INTUITIONISTIC FUZZY EQUATIONS AND ITS APPLICATION ON RELIABILITY EVALUATION

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ABSTRACT

Reliability analysis using Intuitionistic Fuzzy Numbers (IFNs) is studied by many researchers, because of its importance in wide range applications in real world. In this paper, due to the lack of a well-established theory of Intuitionistic Fuzzy Equations, we intend to characterize some properties of Intuitionistic Fuzzy Equations by discussing equations of two very simple types : $\tilde{A}^i + \tilde{X}^i = \tilde{B}^i$ *and* $\tilde{C}^i . \tilde{X}^i = \tilde{D}^i$ *where :* $\tilde{A}^i, \tilde{B}^i, \tilde{C}^i, \tilde{D}^i$ *are Intuitionistic Fuzzy Numbers and* $\tilde{X}^i$ *is an unknown Intuitionistic Fuzzy Number for which either of the equations is to be satisfied. An approach to evaluate the unknown components of system failure using Intuitionistic Fuzzy Numbers is presented by using Intuitionistic Fuzzy Fault tree analysis.*

Keywords* : *Fuzzy set, Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Equations, System Reliability

I INTRODUCTION

Research on the theory of fuzzy sets has been growing steadily since the inception of the theory by L.A. Zadeh [1] and has meaningful applications in many fields like engineering, medical science, social science, graph theory etc. In fuzzy set theory, the membership of an element to a fuzzy set is a single value in [0, 1] and it represents the degree of belongingness of the element to the fuzzy set. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to one minus the membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was proposed by K. Atanassov [2] as Intuitionistic fuzzy sets. Intuitionistic fuzzy sets (IFS) are sets whose elements have degrees of membership and non-membership which is an extension of L. Zadeh's [1] notion of fuzzy set. Atanassov (1999, 2012) [2-8] carried out rigorous research based on the theory and applications of Intuitionistic fuzzy sets. Szmidt and Kacrzyk [9], Cornelis, Deschrijver and Kerre[10], Buhaesku [11], Stoyanova and Atanassov [12], Stoyanova [13], Deschrijver and Kerre [14] are also contributed much in Intuitionistic Fuzzy Sets. Burillo [15] et al proposed definition of Intuitionistic Fuzzy Number (IFN) and studied perturbations of IFN and the first properties of the correlation between these numbers. Mitchell [16] considered the problem of ranking a set of intuitionistic fuzzy numbers to define a fuzzy rank and a characteristic vagueness factor for each intuitionistic fuzzy number.

In real world problems, the collected data or system parameters are often imprecise because of incomplete or non-obtainable information, and the probabilistic approach to the conventional reliability analysis is inadequate to account for such built-in uncertainties in data. Therefore concept of fuzzy reliability has been introduced and formulated either in the context of the possibility measures or as a transition from fuzzy success state to fuzzy failure state [17]. Cheng and Mon [18] considered that components are with different membership functions, then interval arithmetic and α-cuts were used to evaluate fuzzy system reliability. G. S. Mahapatra and T. K. Roy[19] introduced intuitionistic fuzzy number and its arithmetic operations based on extension principle of intuitionistic fuzzy sets. They also presented that the arithmetic operation of two or more IFN is again an IFN.

The Intuitionistic theory is a relatively new branch of the fuzzy set theory and so there are many unsolved or unformulated problems in it. In the Intuitionistic Fuzzy Theory, there are a bunch of open problems. One area of such theory in which IFNs and arithmetic operations on IFNs play a fundamental role are Intuitionistic fuzzy equations. In this paper, Intuitionistic Fuzzy Equations according to the approach of arithmetic operations on IFNs is presented. Trapezoidal intuitionistic fuzzy number (TrIFN) is defined and simple Intuitionistic Fuzzy Equations (IFE) are solved. The difficulty in solving such equations arises due to the fact that arithmetic operations on these equation does not lead to the exact solution. The example presented by G. S. Mahapatra and T. K. Roy [20] 'Application of system failure using Intuitionistic Fuzzy number' is used to verify the concept presented in this paper. Intuitionistic fuzzy equations using TrIFNs are used to evaluate the unknown components of imprecise system failure by Intuitionistic fuzzy fault tree analysis.

II BASIC CONCEPT OF INTUTIONISTIC FUZZY NUMBERS

Definition : 2.1 An intuitionistic fuzzy set $\tilde{A}^i$ [Attanassov, 1986] on X is given by

$\tilde{A}^i = \{< x, \mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x) >: x \epsilon X\}$ with $\mu_{\tilde{A}^i}(x) : X \to [0,1]$ and $\nu_{\tilde{A}^i}(x) : X \to [0,1]$ such that

$0 \le \mu_{\tilde{A}^i}(x) + \nu_{\tilde{A}^i}(x) \le 1$ for all $x \epsilon X$.

The value $\mu_{\tilde{A}^i}(x)$ is a lower bound on the degree of membership of x derived from the evidence for x and $\nu_{\tilde{A}^i}(x)$ is a lower bound on the negation of x derived from the evidence against x. We will call ,

$\pi_{\tilde{A}^i}(x) = 1 - \mu_{\tilde{A}^i}(x) - \nu_{\tilde{A}^i}(x)$, $x \in X$, the intuitionistic index of x. It is the hesitancy of x in $\tilde{A}^i$, and expressed lack of knowledge of whether $x \in X$ or not.

