Common Random Fixed Point of Asymptotically Regular Mappings in Polish Spaces

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ABSTRACT

The purpose of this paper to establish a random fixed point theorem for three random multivalued operators satisfying general contractive condition for asymptotically regular mappings in Polish spaces.

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Keywords -- Random Multivalued Operator, Asymptotically Regular, Random Fixed Point, Measurable Mapping

I. INTRODUCTION

Random fixed point theorem for contraction mappings in Polish spaces are of fundamental importance in Probabilistic functional analysis. Their study was initiated by the Prague School of Probabilistic with work of Spacek [16] and Hans [10, 11]. Bharucha-Reid [9], Itoh [12] proved several random fixed point theorems and gave their applications to random differential equations in Banach spaces. Lin[13],Sehgal and Singh[15],proved random approximation and random fixed point theorem for non self map and set valued mappings. Papageorgiou [14], proved random fixed point theorems for measurable multifunction in Banach spaces. Beg and Azam [6] prove some results on fixed points of asymptotically regular multivalued mappings. Random coincidence point theorems and random fixed point theorems are stochastic generalization of classical coincidence point theorems and classical fixed point theorems. Beg and Shahzad [7], Beg and Abbas [5] studied the structure of common random fixed points and random coincidence points of compatible random operators. Beg, Abbas and Azam [8] obtained sufficient conditions for existence of random fixed point of a non expansive rotative random operator and establish the existence of random periodic points for random single valued ε-contractive and ε-expansive random operators. Badshah and Sayyed [3], Badshah and Gagrani[1], proved some random fixed point theorems for random multivalued operators in Polish spaces. Afterwards, Badshah and Shrivastava [4] introduce the concept of semi-compatibility in Polish spaces and proved some random fixed point theorems for random multivalued operators in Polish spaces. Recently, Badshah and Pariya[2] establish a common random fixed point theorem for six random operators using the concept of semi compatibility , weak compatibility and commutativity of random operators in Polish spaces.

II. PRELIMINARIES

Let (X,d) be a Polish Space, that is a separable complete metric space, and let (Ω, Σ) be a measurable space. Let 2^X be a family of all subset of X and CB(X) denote the family of all non-empty bounded closed subsets of X. A mapping $T: \Omega \to 2^X$ is called measurable if for all open subset C of X. $T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \emptyset\} \in \Sigma$. A mapping $\xi : \Omega \to X$ is said to be measurable selector of a measurable mapping $T: \Omega \to 2^X$ if ξ is measurable and $\xi(\omega) \in T(\omega)$ for all $\omega \in \Omega$.

Definition 2.1. A mapping $f : \Omega \times X \to X$ is called a random operator if for all $x \in X$, f(., x) is measurable.

Definition 2.2. A mapping $T : \Omega \times X \to CB(X)$ is called a random multivalued operator if for every $x \in X$, T(. , x) is measurable.

Definition 2.3.A measurable mapping $\xi : \Omega \to X$ is called random fixed point of a random multivalued operator $T : \Omega \times X \to CB(X)$ $(f : \Omega \times X \to X)$, if for every $\omega \in \Omega$, $\xi(\omega) \in T(\omega, \xi(\omega))$ $(f(\omega, \xi(\omega)) = \xi(\omega))$.

Definition 2.4. Let $T : \Omega \times X \to CB(X)$ be a random operator and $\{\xi_n\}$ a sequence of measurable mappings $\xi_n : \Omega \to X$. The sequence $\{\xi_n\}$ is said to be asymptotically T-regular if $d\left(\xi_n(\omega), T\left(\omega, \xi_n(\omega)\right)\right) \to 0$.

In this paper, the existence of a common random fixed point of three random multivalued operators with asymptotically regular mappings on Polish spaces proved. The result extends and generalized the result of Beg and Azam [6], Badshah and Sayyed [3] and previously known results.

