulation schemes utilising an 8 PSK scheme versus the complexity at a BER of 10^{-4}. The coding gain is measured by comparing to that of the uncoded QPSK scheme which exhibit a BER of 10^{-4} =8.5 dB and 34.8 dB for transmission over the AWGN and uncorrelated Rayleigh channel respectively. The overall complexity of TCM, BICM ,BICM-ID,TTCM is given by the S-State component and number of iteration respectively.

Figure 8 and Figure 7 repersents the coding gain versus the Decoding complex for the 8 PSK transmission over AWGN. At a Low Decoding complexity as of 8, the non-iterative decoding TCM exhibits the highest decoding gain for AWGN channel as it can be seen. By contrast, the BICM purposed will be performing much better in uncorrelated Rayleigh channels.

For iterative decoding schemes such as BICM-ID and TTCM different combination of t and S yields different performance at the same Decoding complexity. The coding gain of BICM-ID which uses $t.S = 8 * 8$ is always better than that of $t.S = 4 * 16$ since, the states which uses S=16 and t=4 cannot reach its ideal performance at t=4. In general, the coding gain of TTCM is the highest for the both the channels.

3.6 Area/Capacity vs SNR

The area/capacity plots are obtained by generating the area beneath the EXIT function for the inner codes. Here, the maximum capacity it can achieve is three because we are taking the reference signal as 8 PSK. The Figure 9

gives the comparison of the DCMC channel capacity with the TCM/TTCM and BICM/BICM-ID. Its evident from the figure BICM/BICMID can achieve a almost equal to 3 bits/symbol which is the desired DCMC capacity it can reach at an SNR equivalent to 12 dB, while TTCM suffer a loss and can only achieve a capacity of 2 bits/symbol.

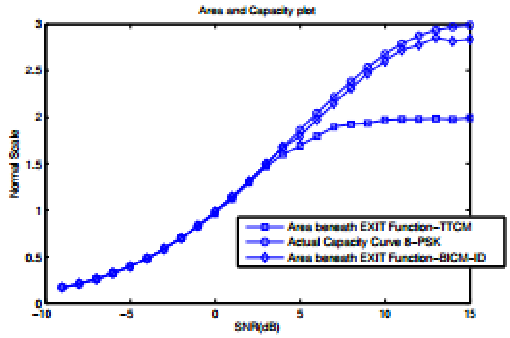

Figure 9: Area beneath EXIT function and DCMC Capacity plots in AWGN channel

4 Conclusion

This paper studies comparative study of the four coded modulation for transmission over the AWGN and uncorrelated Raleigh channel. Each of the four coded modulation was studied separately in term of their interleaving frame length, coded memory elements and with the aid of the EXIT charts. The following conclusion were obtained from the simulation that was carried out, for a given complexity TCM performed better than BICM in AWGN channel while the TCM's performance was worse than BICM in uncorrelated channel. However, when considering the iterative coding schemes BICM-ID almost outperformed both the TCM and BICM over the both the channel maintaining the same complexity. TTCM was the clear winner and has almost shown its dominance in its comparative study. Both the BICM/ID and TTCM had its share of advantage and disadvantages, as BICM/BICM-ID matched the theoretical channel capacity while TTCM suffer a loss in channel capacity. TTCM EXIT tunnel function found to be opened at a much lower SNR than BICM/ID.

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