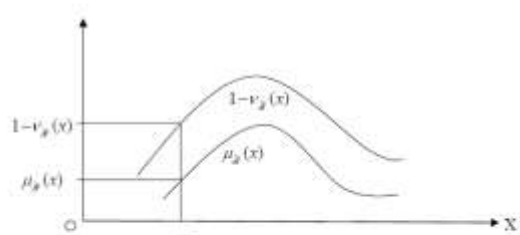


Figure 1. Intuitionistic fuzzy set explanation of real number R

An IFS $\tilde{A}^i$ in X is characterized by a membership function $\mu_{\tilde{A}^i}(x)$ and non-membership function $\nu_{\tilde{A}^i}(x)$. Here $\mu_{\tilde{A}^i}(x)$ and $\nu_{\tilde{A}^i}(x)$ are associated with each point in X, a real number in [0,1] with the value of $\mu_{\tilde{A}^i}(x)$ and $\nu_{\tilde{A}^i}(x)$ at X representing the grade of membership and non-membership of x in $\tilde{A}^i$. Thus closure the value of $\mu_{\tilde{A}^i}(x)$ to unity and the value of $\nu_{\tilde{A}^i}(x)$ to zero; higher the grade of membership and lower the grade of non-membership of x. When $\tilde{A}^i$ is an ordinary set its membership function or non-membership function can take on only two values 0 and 1. If $\mu_{\tilde{A}^i}(x) = 0$ and $\nu_{\tilde{A}^i}(x) = 1$ the element x does not belong to $\tilde{A}^i$. An IFS becomes a fuzzy set $\tilde{A}^i$ when $\nu_{\tilde{A}^i}(x) = 0$ but $\mu_{\tilde{A}^i}(x) \epsilon [0,1]\ \forall\ x \in \tilde{A}^i$.

Definition : 2.2 A set of (α, β) – cut, generated by IFS $\tilde{A}^i$, where α, β ∈ $[0,1]$ are fixed numbers such that α + β ≤ 1 is defined as

$$\tilde{A}^i{}_{\alpha,\beta} = \left\{ \begin{matrix} \left(x, \mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x)\right) : x \epsilon X, \\ \mu_{\tilde{A}^i}(x) \geq \alpha, \nu_{\tilde{A}^i}(x) \leq \beta;\ \alpha, \beta \epsilon [0,1] \end{matrix} \right\}$$

(α, β) – level interval or (α, β) – cut, denoted by $\tilde{A}^i{}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\tilde{A}^i$ atleast to the degree α and which does belong to $\tilde{A}^i$ at most to the degree β.

Definition 2.3 Intuitionistic Fuzzy Number (IFN):

An IFN $\tilde{A}^i$ is defined as follows:

i) An intuitionistic fuzzy subset of the real line.

ii) Normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$ (so $\nu_{\tilde{A}^i}(x_0) = 0$)

iii) A convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right)\ \forall\ x_1, x_2 \in R,\ \lambda \in [0,1]$$

iv) A concave set for the non – membership function $\nu_{\tilde{A}^i}(x)$, i.e.,

$$\nu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \leq \max\left(\nu_{\tilde{A}^i}(x_1), \nu_{\tilde{A}^i}(x_2)\right)\ \forall\ x_1, x_2 \in R,\ \lambda \in [0,1].$$

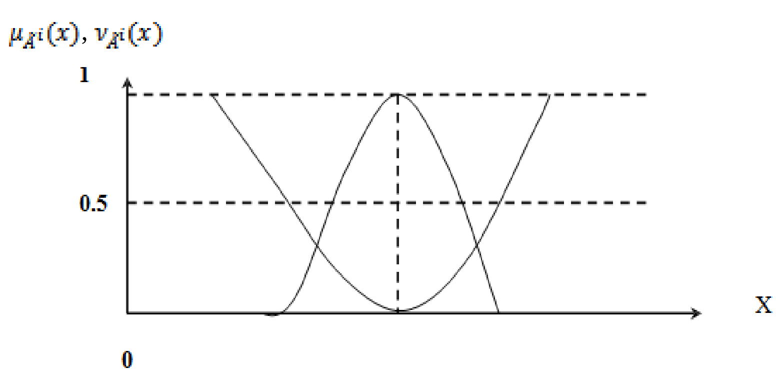


Fig. 2 Membership and non membership functions of $\tilde{A}^i$

Definition 2.4 Trapezoidal Intuitionistic Fuzzy Number (TrIFN):

A TrIFN (Fig.1) $\tilde{A}^i$ is a subset of IFS in R with membership function and non - membership function as follows

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & for\ a_3 \le x \le a_4 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & for\ a_1' \le x \le a_2 \\ 0 & for\ a_2 \le x \le a_3 \\ \frac{x-a_3}{a_4'-a_3} & for\ a_3 \le x \le a_4' \\ 1 & otherwise \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ and TrIFN is denoted by $\tilde{A}^i_{TrIFN} = (a_1, a_2, a_3, a_4;\ a_1', a_2, a_3, a_4')$

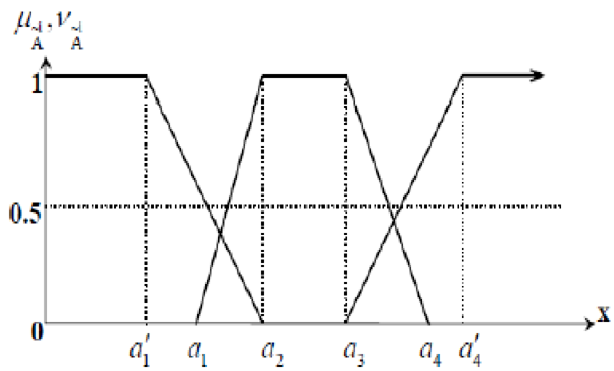


Figure 2: Membership and non-membership function of TrIFN

Note : Here $\mu_{\tilde{A}^i}$(x) increases with constant rate for $x \in [a_1, a_2]$ and decreases with constant rate for $x \in [a_3, a_4]$ but $\nu_{\tilde{A}^i}$(x) decreases with constant rate for $x \in [a_1', a_2]$ and increases with constant rate for $x \in [a_3, a_4']$.

III ARITHMETIC OPERATIONS AND SOME PROPERTIES ON INTUITIONISTIC FUZZY NUMBERS

In this section, the arithmetic operations of IFNs based on intuitionistic fuzzy extension principle and approximation ((α,β)-cuts) method introduced by G.S. Mahapatra, T.K. Roy [15] is presented.

