

IMAGE DENOISING USING CURVELET TRANSFORM AND WEINER FILTERING

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ABSTRACT

In this paper we propose a new method to reduce noise in digital image. Images corrupted by Gaussian Noise are still a classical problem. To reduce the noise or to improve the quality of image we have used two parameters i.e. quantitative and qualitative. For quantity we will compare peak signal to noise ratio (PSNR). Higher the PSNR better the quality of the image.

The Curvelet transform is a higher dimensional generalization of the Wavelet transform designed to represent images at different scales and different angles. In this paper we proposed a Curvelet Transformation based image denoising, which is combined with wiener filter in place of the low pass filtering in the transform domain. We demonstrated through simulations with images contaminated by three different noise i.e. Gaussian, salt and pepper and speckle. Experimental results show denoising of an image is done by processing an image through Wiener filter and using curvelet transform [1], [6]. Experimental results show that proposed denoising technique performs better in terms of the PSNR. Simple de-noising algorithms that use the curvelet transform consist of three steps.

- Calculate the curvelet transform of the noisy signal.
- Modify the noisy curvelet coefficients according to some rule.
- Compute the inverse transform using the modified coefficients.

Keywords: Curvelet Transform, Discrete Wavelet Transform, Discrete Curvelet Transform, Filter, PSNR.

I. INTRODUCTION

In this paper, we propose a novel denoising method. The method uses curvelet transform and Wiener filtering [8] to denoise an image. Initially we obtain a noisy image by degrading it by adding additive Gaussian noise (most common type of noise). Then we implement our algorithm, which firstly passes it through a Wiener filter. And then the output of which is then applied by curvelet transform. The resultant image is denoised and also retains the important image information. Fig.1 shows the flow of the proposed algorithm. Discrete curvelet transform is one of the most powerful approaches in capturing edge curves in an image. Related works on curvelet features are also investigated. In this research, we generate a texture features descriptor using wrapping based discrete curvelet transform. This descriptor is used to represent images in a large database in terms of their features and to measure the similarity between images. The retrieval outcome shows, the proposed curvelet texture feature descriptor outperforms the Gabor filters in both retrieval accuracy and efficiency. The optimal level of curvelet decomposition is also investigated to obtain the highest retrieval outcome in terms of

effectiveness and efficiency. From the experimental results, we find that curvelet texture feature is robust to a reasonable scale distortion

The initial approach of curvelet transform implements the concept of discrete ridgelet transform [24]. Since its creation in 1999 [12], ridgelet based curvelet transform has been successfully used as an effective tool in image denoising [19], image decomposition [61], texture classification [35], image deconvolution [32], astronomical imaging [36] and contrast enhancement [37], etc. But ridgelet based curvelet transform is not efficient as it uses complex ridgelet transform [17]. In 2005, Candès et al. proposed two new forms of curvelet transform based on different operations of Fourier samples [18], namely, unequally-spaced fast Fourier transform (USFFT) and wrapping

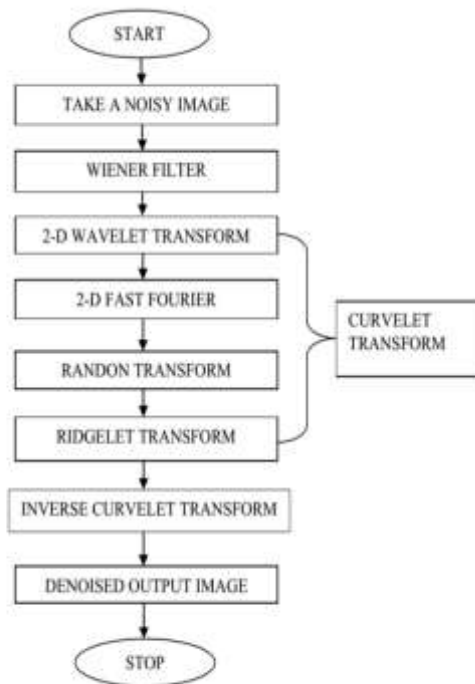


Fig.1. Proposed Algorithm

based fast curvelet transform. Wrapping based curvelet transform is faster in computation time and more robust than ridgelet and USFFT based curvelet transform [17]. To our knowledge, wrapping based curvelet transform has not been used in CBIR and there is no work on a systematic evaluation of curvelet in CBIR.

