

# CHANNEL EQUALIZATION OF ADAPTIVE FILTERS USING LMS AND RLS ALGORITHMS

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## ABSTRACT

*The recent digital transmission systems impose the application of channel equalizers with short training time and high tracking rate. Equalization techniques compensate for the time dispersion introduced by communication channels and combat the resulting inter-symbol interference (ISI) effect. Given a channel of unknown impulse response, the purpose of an adaptive equalizer is to operate on the channel output such that the cascade connection of the channel and the equalizer provides an approximation to an ideal transmission medium. Typically, adaptive equalizers used in digital communications require an initial training period, during which a known data sequence is transmitted. This paper describes the adaptive equalization problem for Rayleigh frequency selective fading (FSF) channel model and analyses the performance improvement with FIR and IIR adaptive equalization in Rayleigh FSF channel. We use linear equalization with both LMS (least mean squares) and RLS (Recursive Least Squares) algorithms to compare the different methods. In the receiver part, we have received distorted image exposed to random noise, block noise, and ISI. The adaptive equalization removes the noisy, distorted etc. effects of the wireless channel and allows subsequent symbol demodulation. In this paper three types of PSK modulation (BPSK, QPSK and 8PSK) have been applied to system to compare BER&SER (Bit Error Rate & Symbol Error Rate) performance of our model.*

**Keywords—FSF, LMS, RLS, ISI, BPSK, QPSK, 8PSK.**

## I. INTRODUCTION

In the digital signal processing the major problem occurs while designing the filter, at the receiver processing in order to transmit enormous amount of data within the filter band. Tighter filter parameters are the need of the day. One of the most important advantages of the digital transmission systems for voice, data and video communications is their higher reliability in noise environment in comparison with that of their analog counterparts. Unfortunately most often the digital transmission of information is accompanied with a phenomenon known as intersymbol interference (ISI) [1]. This work represents the performance analysis and comparison between the LMS and RLS Adaptive FIR filter. Interest in adaptive filters continues to grow as they begin to find practical real-time applications in areas such as channel equalization, echo cancellation, noise cancellation and many other adaptive signal processing applications. On the contrary in the case of the Adaptive Filters, they are implemented where ever there is the need for the digital filter's characteristics to be variable,

adapting to changing signal. Adaptive filtering consists of two basic operations; the filtering process which generates an output signal from an input data signal, and the adaptation process which adjusts the filter coefficients in a way to minimize a desired cost function. Basically, there are a large number of filter structures and algorithms that have been used in adaptive filtering applications.

Adaptive filters are an important part of signal processing. Adaptive filter is a nonlinear filter since its characteristics are dependent on the input signal and consequently the homogeneity and additivity conditions are not satisfied [2]. The key to successful adaptive signal processing understands the fundamental properties of adaptive algorithms such as LMS, RLS etc. Application of adaptive filter is the cancellation of the noise component, an undesired signal in the same frequency range.

## II. CHANNEL EQUALIZATION

As mentioned in the introduction the intersymbol interference imposes the main obstacles to achieving increased digital transmission rates with the required accuracy. ISI problem is resolved by channel equalization [3] in which the aim is to construct an equalizer. In this paper, we will work on the problem of adaptive channel equalization. Adaptive equalizers are necessary for reliable communication of digital data across non-ideal channels. The basic operation of a digital communication system may be described as follows. Let  $d(n)$  be a digital sequence that is to be transmitted across a channel, with  $d(n)$  having values of plus or minus one. This sequence is input to a pulse generator, which produces a pulse of amplitude  $A$  at time  $n$  if  $d(n) = 1$  and a pulse of amplitude  $-A$  if  $d(n) = -1$ . This sequence of pulses is then modulated and transmitted across a channel to a remote receiver. The receiver demodulates and samples the received waveform, which produces a discrete-time sequence  $x(n)$ . Although ideally  $x(n)$  will be equal to  $d(n)$  in practice this will rarely be the case. There are two reasons for this.

a. Since the channel is never ideal, it will introduce some distortion. One common type of distortion is *channel dispersion* that is the result of the nonlinear phase characteristics of the channel. This dispersion causes a distortion of the pulse shape, thereby causing neighboring pulses to interfere with each other, resulting in an effect known as *intersymbol interference*.

b. The second reason is that the received waveform will invariably contain noise. This noise may be introduced by the channel or it may be the result of nonideal elements in the transmitter and receiver.