3.1 Arithmetic Operations of Intuitionistic Fuzzy Numbers Based on Extension Principle

The arithmetic operation (*) of two IFNS is a mapping of an input vector X=$[x_1, x_2]^T$ define in the Cartesian product space RxR onto an output y define in the real space R. If $\tilde{A}_1{}^i$ *and* $\tilde{A}_2{}^i$ are IFN then their outcome of arithmetic operation is also a IFN determined with the formula

$$(\tilde{A}_1^i * \tilde{A}_2^i)(y) = \left\{\left(y, \sup_{y=x_1*x_2} \left[min\left(\mu_{\tilde{A}_1^i}(x_1), \mu_{\tilde{A}_2^i}(x_2)\right)\right], \inf_{y=x_1*x_2} \left[max\left(\nu_{\tilde{A}_1^i}(x_1), \nu_{\tilde{A}_2^i}(x_2)\right)\right]\right) \forall x_1, x_2, y \in R\right\}.$$

To calculate the arithmetic operation of IFNs it is sufficient to determine the membership function and non-membership function as follows

$$\mu_{\tilde{A}_1^i * \tilde{A}_2^i}(y) = \sup_{y=x_1*x_2} \left[min\left(\mu_{\tilde{A}_1^i}(x_1), \mu_{\tilde{A}_2^i}(x_2)\right)\right] \text{ and } \quad \nu_{\tilde{A}_1^i * \tilde{A}_2^i}(y) = \inf_{y=x_1*x_2} \left[max\left(\nu_{\tilde{A}_1^i}(x_1), \nu_{\tilde{A}_2^i}(x_2)\right)\right].$$

3.2 Arithmetic Operations of Intuitionistic Fuzzy Numbers Based on (α, β)-cuts Method

If $\tilde{A}_1^{\,i}$ is an IFN, then (α, β) - cut is given by

$$A_{\alpha,\beta} = \begin{cases} [A_1(\alpha), A_2(\alpha)] & \text{for degree of acceptance } \alpha\epsilon[0,1] \\ [A_1'(\beta), A_2'(\beta)] & \text{for degree of rejection } \beta\epsilon[0,1] \end{cases} \quad \text{with } \alpha + \beta \leq 1.$$

Here (i) $\frac{dA_1(\alpha)}{d\alpha} > 0, \frac{dA_2(\alpha)}{d\alpha} < 0$ for all α∈(0, 1), $A_1(1) \leq A_2(1)$ and

(ii) $\frac{dA'_1(\beta)}{d\beta} < 0, \frac{dA'_2(\beta)}{d\beta} > 0$ for all β ∈(0, 1), $A'_1(0) \leq A'_2(0)$.

It is expressed as $A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1'(\beta), A_2'(\beta)]\}$, $\alpha + \beta \leq 1$, α, β∈[0, 1].

For instance, if $\tilde{A}^i = (a_1, a_2, a_3, a_4\,;\, a_1', a_2, a_3, a_4')$ is a TrIFN, then (α, β)-level intervals or (α, β)-cuts is

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1'(\beta), A_2'(\beta)]\}, \alpha + \beta \leq 1, \alpha, \beta \in [0, 1]$$

Where $A_1(\alpha) = a_1 + \alpha(a_2 - a_1), A_2(\alpha) = a_4 - \alpha(a_4 - a_3)$;

$$A_1'(\beta) = a_2 - \beta(a_2 - a_1'), A_2'^{(\beta)} = a_3 + \beta(a_4' - a_3)$$

G.S. Mahapatra and T.K. Roy [20] presented that the arithmetic operation of two or more intuitionistic fuzzy number is again an intuitionistic fuzzy number. The properties introduced by them are mentioned below which forms the basic for this paper.

Property 3.1

(a) If TrIFN $\tilde{A}^i = (a_1, a_2, a_3, a_4\,;\, a_1', a_2, a_3, a_4')$ and y = ka (with k>0), then $\tilde{Y}^i = k\tilde{A}^i$ is a TrIFN $(ka_1, ka_2, ka_3, ka_4\,;\, ka_1', ka_2, ka_3, ka_4')$.

(b) If y = ka (with k<0, i.e., k is negative), then $\tilde{Y}^i = k\tilde{A}^i$ is a TrIFN $(ka_4, ka_3, ka_2, ka_1\,;\, ka_4', ka_3, ka_2, ka_1')$.

Property 3.2

If $\tilde{A}^i = (a_1, a_2, a_3, a_4\ ;\ a_1{}', a_2, a_3, a_4{}')$ and $\tilde{B}^i = (b_1, b_2, b_3, b_4\ ;\ b_1{}', b_2, b_3, b_4{}')$ are two TrIFNs, then $\tilde{C}^i = \tilde{A}^i \oplus \tilde{B}^i$ is also TrIFN $\tilde{A}^i \oplus \tilde{B}^i = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4\ ;\ a_1' + b_1{}', a_2 + b_2, a_3 + b_3, a_4' + b_4{}')$.

Note: If we have the transformation $\tilde{C}^i = k_1\tilde{A}^i + k_2\tilde{B}^i$ (k_1, k_2 are real numbers, not all zero), then the IFS $\tilde{C}^i = k_1\tilde{A}^i + k_2\tilde{B}^i$ is the following TrIFN:

(i) $(k_1a_1 + k_2b_1, k_1a_2 + k_2b_2, k_1a_3 + k_2b_3, k_1a_4 + k_2b_4;\ k_1a'_1 + k_2b'_1, k_1a_2 + k_2b_2, k_1a_3 + k_2b_3, k_1a'_4 + k_2b'_4)$ $\quad if\ k_1 > 0, k_2 \geq 0\ \ or\ k_1 \geq 0, k_2 > 0$

(ii) $(k_1a_1 + k_2b_4, k_1a_2 + k_2b_3, k_1a_3 + k_2b_2, k_1a_4 + k_2b_1;\ k_1a'_1 + k_2b'_4, k_1a_2 + k_2b_3, k_1a_3 + k_2b_2, k_1a'_4 + k_2b'_1)$ $\quad if\ k_1 > 0, k_2 \leq 0\ \ or\ k_1 \geq 0, k_2 < 0$