III. MAIN RESULT

Theorem 3.1. Let X be a Polish space and $A, B, S: \Omega \times X \to CB(X)$ be three continuous random multivalued operators satisfying

(i) $B(\omega, X) \subset A(\omega, X)$ and $S(\omega, X) \subset A(\omega, X)$

(ii) $H(B(\omega, x), S(\omega, y)) \leq \alpha(\omega) \frac{[d(A(\omega,y), S(\omega,y))]^2}{d(A(\omega,x), B(\omega,x)) + d(A(\omega,x), S(\omega,y))} + \beta(\omega) d(A(\omega, x), A(\omega, y))$

where $\alpha, \beta: \Omega \to (0,1)$ are measurable mappings such that $\alpha(\omega) + \beta(\omega) < 1$ for

$x, y \in X$ and each $\omega \in \Omega$. If $\xi_n : \Omega \to X$ is sequence of asymptotically A-regular,B-regular and S-regular measurable mappings then there exists measurable mapping $\xi: \Omega \to X$ is random fixed point of random multivalued operators A, B,and S such that

$\xi(\omega) \in A(\omega, \xi(\omega))$, $\xi(\omega) \in B(\omega, \xi(\omega))$ and $\xi(\omega) \in S(\omega, \xi(\omega))$ for each $\omega \in \Omega$.

Moreover $A(\omega, \xi_n(\omega)) \to A(\omega, \xi(\omega))$, $B(\omega, \xi_n(\omega)) \to B(\omega, \xi(\omega))$ and $S(\omega, \xi_n(\omega)) \to S(\omega, \xi(\omega))$

(Here H represent the Hausdroff metric on CB(X) induced by the metric d.)

Proof. Let $\xi_0: \Omega \to X$ be an arbitrary measurable mapping and choose measurable mapping $\xi_1, \xi_2: \Omega \to X$ such that for each $\omega \in \Omega$

$B(\omega,\xi_0(\omega)) = A(\omega,\xi_1(\omega))$ and $S(\omega,\xi_1(\omega)) = A(\omega,\xi_2(\omega))$

Then for each $\omega \in \Omega$

$$d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right) = H\left(B(\omega,\xi_0(\omega)),S(\omega,\xi_1(\omega))\right)$$

$$\leq \alpha(\omega)\frac{\left[d\left(A(\omega,\xi_1(\omega)),S(\omega,\xi_1(\omega))\right)\right]^2}{d\left(A(\omega,\xi_0(\omega)),B(\omega,\xi_0(\omega))\right)+d\left(A(\omega,\xi_0(\omega)),S(\omega,\xi_1(\omega))\right)} + \beta(\omega)d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)$$

$$= \alpha(\omega)\frac{\left[d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right)\right]^2}{d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)+d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_2(\omega))\right)} + \beta(\omega)d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)$$

$$\leq \alpha(\omega)\frac{\left[d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right)\right]^2}{d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right)} + \beta(\omega)d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)$$

$$\leq \alpha(\omega)d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right) + \beta(\omega)d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)$$

$$\Rightarrow (1-\alpha(\omega))\, d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right) \leq \beta(\omega)d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)$$

$$\Rightarrow d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right) \leq \frac{\beta(\omega)}{1-\alpha(\omega)}d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right)$$

$$\Rightarrow d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right) \leq k\, d\left(A(\omega,\xi_0(\omega)),A(\omega,\xi_1(\omega))\right) \quad ..(1)$$

Where $k = \frac{\beta(\omega)}{1-\alpha(\omega)} < 1$

In the same manner there exists a measurable mapping $\xi_3 : \Omega \to X$ such that

$$d\left(A(\omega,\xi_2(\omega)),A(\omega,\xi_3(\omega))\right) = d\left(B(\omega,\xi_1(\omega)),S(\omega,\xi_2(\omega))\right)$$

$$\leq \alpha(\omega)\frac{\left[d\left(A(\omega,\xi_2(\omega)),S(\omega,\xi_2(\omega))\right)\right]^2}{d\left(A(\omega,\xi_1(\omega)),B(\omega,\xi_1(\omega))\right)+d\left(A(\omega,\xi_1(\omega)),S(\omega,\xi_2(\omega))\right)} + \beta(\omega)d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right)$$

$$\Rightarrow d\left(A(\omega,\xi_2(\omega)),A(\omega,\xi_3(\omega))\right) \leq \frac{\beta(\omega)}{1-\alpha(\omega)}d\left(A(\omega,\xi_1(\omega)),A(\omega,\xi_2(\omega))\right)$$

$$\Rightarrow d\left(A\left(\omega,\xi_2(\omega)\right),A\left(\omega,\xi_3(\omega)\right)\right) \le k\, d\left(A\left(\omega,\xi_1(\omega)\right),A\left(\omega,\xi_2(\omega)\right)\right)$$

$$\le k^2\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right) \qquad \text{.. by (1)}$$