In the following section, we first describe the curvelet transform approaches and their advantages in texture representation over other spectral approaches. Then, we provide a brief description of the related works already done using curvelet transform.

II. CURVELET TRANSFORM

The curvelet are based on multiscale ridgelet combined with a spatial band pass filtering operation. A 2-D wavelet transform is used to isolate the image at different scales and spatial partitioning is used to break each scale into blocks. Large size blocks are used to partition the large scale wavelet transform components and small size blocks are used to partition the small scale components. Finally, the ridgelet transform is applied to each block. In this way, the image edges at a certain scale can be represented efficiently by the ridgelet transform because the image edges are almost like straight lines at that scale. The Curvelet transform can sparsely characterize the high-dimensional signals which have lines, curves or hyper plane singularities [48]

Curvelets are based on multiscale ridgelets combined with a spatial bandpass filtering operation to isolate different scale. Like ridgelets, curvelets occur at all scales, locations, and orientations. However, while ridgelets all have global length and variable widths, curvelets in addition to a variable width have a variable length and so a variable anisotropy. The length and width at fine scales are related by a scaling law width equal to length² and so the anisotropy increases with decreasing scale like a power law. Recent work shows that thresholding of discrete curvelet coefficients provide near optimal N-term representations of otherwise smooth objects with discontinuities along curves. Thus for understanding curvelet, we should know about ridgelet and radon transform.

2.1 Continuous Ridgelet Transform

The ridgelet transform [3], [7] of two-dimensional function $f(x, y)$ allows the sparse representation of both smooth function and of straight edges by superposition of ridgelet function. For every $L_2(R^2)$ we get its ridgelet coefficients $Rf(a, b, \theta)$ by an inner product with the frame like function $\Psi_{a, b, \theta}(x)$ which is a wavelet in transverse orientation constant along the line $x_1 \cos \theta + x_2 \sin \theta = \text{constant}$. For every $a > 0$, each $b \in R$ and each $\theta \in [0, 2\pi)$, the bivariate ridgelet $\Psi_{a, b, \theta}$ is given by

$$\frac{1}{\sqrt{a}} \psi\left(\frac{x \cos \theta + y \sin \theta - b}{a}\right)$$

This function is constant along the lines $x_1 \cos \theta + x_2 \sin \theta = \text{constant}$. Transverse to these ridges it is a wavelet. Given an integrable bivariate function $f(x)$, the ridgelet coefficients by

$$\mathcal{R}_f(a, b, \theta) = \int \psi_{a, b, \theta}(x) f(x) dx.$$

The reconstruction formulae is given as

$$f(x) = \int_0^{2\pi} \int_{-\infty}^{\infty} \int_0^{\infty} \mathcal{R}_f(a, b, \theta) \psi_{a, b, \theta}(x) \frac{da}{a^3} db \frac{d\theta}{4\pi}$$

The above equation (4) is valid for both integrable and square integrable. Furthermore, this formula is stable as one has a Parseval relation is given by

$$\int |f(x)|^2 dx = \int_0^{2\pi} \int_{-\infty}^{\infty} \int_0^{\infty} |\mathcal{R}_f(a, b, \theta)|^2 \frac{da}{a^3} db \frac{d\theta}{4\pi}.$$

Hence, much like the wavelet or Fourier transforms, the identity expresses the fact that one can represent any arbitrary function as a continuous superposition of ridgelet.