(iii) $(k_1a_4 + k_2b_1, k_1a_3 + k_2b_2, k_1a_2 + k_2b_3, k_1a_2 + k_2b_4;\ k_1a'_4 + k_2b'_1, k_1a_3 + k_2b_2, k_1a_2 + k_2b_3, k_1a'_1 + k_2b'_4)$ $\quad if\ k_1 < 0, k_2 \geq 0\ \ or\ k_1 \leq 0, k_2 > 0$

(iv) $(k_1a_4 + k_2b_4, k_1a_3 + k_2b_3, k_1a_2 + k_2b_2, k_1a_1 + k_2b_1;\ k_1a'_4 + k_2b'_4, k_1a_3 + k_2b_3, k_1a_2 + k_2b_2, k_1a'_1 + k_2b'_1)$ $\quad if\ k_1 < 0, k_2 \leq 0\ \ or\ k_1 \leq 0, k_2 < 0$

Property 3.3

If $\tilde{A}^i = (a_1, a_2, a_3, a_4\ ;\ a_1{}', a_2, a_3, a_4{}')$ and $\tilde{B}^i = (b_1, b_2, b_3, b_4\ ;\ b_1{}', b_2, b_3, b_4{}')$ are two TrIFNs, then $\tilde{P}^i = \tilde{A}^i \otimes \tilde{B}^i$ is also TrIFN $\tilde{A}^i \otimes \tilde{B}^i = (a_1b_1,\ a_2b_2,\ a_3b_3,\ a_4b_4\ ;\ a_1'b_1',\ a_2b_2,\ a_3b_3,\ a_4'b_4{}')$.

Property 3.4

If $\tilde{A}^i = (a_1, a_2, a_3, a_4\ ;\ a_1{}', a_2, a_3, a_4{}')$ and $\tilde{B}^i = (b_1, b_2, b_3, b_4\ ;\ b_1{}', b_2, b_3, b_4{}')$ are two TrIFNs, then $\tilde{D}^i = \tilde{A}^i \div \tilde{B}^i$ is also TrIFN $\tilde{A}^i \div \tilde{B}^i = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\ ;\ \frac{a'_1}{b'_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a'_4}{b'_1}\right)$.

IV INTUITIONISTIC FUZZY EQUATIONS

Intuitionistic fuzzy equations are equations in which coefficients and unknowns are Trapezoidal Intuitionistic fuzzy number (TrIFN) and formulas are constructed by arithmetic operations on Intuitionistic fuzzy number. Here, we only intend to characterize some properties of Intuitionistic fuzzy equations by discussing equations of two very simple types

$$\tilde{A}^i + \tilde{X}^i = \tilde{B}^i \quad \text{and} \quad \tilde{C}^i . \tilde{X}^i = \tilde{D}^i$$

where $\tilde{A}^i, \tilde{B}^i, \tilde{C}^i$ *and* $\tilde{D}^i$ are TrIFNs and X is an unknown Trapezoidal Intuitionistic fuzzy number for which either of the equations is to be satisfied.

Property 4.1 The equation $\tilde{A}^i + \tilde{X}^i = \tilde{B}^i$ …(1) has a solution if and only if

$b_1' - a_1' \leq b_1 - a_1 \leq b_2 - a_2 \leq b_3 - a_3 \leq b_4 - a_4 \leq b_4' - a_4'$

Where $\tilde{A}^i = (a_1, a_2, a_3, a_4\,;\, a_1', a_2, a_3, a_4')$ and $\tilde{B}^i = (b_1, b_2, b_3, b_4\,;\, b_1', b_2, b_3, b_4')$ are two TrIFNs.

Proof. We first prove that $\tilde{X}^i = \tilde{B}^i - \tilde{A}^i$ is not the solution.

i.e., $\tilde{B}^i - \tilde{A}^i = (b_1 - a_4,\ b_2 - a_3,\ b_3 - a_2, b_4 - a_1\,;\, b_1' - a_4',\ b_2 - a_3,\ b_3 - a_2,\ b_4' - a_1')$

substituting the value for $\tilde{X}^i$ in (1) we get

$$\tilde{A}^i + (\tilde{B}^i - \tilde{A}^i) = (a_1, a_2, a_3, a_4\,;\, a_1', a_2, a_3, a_4') - (b_1 - a_4, b_2 - a_3, b_3 - a_2, b_4 - a_1\,;\, b_1' - a_4',\ b_2 - a_3,\ b_3 - a_2,\ b_4' - a_1')$$

$$= (a_1 + (b_1 - a_4),\ a_2 + (b_2 - a_3),\ a_3 + (b_3 - a_2),\ a_4 + (b_4 - a_1);$$

$$a_1' + (b_1' - a_4'),\ a_2 + (b_2 - a_3),\ a_3 + (b_3 - a_2),\ a_4' + (b_4' - a_1'))$$

$$\neq (b_1, b_2, b_3, b_4\,;\, b_1', b_2, b_3, b_4') = \tilde{B}^i$$

i.e., $\tilde{A}^i + (\tilde{B}^i - \tilde{A}^i) \neq \tilde{B}^i$ whenever $a_1 \neq a_4,\ a_2 \neq a_3$ *and* $a_1' \neq a_4'$

$\therefore\ \tilde{X}^i = \tilde{B}^i - \tilde{A}^i$ is not a solution of the equation.