Similarly proceeding in the same way by induction, we produce a sequence of measurable mapping $\xi_n : \Omega \to X$ such that

$$d\left(A\left(\omega,\xi_n(\omega)\right),A\left(\omega,\xi_{n+1}(\omega)\right)\right) \le k^n\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right)$$

furthermore for $m > n$

$$d\left(A\left(\omega,\xi_n(\omega)\right),A\left(\omega,\xi_m(\omega)\right)\right)$$

$$\le d\left(A\left(\omega,\xi_n(\omega)\right),A\left(\omega,\xi_{n+1}(\omega)\right)\right) + d\left(A\left(\omega,\xi_{n+1}(\omega)\right),A\left(\omega,\xi_{n+2}(\omega)\right)\right) + \dots\dots + d\left(A\left(\omega,\xi_{m-1}(\omega)\right),A\left(\omega,\xi_m(\omega)\right)\right)$$

$$\le k^n\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right) + k^{n+1}\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right) + \dots\dots$$

$$+k^{m-1}\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right)$$

$$= [k^n + k^{n+1} + \dots\dots + k^{m-1}]\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right)$$

$$= k^n[1 + k + k^2 + \dots\dots + k^{m-n-1}]\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right)$$

$$\le \frac{k^n}{1-k}\, d\left(A\left(\omega,\xi_0(\omega)\right),A\left(\omega,\xi_1(\omega)\right)\right)$$

which tends to zero as $n \to \infty$. It follows that $\{A\left(\omega,\xi_n(\omega)\right)\}$ is Cauchy sequence in CB(X) so $\{B\left(\omega,\xi_n(\omega)\right)\}$ and $\{S\left(\omega,\xi_n(\omega)\right)\}$ also convergence in CB(X) and hence exists a measurable mapping $\xi : \Omega \to X$ such that $A\left(\omega,\xi_n(\omega)\right) \to \xi(\omega)$, $B\left(\omega,\xi_n(\omega)\right) \to \xi(\omega)$ and $S\left(\omega,\xi_n(\omega)\right) \to \xi(\omega)$ therefore there exits $A(\omega) \in CB(X)$ such that $H\left(A\left(\omega,\xi_n(\omega)\right),A(\omega)\right) \to 0$. (By Itoh proposition(1) $A : \Omega \to CB(X)$ is measurable).

Let $\xi : \Omega \to X$ be a measurable mapping such that for each $\omega \in \Omega$, $\xi(\omega) \in A(\omega)$.

Thus for any $\omega \in \Omega$ and by A, B and S asymptotically regularity of sequences $\xi_n(\omega)$.

We have

$$d\left(\xi(\omega),A(\omega,\xi(\omega))\right) \le H\left(A(\omega),A(\omega,\xi(\omega))\right)$$

$$= \lim_{n\to\infty} H\left(A\left(\omega,\xi_n(\omega)\right),A(\omega,\xi(\omega))\right)$$

$$= \lim_{n\to\infty} H\left(B\left(\omega,\xi_n(\omega)\right),S(\omega,\xi(\omega))\right)$$

$$\leq \lim_{n\to\infty}\left[\alpha(\omega)\frac{\left[d\left(A(\omega,\xi(\omega)),S(\omega,\xi(\omega))\right)\right]^2}{d\left(A\left(\omega,\xi_n(\omega)\right),B\left(\omega,\xi_n(\omega)\right)\right)+d\left(A\left(\omega,\xi_n(\omega)\right),S(\omega,\xi(\omega))\right)}\right.$$
$$\left.+\beta(\omega)d\left(A\left(\omega,\xi_n(\omega)\right),A(\omega,\xi(\omega))\right)\right]$$

Proceeding in the same way, we obtain

$\Rightarrow d\left(\xi(\omega),A(\omega,\xi(\omega))\right)\leq 0$ for each $\omega\in\Omega$.