2.2 Radon Transform

A basic tool for calculating ridgelet coefficients is to view ridgelet analysis as a form of wavelet analysis in the Radon domain. The Radon transform of an object f is the collection of line integrals indexed by $(\theta, t) \in [0, 2\pi) \times R$.

$$Rf(\theta, t) = \int f(x_1, x_2) \delta(x_1 \cos \theta + x_2 \sin \theta - t) dx_1 dx_2$$

Where δ is the Dirac distribution. The ridgelet coefficients $Rf(a, b, \theta)$ of an object f are given by analysis of the Radon transform via

$$\mathcal{R}_f(a, b, \theta) = \int Rf(\theta, t) a^{-1/2} \psi((t - b)/a) dt.$$

Hence, the ridgelet transform [2] is precisely the application of a one-dimensional (1-D) wavelet transform to the slices of the Radon transform where the angular variable θ is constant and t is varying.

2.3 Discrete Curvelet Transform of Continuum Function

Basically, curvelet transform extends the ridgelet transform to multiple scale analysis.

Therefore, we start from the definition of ridgelet transform. Given an image the continuous ridgelet coefficients are expressed as [19]:

$$f(a, b, \theta) = \iint \psi_{a, b, \theta}(x, y) f(x, y) dx dy$$

Here, a is the scale parameter where $a > 0$, $b \in \mathbb{R}$ is the translation parameter And $\theta \in [0, 2\pi)$

A ridgelet can be defined as [19]:

$$a, b, \theta(x, y) = a^{-\frac{1}{2}} \psi\left(\frac{x \cos \theta + y \sin \theta - b}{a}\right)$$

where θ is the orientation of the ridgelet. Ridgelet are constant along the lines.

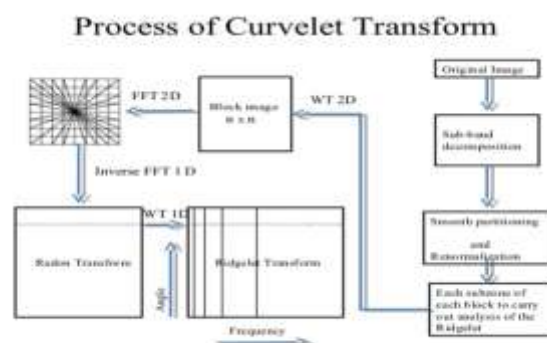


Figure 2 Process of Curvelet Transform

III. DISCRETE WAVELET TRANSFORM

Wavelet transform is introduced with the advancement in multiresolution transform research. Discrete wavelet transform is one of the most promising multiresolution approaches used in CBIR. It has the advantage of a time-frequency representation of signals where Fourier transform is only frequency localized. The location, at which a frequency component of an image exists, is important as it draws the discrimination line.

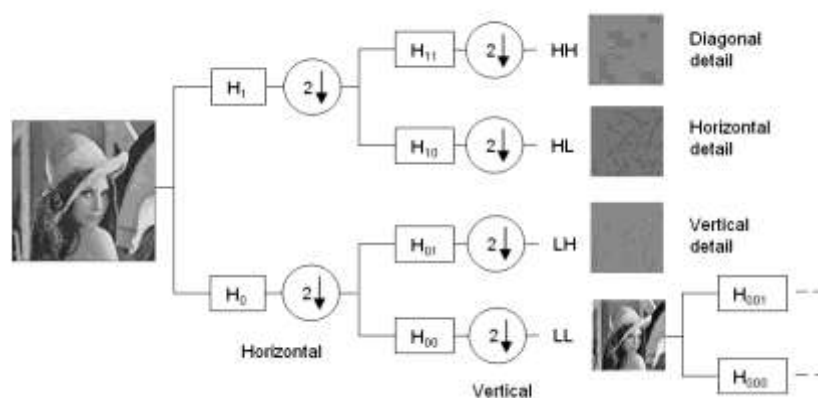


Fig 3 Decomposition of Image .

Unlike the FT and STFT, the window size varies at each resolution level when the wavelet transform is applied to an image. In discrete wavelet transform, the original image is high-pass filtered yielding three detail images, describing the local changes in horizontal, vertical and diagonal direction of the original image. The image is

then low- pass filtered yielding an approximation image which is again filtered in the same manner to generate high and low frequency subbands at the next lower resolution level This process is continued until the whole image is processed or a level is determined as the lowest to stop decomposition. This continuing decomposition process is known as down sampling and shown in Fig. 3

IV. PERFORMANCE CALCULATION

In this paper, we presented a strategy for denoising a noisy image by the wiener and the curvelet transforms. The noise level in an image can be estimated by calculating the PSNR value. Therefore, performance can be measured by comparing the PSNR values of the noisy, curvelet and the proposed algorithm.