Let $\tilde{X}^i = (x_1, x_2, x_3, x_4\,;\, x_1', x_2, x_3, x_4')$. Then the Intuitionistic fuzzy equation $\tilde{A}^i + \tilde{X}^i = \tilde{B}^i$ can be given by

$$(a_1 + x_1, a_2 + x_2, a_3 + x_3, a_4 + x_4\,;\, a_1' + x_1', a_2 + x_2, a_3 + x_3, a_4' + x_4')$$

$$= (b_1, b_2, b_3, b_4\,;\, b_1', b_2, b_3, b_4')$$

$.\Rightarrow\ a_1 + x_1 = b_1,\ a_2 + x_2 = b_2,\ a_3 + x_3 = b_3,\ a_4 + x_4 = b_4,$

$a_1' + x_1' = b_1',\ a_4' + x_4' = b_4'.$

From this we get the solution, $\tilde{X}^i = (x_1, x_2, x_3, x_4\,;\, x_1', x_2, x_3, x_4')$ as

$x_1 = b_1 - a_1, x_2 = b_2 - a_2, x_3 = b_3 - a_3, x_4 = b_4 - a_4, x_1' = b_1' - a_1', x_4' = b_4' - a_4'.$

Since we have $\tilde{X}^i$ as a TrIFN, we should have $x_1' \leq x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_4'$.

i.e., the equation has a solution if and only if $b_1' - a_1' \leq b_1 - a_1 \leq b_2 - a_2 \leq b_3 - a_3 \leq b_4 - a_4 \leq b_4' - a_4'$

Property 4.2: The equation $\tilde{C}^i . \tilde{X}^i = \tilde{D}^i$ …(2) has a solution if and only if

$d_1'/c_1' \leq d_1/c_1 \leq d_2/c_2 \leq d_3/c_3 \leq d_4/c_4 \leq d_4'/c_4'$

Where $\tilde{C}^i = (c_1, c_2, c_3, c_4 ; c_1', c_2, c_3, c_4')$ and $\tilde{D}^i = (d_1, d_2, d_3, d_4 ; d_1', d_2, d_3, d_4')$ are two TrIFNs.

Proof. We first prove that $\tilde{X}^i = \tilde{D}^i / \tilde{C}^i$ is not the solution.

i.e., $\tilde{D}^i / \tilde{C}^i = \left(\frac{d_1}{c_4}, \frac{d_2}{c_3}, \frac{d_3}{c_2}, \frac{d_4}{c_1}; \frac{d_1'}{c_4'}, \frac{d_2}{c_3}, \frac{d_3}{c_2}, \frac{d_4'}{c_1'}\right)$

substituting the value for $\tilde{X}^i$ in (2) we get,

$$\tilde{C}^i . \left(\tilde{D}^i / \tilde{C}^i\right) = (c_1, c_2, c_3, c_4 ; c_1', c_2, c_3, c_4') . \left(\frac{d_1}{c_4}, \frac{d_2}{c_3}, \frac{d_3}{c_2}, \frac{d_4}{c_1}; \frac{d_1'}{c_4'}, \frac{d_2}{c_3}, \frac{d_3}{c_2}, \frac{d_4'}{c_1'}\right)$$

$$= \left(c_1 . \frac{d_1}{c_4}, c_2 . \frac{d_2}{c_3}, c_3 . \frac{d_3}{c_2}, c_4 . \frac{d_4}{c_1}; c_1' . \frac{d_1'}{c_4'}, c_2 . \frac{d_2}{c_3}, c_3 . \frac{d_3}{c_2}, c_4' . \frac{d_4'}{c_1'}\right)$$

$$\neq (d_1, d_2, d_3, d_4 ; d_1', d_2, d_3, d_4') = \tilde{D}^i$$

i.e., $\tilde{C}^i . \left(\tilde{D}^i / \tilde{C}^i\right) \neq \tilde{D}^i$ $\therefore$ $\tilde{X}^i = \tilde{D}^i / \tilde{C}^i$ is not a solution of the equation (2).

Let $\tilde{X}^i = (x_1, x_2, x_3, x_4 ; x_1', x_2, x_3, x_4')$. Then the Intuitionistic fuzzy equation $\tilde{C}^i . \tilde{X}^i = \tilde{D}^i$ can be given by

$(c_1x_1, c_2x_2, c_3x_3, c_4x_4 ; c_1'x_1', c_2x_2, c_3x_3, c_4'x_4') = (d_1, d_2, d_3, d_4 ; d_1', d_2, d_3, d_4')$

$. \Rightarrow c_1x_1 = d_1 , c_2x_2 = d_2 , c_3x_3 = d_3 , c_4x_4 = d_4 , c_1'x_1' = d_1', c_4'x_4' = d_4'.$

From this we get the solution, $\tilde{X}^i = (x_1, x_2, x_3, x_4 ; x_1', x_2, x_3, x_4')$ as

$x_1 = d_1/c_1 , x_2 = d_2/c_2 , x_3 = d_3/c_3 , x_4 = d_4/c_4 , x_1' = d_1'/c_1' , x_4' = d_4'/c_4'.$

Since we have $\tilde{X}^i$ as a TrIFN, we should have $x_1' \leq x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_4'$.

i.e., the equation has a solution if and only if $d_1'/c_1' \leq d_1/c_1 \leq d_2/c_2 \leq d_3/c_3 \leq d_4/c_4 \leq d_4'/c_4'$

In addition, Any Intuitionistic Fuzzy Number can be uniquely represented by its (α, β) cuts. Hence the described procedure can be applied to (α, β) cuts of arbitrary TrIFNs.

i.e., The (α, β) cut of the Intuitionistic fuzzy equation, $A_{\alpha,\beta} + X_{\alpha,\beta} = B_{\alpha,\beta}$ has a solution,

$$X_{\alpha,\beta} = \{[X_1(\alpha), X_2(\alpha)];\ [X_1'(\beta), X_2'(\beta)]\},\ \alpha+\beta\leq 1,\ \alpha,\beta\in[0,1]$$

Where $X_1(\alpha) = x_1 + \alpha(x_2 - x_1)$, $X_2(\alpha) = x_4 - \alpha(x_4 - x_3)$,

$X_1'(\beta) = x_2 - \beta(x_2 - x_1')$ and $X_2'(\beta) = x_3 + \beta(x_4' - x_3)$.