Therefore for all $\omega\in\Omega$, $d\left(\xi(\omega),A(\omega,\xi(\omega))\right)=0$

Hence $\xi(\omega)\in A(\omega,\xi(\omega))$.

Similarly

$$d\left(\xi(\omega),B(\omega,\xi(\omega))\right)\leq H\left(A(\omega),B(\omega,\xi(\omega))\right)$$
$$=\lim_{n\to\infty}H\left(A\left(\omega,\xi_n(\omega)\right),B(\omega,\xi(\omega))\right)$$
$$=\lim_{n\to\infty}H\left(S\left(\omega,\xi_n(\omega)\right),B(\omega,\xi(\omega))\right)$$
$$=\lim_{n\to\infty}H\left(B(\omega,\xi(\omega)),S\left(\omega,\xi_n(\omega)\right)\right)$$

$$\leq \lim_{n\to\infty}\left[\alpha(\omega)\frac{\left[d\left(A\left(\omega,\xi_n(\omega)\right),S\left(\omega,\xi_n(\omega)\right)\right)\right]^2}{d\left(A(\omega,\xi(\omega)),B(\omega,\xi(\omega))\right)+d\left(A(\omega,\xi(\omega)),S\left(\omega,\xi_n(\omega)\right)\right)}\right.$$
$$\left.+\beta(\omega)d\left(A(\omega,\xi(\omega)),A\left(\omega,\xi_n(\omega)\right)\right)\right]$$

i.e.
$d\left(\xi(\omega),B(\omega,\xi(\omega))\right)\leq 0.$

$\Rightarrow \xi(\omega)\in B(\omega,\xi(\omega))$, for each $\omega\in\Omega$.

Similarly

$$d\left(\xi(\omega),S(\omega,\xi(\omega))\right)\leq H\left(A(\omega),S(\omega,\xi(\omega))\right)$$
$$=\lim_{n\to\infty}H\left(A\left(\omega,\xi_n(\omega)\right),S(\omega,\xi(\omega))\right)$$

Hence

$\xi(\omega)\in S(\omega,\xi(\omega))$, for each $\omega\in\Omega$.

Also it's follows that

$A(\omega,\xi(\omega)) = A(\omega) = \lim_{n\to\infty} A(\omega,\xi_n(\omega))$, for each $\omega \in \Omega$,

$S(\omega,\xi(\omega)) = A(\omega) = \lim_{n\to\infty} S(\omega,\xi_n(\omega))$, for each $\omega \in \Omega$ and

$B(\omega,\xi(\omega)) = A(\omega) = \lim_{n\to\infty} B(\omega,\xi_n(\omega))$, for each $\omega \in \Omega$.

This is complete proof of theorem.

Theorem 3.2. Let X be a Polish space and $A, B, S: \Omega \times X \to CB(X)$ be three continuous random multivalued operators satisfying

(i) $B(\omega, X) \subset A(\omega, X)$ and $S(\omega, X) \subset A(\omega, X)$

(ii) $H(B(\omega,x), S(\omega,y)) \le \alpha(\omega) \frac{[d(A(\omega,y),S(\omega,y))]^2}{d(A(\omega,x),B(\omega,x))+d(A(\omega,x),S(\omega,y))} + \beta(\omega) d(A(\omega,x), A(\omega,y))$

Where $\alpha, \beta: \Omega \to (0,1)$ are measurable mappings such that $\alpha(\omega) + \beta(\omega) < 1$ for $x, y \in X$ and each $\omega \in \Omega$.

then there exists a common random fixed point of random multivalued operators A, B and S.

(Here H represent the Hausdroff metric on CB(X) induced by the metric d.)

Proof. The proof of theorem follows from theorem 3.1.

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