- a. To study all the methods of the Denoising.
- b. Attenuate the color frequencies using Gabor Filter and Wiener Filter
- c. Applying the Curvelet Transform.
- d. Compare the image quality using PSNR Tool Proposed algorithm steps:
- e. Step I: Take noisy image.
- f. Step II: Applying Curvelet Transform as under:
 - (i) Sub Band Decomposition
 - (ii) Smooth Partitioning
 - (iii) Renormalization
 - (iv) Ridgelet Analysis
- g. Step III: In Sub Band Decomposition
 - (i) Divide image into resolution layers
 - (ii) Each layer contains details of different frequencies.
 - (iii) These frequencies are attenuates and approximate with the help of Gabor Filter.
- h. Step VI: For Reconstruction inverse of curvelet transform is performed.
- i. Step V: Output is the final Denoised image.

V. RESULT AND DISCUSSION

We test our algorithm with Peppers image. The tested images are corrupted with Gaussian white noise. Table1 shows the simulation results in terms of PSNR measure, for standard pepper image. As in Table 1, our proposed scheme provided the best PSNR value for the Peppers image at various noise standard deviations. However, curvelet transform has given considerably low value. PSNR values of proposed algorithm along with curvelet transform output and noisy image. We observed that the PSNR value of our proposed method is higher compare to curvelet transform and it works well for all the level of noises.

Table 1 for Parameter Calculation

S. No	Parameters	Values
	Noise	PSNR
1	Gaussian	89.0425
2	Salt and Peeper	98.6528
3	Speckle	93.6640



Noise with Gaussian



Output figure



Noise with. Salt and Pepper



Output figure



Noise with Speckle



Output figure

Fig.4 Images with different noises and relative output image

REFERENCES

- [1] F. Long, H. J. Zhang, and D. D. Feng, "Fundamentals of Content-based Image Retrieval," in Multimedia Information Retrieval and Management, D. Feng Eds, Springer, 2003.
- [2] accessed <http://au.images.search.yahoo.com/> on 09 November, 2008.
- [3] "Content-based image retrieval," <http://en.wikipedia.org/wiki/CBIR>.
- [4] H. Tamura, S. Mori, and T. Yamawaki, "Texture Features Corresponding to Visual Perception," IEEE Trans. on Systems, Man and Cybernetics, vol. 8(6), pp. 460-473, 1978.
- [5] Y. L. Huang, "A Fast Method for Textural Analysis of DCT-Based Image," Journal of Information Science and Engineering, vol. 21(1), pp. 181-194, 2005.
- [6] L. Chen, G. Lu, and D. S. Zhang, "Effects of Different Gabor Filter Parameters on Image Retrieval by Texture," in Proc. of IEEE 10th International Conference on Multi-Media Modelling, Australia, 2004, pp. 273-278.
- [7] B. S. Manjunath and W. Y. Ma, "Texture Features for Browsing and Retrieval of Image Data," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 18(8), pp. 837-842, 1996.
- [8] B. S. Manjunath et al, "Introduction to MPEG-7," John Wiley & Son Ltd., 2002.
- [9] J. Z. Wang, J. Li, and G. Wiederhold, "SIMPLiCity: Semantics-sensitive Integrated Matching for Picture