4.1 An Example to Find The Solution of Intuitionistic fuzzy equations:

Let $\tilde{A}^i, \tilde{B}^i$ be two TrIFNs whose membership an non-membership functions are given by

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x-1.5}{1.5}, & 1.5 \le x \le 3 \\ 1, & 3 \le x \le 4 \\ \frac{5.2-x}{1.2}, & 4 \le x \le 5.2 \\ 0, & otherwise \end{cases}, \qquad \nu_{\tilde{A}^i}(x) = \begin{cases} \frac{1.5-x}{1.9}, & 1.1 \le x \le 3 \\ 1, & 3 \le x \le 4 \\ \frac{x-4}{1.9}, & 4 \le x \le 5.9 \\ 1, & otherwise \end{cases},$$

$$\mu_{\tilde{B}^i}(x) = \begin{cases} \frac{x-3.6}{2.6}, & 3.6 \le x \le 6.2 \\ 1, & 6.2 \le x \le 8.3 \\ \frac{10.9-x}{2.6}, & 8.3 \le x \le 10.9 \\ 0, & otherwise \end{cases}, \quad \nu_{\tilde{B}^i}(x) = \begin{cases} \frac{3.6-x}{4.1}, & 2.1 \le x \le 6.2 \\ 0, & 6.2 \le x \le 8.3 \\ \frac{x-8.3}{4.1}, & 8.3 \le x \le 12.4 \\ 1, & otherwise \end{cases}$$

Solve the following Intuitionistic fuzzy equations for $\tilde{X}^i$ and find the membership and non-membership function for $\tilde{C}^i$

a) $\tilde{A}^i + \tilde{X}^i = \tilde{B}^i$
b) $\tilde{B}^i.\tilde{X}^i = \tilde{C}^i$

Solution:

From the membership and non-membership function of $\tilde{A}^i$and $\tilde{B}^i$. We can formulate the TrIFN as

$\tilde{A}^i = (1.5, 3, 4, 5.2; 1.1, 3, 4, 5.9)$

$$\tilde{B}^i = (3.6, 6.2, 8.3, 10.9; 2.1, 6.2, 8.3, 12.4)$$

First, let us consider the Intuitionistic fuzzy equation $\tilde{A}^i + \tilde{X}^i = \tilde{B}^i$ --------- (1)

Here the solution $\tilde{X}^i = (x_1, x_2, x_3, x_4;\ x_1' x_2, x_3, x_4')$

$= (b_1 - a_1, b_2 - a_2, b_3 - a_3, b_4 - a_4;\ b_1' - a_1', b_2 - a_2, b_3 - a_3, b_4' - a_4')$ is possible iff

$x_1' \le x_1 \le x_2 \le x_3 \le x_4 \le x_4'$

Hence (1) has a solution as

$\tilde{X}^i = (2.1, 3.2, 4.3, 5.7; 1, 3.2, 4.3, 6.5)$

Now, Let us consider the Intuitionistic fuzzy equation $\tilde{B}^i . \tilde{X}^i = \tilde{C}^i$ ---------- (2)

Substituting the solution $\tilde{X}^i$ in (2) and by means of arithmetic operations on TrIFN we can calculate $\tilde{C}^i$ as

$\tilde{C}^i = (b_1 x_1, b_2 x_2, b_3 x_3, b_4 x_4;\ b_1' x_1', b_2 x_2, b_3 x_3, b_4' x_4')$

$= (7.56, 19.84, 35.69, 62.13; 2.1, 19.84, 35.69, 80.6)$

Hence the membership and non-membership function of $\tilde{C}^i$ is given by

$$\mu_{\tilde{C}^i}(x) = \begin{cases} \frac{x-7.56}{12.28}, & 7.56 \le x \le 19.84 \\ 1, & 19.84 \le x \le 35.69 \\ \frac{62.13-x}{26.44}, & 35.69 \le x \le 62.13 \\ 0, & otherwise \end{cases},$$

$$\nu_{\tilde{C}^i}(x) = \begin{cases} \frac{19.84-x}{17.74}, & 2.1 \le x \le 19.84 \\ 0, & 19.84 \le x \le 35.69 \\ \frac{x-35.69}{44.91}, & 35.69 \le x \le 80.6 \\ 1, & otherwise \end{cases}$$

Note: The (α, β) cut of the Intuitionistic fuzzy equation is given by

$A_{\alpha,\beta} + X_{\alpha,\beta} = B_{\alpha,\beta}$ where $X_{\alpha,\beta} = \{[X_1(\alpha), X_2(\alpha)]; [X_1'(\beta), X_2'(\beta)]\}$, $0 \le \alpha + \beta \le 1$, $\alpha, \beta \in [0,1]$

Let $\alpha = 0.25$, $\beta = 0.5$ then $X_1(\alpha) = x_1 + \alpha(x_2 - x_1) = 2.1 + 0.25(3.2 - 2.1) = 2.375$

$X_2(\alpha) = 5.7 - 0.25(5.7 - 4.3) = 5.35$

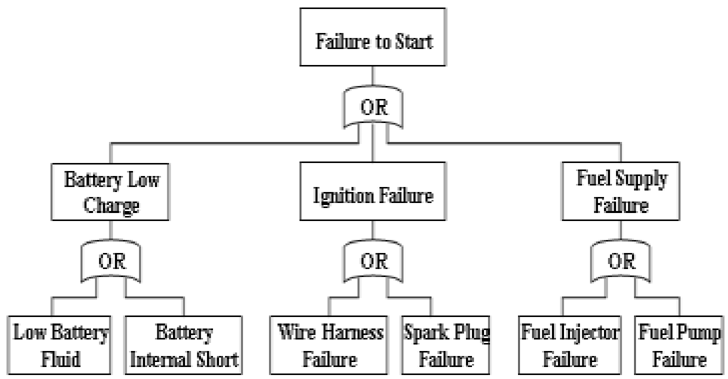
$X_1'(\beta) = 3.2 - 0.5(3.2 - 1) = 2.1$

$X_2'(\beta) = 4.3 + 0.5(6.5 - 4.3) = 5.4$

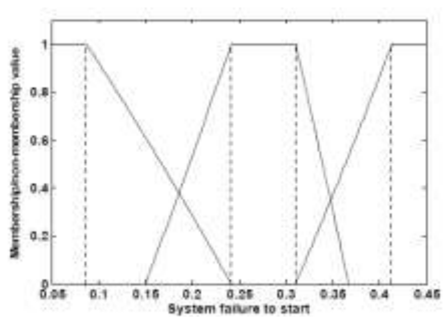
Hence $X_{\alpha,\beta} = \{[2.375, 5.35]; [2.1, 5.4]\}$.