Assuming a linear dispersive channel, a model for the received sequence  $x(n)$  is;

$$x(n) = \sum_{k=-\infty}^n d(k)h(n-k) + v(n) \quad 1$$

where  $h(n)$  is the unit sample response of the channel and  $v(n)$  is additive noise. Given the received sequence  $x(n)$ . The receiver then makes a decision as to whether a plus one or a minus one was transmitted at time  $n$ . This decision is typically made with a simple threshold device, i.e.,

$$\hat{d}(n) = \begin{cases} 1 & : x(n) \geq 0 \\ -1 & : x(n) < 0 \end{cases} \quad 2$$

In order to reduce the error, the receiver will often employ an equalizer in order to reduce the effects of channel distortion. Since the precise characteristics of the channel are unknown, and possibly time-varying, the equalizer is typically an adaptive filter. One of the challenges in the design of an adaptive filter is generating the desired signal  $d(n)$ , which is required in order to compute the error  $e(n)$ .

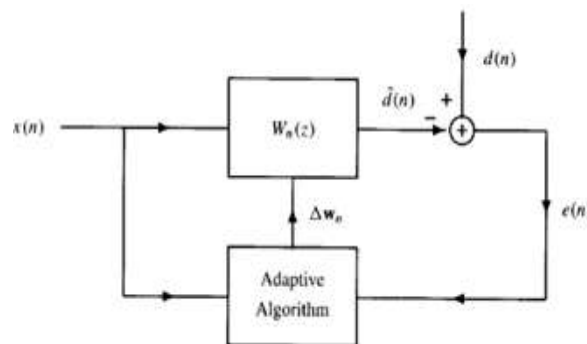
There are some most common Adaptive filters which can be used for equalizations with noticeable performance based on particular application such as LMS (Least Mean Square), RLS (Recursive Least Square) [4] and Kalman filter. The least-mean-square (LMS) algorithm [5] is a member of the family of stochastic gradient algorithm and it is a linear adaptive algorithm [6] that consists of two basic processes filtering process and an adaptive process.

### III. ADAPTIVE FILTERS

Because the signals in almost every application will be nonstationary, the approaches and techniques that we have been considering thus far would be appropriate. Processing nonstationary processes in blocks, over approximately stationary intervals is limited in its effectiveness for several reasons. For rapidly varying processes, the interval over which a process may be assumed to be stationary may be too small to allow for sufficient accuracy or resolution in the estimation of the relevant parameters. This approach would not easily accommodate step changes within the analysis intervals. This solution imposes an incorrect model on the data, i.e., piecewise stationary. So, a better approach is a nonstationarity assumption at the outset. In new model least squares error in place of mean-square error. In a stationary environment, the adaptive filter should produce a sequence of corrections  $\Delta w$ , in such a way that  $w_n$  converges to the solution to the Wiener-Hopf equations. It should not be necessary to know the signal statistics  $r_x(k)$  and  $r_{dx}(k)$  in order to compute  $\Delta w$ . The estimation of these statistics should be "built into" the adaptive filter. For nonstationary signals, the filter should be able to adapt to the changing statistics and "track" the solution as it evolves in time.

The Least Squares Error is;

$$\varepsilon(n) = \sum_{i=0}^n |e(i)|^2 \quad 3$$



**Figure 1.** Block diagram of an adaptive filter consisting of a shift- varying filter  $W_n(z)$  and an adaptive algorithm for updating the filter coefficients  $w_n(k)$ .

$x(n) \rightarrow$ obtained signal       $d(n) \rightarrow$ desired signal       $v(n) \rightarrow$  noise

$$x(n) = d(n) + v(n) \quad 4$$

$$E\{e^2(n)\} = E\{v^2(n)\} + E\{[d(n) - y(n)]^2\} \quad 5$$

Minimizing  $E\{e^2(n)\}$  is equivalent to minimizing  $E\{[d(n) - y(n)]^2\}$ , the mean- square error between  $d(n)$  and the output of the adaptive filter,  $y(n)$ . Thus, the output of the adaptive filter is the minimum mean-square estimate of  $d(n)$ .