V EVALUATING UNKNOWN COMPONENTS IN SYSTEM FAILURE USING INTUITIONISTIC FUZZY EQUATIONS & FUZZY FAULT TREE ANALYSIS

Starting failure of an automobile depends on different facts which is briefly explained by G.S. Mahapatra and T.K. Roy [15]. The facts are battery low charge, ignition failure and fuel supply failure. There are two sub-factors of each of the facts. The fault tree of failure to start of the automobile is shown below.



Suppose we are given the data such that the fuzzy failure to start an automobile and other facts are presented and we are intiated to calculate or compute the value of inner components such as the value for ignition failure, the value for battery internal short, the value for spark plug failure and the value for fuel pump failure. In such case, it is difficult to follow the arithmetic operations defined on TrIFNs as it deviates from actual value. Hence Intuitionistic Fuzzy equations play a vital role in such situations to bring out the appropriate value for the unknown TrIFNs.



TrIFN representing the system failure to start an automobile

$\tilde{F}^{i}_{fs}$ represents the system failure to start of automobile.

$\tilde{F}^{i}_{blc}$ represents the failure to start of automobile due to Battery Low Charge.

$\tilde{F}^{i}_{if}$ represents the failure to start of automobile due to Ignition Failure.

$\tilde{F}^{i}_{fsf}$ represents the failure to start of automobile due to Fuel Supply Failure.

$\tilde{F}^{i}_{lbf}$ represents the failure to start of automobile due to Low Battery Fluid.

$\tilde{F}^i{}_{bis}$ represents the failure to start of automobile due to Battery Internal Short.

$\tilde{F}^i{}_{whf}$ represents the failure to start of automobile due to Wire Harness Failure.

$\tilde{F}^i{}_{spf}$ represents the failure to start of automobile due to Spark Plug Failure.

$\tilde{F}^i{}_{fif}$ represents the failure to start of automobile due to Fuel Injector Failure.

$\tilde{F}^i{}_{fpf}$ represents the failure to start of automobile due to Fuel Pump Failure.

The intuitionistic fuzzy failure to start of an automobile can be calculated when the failures of the occurrence of basic fault events are known. The numerical explanation for starting failure of the automobile using fault tree analysis with intuitionistic fuzzy failure rate is presented below. The components failure rates as TrIFN are given by

$\tilde{F}^i{}_{fs} = (0.149855, 0.241615, 0.310286, 0.366922; 0.086857, 0.241615, 0.310286, 0.413272)$,

$\tilde{F}^i{}_{blc} = (0.0396, 0.0785, 0.107, 0.1258; 0.0199, 0.0785, 0.107, 0.1444)$,

$\tilde{F}^i{}_{fsf} = (0.0688, 0.107, 0.1351, 0.1536; 0.0494, 0.107, 0.1351, 0.1719)$,

$\tilde{F}^i{}_{lbf} = (0.02, 0.05, 0.06, 0.07; 0.01, 0.05, 0.06, 0.08)$,

$\tilde{F}^i{}_{whf} = (0.03, 0.04, 0.05, 0.07; 0.01, 0.04, 0.05, 0.09)$,

$\tilde{F}^i{}_{fif} = (0.03, 0.05, 0.07, 0.08; 0.02, 0.05, 0.07, 0.09)$.

Failure to start of an automobile can be evaluated by using the following steps:

Step 1.

$$\tilde{F}^i{}_{fs} = 1 \ominus (1 \ominus \tilde{F}^i{}_{blc})(1 \ominus \tilde{F}^i{}_{if})(1 \ominus \tilde{F}^i{}_{fsf}) \quad \text{……………………………………..(1)}$$

Using equation (1) and the numerical data presented above, we can compute the unknown value of $\tilde{F}^i{}_{if} = (x_1, x_2, x_3, x_4; x_1', x_2, x_3, x_4')$ as follows.

$$(1 \ominus \tilde{F}^i{}_{if}) = (1 - x_1, 1 - x_2, 1 - x_3, 1 - x_4; 1 - x_1', 1 - x_2, 1 - x_3, 1 - x_4')$$

$$= (y_1, y_2, y_3, y_4; y_1', y_2, y_3, y_4') = \tilde{Y}^i$$

Then equation (1) becomes $\tilde{F}^i{}_{fs} \oplus (1 \ominus \tilde{F}^i{}_{blc})(1 \ominus \tilde{F}^i{}_{fsf}) \tilde{Y}^i = 1$ ……………….(1a)

$$(1 \ominus \tilde{F}^i{}_{blc}) = (0.9604, 0.9215, 0.893, 0.8742; 0.9801, 0.9215, 0.893, 0.8556)$$

$$(1 \ominus \tilde{F}^i{}_{fsf}) = (0.9312, 0.893, 0.8649, 0.8464; 0.9506, 0.893, 0.8649, 0.8281)$$

Using the arithmetic operations, we can compute the value of $(1 \ominus \tilde{F}^i{}_{blc})(1 \ominus \tilde{F}^i{}_{fsf})$, then we substitute the value in (1a) and using the concept of Intuitionistic fuzzy equations we can compute the value of $\tilde{F}^i{}_{if}$ as

$\tilde{F}^i{}_{if} = (0.0494, 0.0784, 0.107, 0.144; 0.0199, 0.0784, 0.107, 0.1719)$.

Step 2.

$$\tilde{F}^i{}_{blc} = 1 \ominus \left(1 \ominus \tilde{F}^i{}_{lbf}\right)\left(1 \ominus \tilde{F}^i{}_{bis}\right)$$

$$\tilde{F}^i{}_{fs} = 1 \ominus \left(1 \ominus \tilde{F}^i{}_{whf}\right)\left(1 \ominus \tilde{F}^i{}_{spf}\right)$$

$$\tilde{F}^i{}_{fs} = 1 \ominus (1 \ominus \tilde{F}^i{}_{fif})(1 \ominus \tilde{F}^i{}_{fpf})$$

By similar procedure explained above, we can able to compute the unknown values of $\tilde{F}^i{}_{bis}, \tilde{F}^i{}_{spf}, \tilde{F}^i{}_{fpf}$ using the concept of intuitionistic fuzzy equations as

$$\tilde{F}^i{}_{bis} = (0.02, 0.03, 0.05, 0.06; 0.01, 0.03, 0.05, 0.07),$$

$$\tilde{F}^i{}_{spf} = (0.02, 0.04, 0.06, 0.08; 0.01, 0.04, 0.06, 0.09),$$

$$\tilde{F}^i{}_{fpf} = (0.04, 0.06, 0.07, 0.08; 0.03, 0.06, 0.07, 0.09).$$

Hence, we can able to compute the unknown values of a inner component of fuzzy system failure to start a system by means of intuitionistic fuzzy equations to get the appropriate value.

VI CONCLUSION

In this paper, Intuitionistic fuzzy equations are defined using the properties of IFNs. The difficulty in solving IFE arise due the fact that normal arithmetic operations defined on IFN does not lead us to the exact or appropriate solution. IFSs separate the positive and negative evidence for the membership of an element in a set. Finally, the result is verified by using the example given by G.s. Mahapatra, T.K. Roy. They computed the Intuitionistic fuzzy failure to start of an automobile when the failures of the occurrence of basic fault events are known. In this paper, we have taken the intuitionistic fuzzy failure to start of an automobile as known and evaluated the unknown basic fault events such as Ignition failure, Battery internal shortage, Spark plug failure and fuel pump failure using Intuitionistic Fuzzy equations. Our computational procedure is very simple to implement for calculations in intuitionistic fuzzy environment for all field of engineering and sciences where vagueness occur.

REFERENCES

[1] Zadeh, L.A., Fuzzy sets, *Information and Control, vol.8, pp.338–353, 1965.*

[2] Atanassov, K.T., Intuitionistic fuzzy sets, *VII ITKR's Session, Sofia, Bulgarian, 1983.*

[3] Atanassov, K.T., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems, vol.20, pp.87–96, 1986.*

[4] Atanassov, K.T., and G. Gargov, Interval-valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems, vol.31, no.3, pp.343–349, 1989.*

[5] Atanassov, K.T., More on intuitionistic fuzzy sets, *Fuzzy Sets and Systems, vol.33, no.1, pp.37–46, 1989.*

[6] Atanassov, K.T., *Intuitionistic Fuzzy Sets*(Physica-Verlag, Heidelberg, New York, 1999).

[7] Atanassov, K.T., Two theorems for Intuitionistic fuzzy sets, *Fuzzy Sets and Systems, vol.110, pp.267–269, 2000.*

[8] Atanassov, K.T., and G. Gargov, Elements of intuitionistic fuzzy logic, Part I, *Fuzzy Sets and Systems, vol.95, no.1, pp.39–52,1998.*

[9] Szmidt, E., and J. Kacprzyk, Distances between intuitionistic fuzzy sets, *Fuzzy Sets and Systems, vol.114, no.3, pp.505–518, 2000.*

[10]Cornelis, C., Deschrijver, G., and E.E. Kerre, Implication in intuitionistic fuzzy and interval- valued fuzzy set theory:construction, application, *International Journal of Approximate Reasoning, 35(2004)55-95.*

[11] Buhaesku, T., Some observations on intuitionistic fuzzy relations, *Itinerant Seminar of Functional Equations, Approximation and Convexity, pp.111–118, 1989.*

[12] Stoyanova, D., and Atanassov, K.T., Relation between operators, defined over intuitionistic fuzzy sets, *IM-MFAIS, (1990), 46-49, Sofia, Bulgaria.*

[13] Stoyanova, D., More on Cartesian product over intuitionistic fuzzy sets, *BUSEFAL, vol.54, pp.9–13, 1993.*

[14] Deschrijver, G., and E.E. Kerre, On the relationship between intuitionistic fuzzy sets and some other extensions of fuzzy set theory, *Journal of Fuzzy Mathematics, vol.10, no.3, pp.711–724, 2002.*

[15] Burillo, P., H. Bustince, and V. Mohedano, Some definition of intuitionistic fuzzy number, *Fuzzy based Expert Systems, Fuzzy Bulgarian Enthusiasts*, 1994.

[16] Mitchell, H.B., Ranking-intuitionistic fuzzy numbers, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, vol.12, no.3, pp.377–386, 2004.*

[17] Cai, K.Y., C.Y. Wen, and M.L. Zhang, Fuzzy reliability modeling of gracefully degradable computing systems, *Reliability Engineering and System Safety, vol.33, pp.141–157, 1991.*

[18] Cheng, C.H., and D.L. Mon, Fuzzy system reliability analysis by interval of confidence, *Fuzzy Sets and Systems, vol.56, pp.9–35, 1993.*

[19] Mahapatra, G.S., and T.K. Roy, Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations,*International Journal of Computational and Mathematical Sciences*, vol.3, no.5, pp.225–232, 2009.

[20] Mahapatra, G.S., and T.K. Roy, Intuitionistic Fuzzy Number and Its Arithmetic Operation with Application on System failure, J*ournal of uncertain systems, vol.7, No.2, pp.92-107, 2013.*