We will exactly focus on the FIR (LMS) and IIR (RLS) adaptive filters. The efficiency of the adaptive filter and its performance in estimating  $d(n)$  will depend on a number of factors including the type of filter (FIR or IIR), the filter structure (direct form, parallel, lattic, etc.), and the way in which the performance measure is defined

(mean-square error, least squares error). Firstly we will introduce FIR adaptive filters, based on the method of steepest descent. Of primary interest will be the LMS adaptive filter,. Secondly, we will focused on the IIR Recursive Least Squares (RLS) algorithm. There is a wide variety of applications in which adaptive filters have been successfully used such as linear prediction, echo cancellation, channel equalization, interference cancellation, adaptive notch filtering, adaptive control, system identification, and array processing. We will use LMS and RLS for channel equalization as application.

### 3.1 LMS Algorithm

Mathematical determination of LMS Algorithm. Filter coefficients  $w_n(k)$  and weight-vector update equation of the steepest descent adaptive filter

$$w_{n+1} = w_n + \mu E\{e(n)x^*(n)\} \quad 6$$

A practical limitation with this algorithm is that the expectation  $E\{e(n)x^*(n)\}$  is generally unknown. Therefore, it must be replaced with an estimate such as the sample mean

$$\hat{E}\{e(n)x^*(n)\} = \frac{1}{L} \sum_{l=0}^{L-1} e(n-l)x^*(n-l) \quad 7$$

Finally, weight-vector update equation;

$$w_{n+1} = w_n + \mu e(n)x^*(n) \quad 8$$

$\mu$  is step size parameter. Then the  $k$ th coefficient is;

$$w_{n+1}(k) = w_n(k) + \mu e(n)x^*(n-k) \quad 9$$

Therefore, an LMS adaptive filter having  $p+1$  coefficients requires  $p+1$  multiplications and  $(p+1)$  additions to update the filter coefficients. In addition, one addition is necessary to compute the error  $e(n) = d(n) - y(n)$  and one multiplication is needed to form the product  $pe(n)$ . Finally,  $p+1$  multiplications and  $p$  additions are necessary to calculate the output,  $y(n)$ , of the adaptive filter. Thus, a total of  $2p+3$  multiplications and  $2p+2$  additions per output point are required.

#### 3.1.1 Convergence of the LMS Algorithm

$$\lim_{n \rightarrow \infty} E\{w_n\} = w = R_x^{-1} r_{dx} \quad 10$$

$$E\{w_{n+1}\} = E\{w_n\} + \mu E\{d(n)x^*(n)\} - \mu E\{x^*(n)x^T(n)w_n\} \quad 11$$

$$0 < \mu < \frac{2}{(p+1)E\{|x(n)|^2\}} \quad 12$$

### 3.2 Recursive Least Squares

We may have different solutions, even if the statistics of the sequences are the same. We will look at the filters that are derived by minimizing a weighted least squares error, and derive an efficient algorithm for performing this minimization known as recursive least squares. There are three different RLS algorithm we can be used. These are the growing window RLS algorithm, exponentially weighted RLS algorithm and the sliding window RLS algorithm. Although RLS is computationally more complex than the LMS algorithm, for WSS processes the growing window RLS algorithm converges much more rapidly. However, for nonstationary process, exponentially weighted RLS algorithm or the sliding window RLS algorithm need to be used. We'll use the exponentially weighted RLS algorithm for our application.

### 3.2.1 Exponentially Weighted RLS

FIR adaptive Wiener filter and find the filter coefficients;

$$w_n = [w_n(0), w_n(1), \dots, w_n(P)]^T \quad 13$$

Minimizing, at time  $n$ , the weighted least squares error;

$$e(n) = \sum_{i=0}^n \lambda^{n-i} |e(i)|^2 \quad 14$$

Where  $0 < \lambda < 1$  is an exponential weighting (forgetting) factor and the error parameter  $e$  at the  $i$ th time;

$$e(i) = d(i) - y(i) = d(i) - w_n^T x(i) \quad 15$$

Matrix form or Wiener-Hopf function of the filter is;

$$R_x(n)w_n = r_{dx}(n) \quad 16$$

$x(i)$  is the data vector such as;

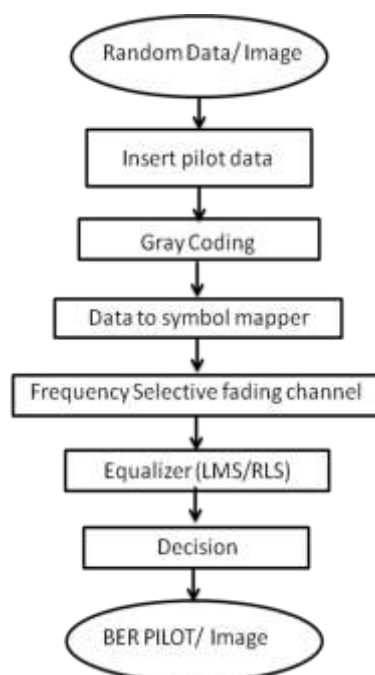
$$x(i) = [x(i), x(i-1), \dots, x(i-p)]^T \quad 17$$

Minimum error alternatively may be expressed as;

$$\{e(n)\}_{\min} = \|d(n)\|_{\lambda}^2 - r_{dx}^H(n)w_n \quad 18$$

### 3.3 Flowchart

The source data may be random data or certain image data. The default value of the source data is 200000 for this project, however it can be modified via program. After the source data is produced as image or random data, the pilot data is inserted into head of source data in each coherence time. Pilot data is used to estimate the random phase shift of the channel. Default value of the pilot data is 8% of the total data. After plotting the data, gray coding is applied to data in our program. After gray coding, data is mapped from binary data to complex data, and each output data represents a point in the constellation diagram. The modulation method of our project is phase shift keying (PSK) modulation. Then three types of PSK modulation (BPSK, QPSK and 8PSK) have been applied to system to compare BER&SER (Bit Error Rate & Sysbol Error Rate) performance of our model.



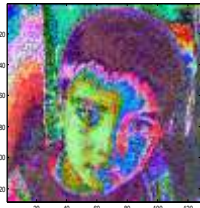
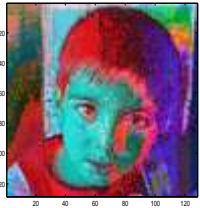
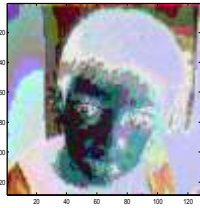

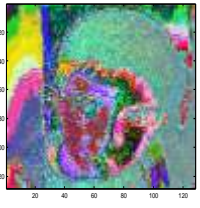
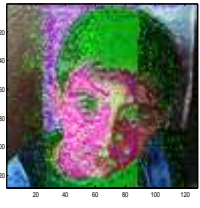
Flow Chart

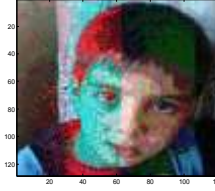
#### IV. RESULTS

The implementation and simulation of Adaptive filter using LMS and RLS algorithm have been done using MATLAB environment and their responses have been studied. The comparison on the adaptive filtering algorithms (namely LMS and RLS) has been carried out based on their BER. It was showed that the BER of LMS equalizer was less. In RLS, the BER decreased by 50.9%, 50.25%, 45.5% for BPSK, QPSK and 8PSK respectively. Thus, RLS meets channel equalization by reducing channel effects; however, its performance can be further improved by using neural network equalization.

**Comparison Table**

Modulation Type	Equalization Algorithm	BER Before Equalization	BER After Equalization
BPSK	LMS	0.88	0.18
QPSK	LMS	0.63	0.18
8PSK	LMS	0.6	0.35
BPSK	RLS	0.82	0.061
QPSK	RLS	0.43	0.18
8PSK	RLS	0.54	0.11

Algorithm	Modulation	Received image	Equalized image
			
LMS	BPSK		
	QPSK		

	8PSK		
RLS	BPSK		
	QPSK		
	8PSK		

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