- Libraries," IEEE Trans. Pattern and Machine Intelligence, vol. 23(9), pp. 947-963, 2001.
- [10] S. Bhagavathy and K. Chhabra, "A Wavelet-based Image Retrieval System," in Technical Report—ECE278A: Vision Research Laboratory, University of California, Santa Barbara, 2007.
- [11] N. Suematsu, Y. Ishida, A. Hayashi, and T. Kanbara, "Region-Based Image Retrieval using Wavelet Transform," in Proc. 15th International Conf. on Vision Interface, Calgary, Canada, May, 2002, pp. 9-16.
- [12] E. J. Candès and D. L. Donoho, "Curvelets - a surprisingly effective nonadaptive representation for objects with edges," in Curve and Surface Fitting: Saint-Malo, A. Cohen, C. Rabut, and L. L. Schumaker, Eds. Nashville, TN: Vanderbilt University Press, 1999.
- [13] J.-L. Starck and M. J. Fadili, "Numerical Issues When Using Wavelets," Encyclopedia of Complexity and System Science, in press.
- [14] J. G. Daugman, "Complete Discrete 2-D Gabor Transforms by Neural Networks for Image Analysis and Compression," IEEE Trans. Acoust. Speech, Signal Processing, vol. 36(7), pp. 1169-1179, July, 1988.
- [15] D. Zhang, A. Wong, M. Indrawan, and G. Lu, "Content-based Image Retrieval Using Gabor Texture Features," in Proc. of First IEEE Pacific-Rim Conference on Multimedia (PCM'00), Sydney, Australia, December 13-15, 2000, pp. 1139-1142.
- [16] W.-Y. Ma and H. Zhang, "Benchmarking of Image Features for Content-based Retrieval," in Signals, Systems & Computers, 1998. Conference Record of the Thirty-Second Asilomar Conference, Pacific Grove, CA, USA., 1998, pp. 253-257.
- [17] M. J. Fadili and J.-L. Starck, "Curvelets and Ridgelets," Encyclopedia of Complexity and System Science, in press., 2007.
- [18] E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying, "Fast Discrete Curvelet Transforms," Multiscale Modeling and Simulation, vol. 5, pp. 861-899, 2005.
- [19] J.-L. Starck, E. J. Candès, and D. L. Donoho, "The Curvelet Transform for Image Denoising," IEEE Transactions on Image Processing, vol. 11(6), pp. 670-684, 2002.
- [20] A. Majumdar, "Bangla Basic Character Recognition Using Digital Curvelet Transform," Journal of Pattern Recognition Research, vol. 1, pp. 17-26, 2007.
- [21] G. Joutel et al., "Curvelets Based Feature Extraction of handwritten shapes for ancient manuscripts classification," in Proc. of SPIE-IS&T Electronic Imaging, SPIE, 2007.
- [22] <http://www.corel.com>.
- [23] P. Brodatz, "Textures: A Photographic Album for Artists and Designers," Dover Publications, 1966.
- [24] J. Zhang and T. Tan, "Brief Review of Invariant Texture Analysis Methods," Pattern Recognition, vol. 35, pp. 735-747, 2002.
- [25] B. S. Manjunath, P. Wu, S. Newsam, and H. D. Shin, "A Texture Descriptor for Browsing and Similarity Retrieval," Signal Processing: Image Communication, vol. 16(1-2), pp. 33-43, Sep. 2000.
- [26] W. K. Leow and S. Y. Lai, "Scale and Orientation-Invariant Texture Matching for Image Retrieval," in Texture Analysis in Machine Vision, M. K. Pietikainen, Ed.: World Scientific, 2000, pp. 151-164.
- [27] C.-Y. Chiu, H.-C. Lin, and S.-N. Yang, "Texture Retrieval with Linguistic Descriptors," in Proc. of IEEE Pacific Rim Conf. on Multimedia, 2001, pp. 308-315.
- [28] W. Y. Ma and B. S. Manjunath, "A Comparison of Wavelet Transform Features for Texture Image Annotation," in IEEE International Conference on Image Processing (ICIP), 1995, pp. 256-259.

- [29] K. L. Lee and L. H. Chen, "A New Method for Coarse Classification of Textures and Class Weight Estimation for Texture Retrieval," *Pattern Recognition and Image Analysis*, vol. 12, No. 4, pp. 400–410, 2002.
- [30] Y. Li, M. J. Kyan, and L. Guan, "Improving Shape-Based CBIR for Natural Image Content Using a Modified GFD," in *2nd International Conference on Image Analysis and Recognition*, Toronto, Canada, 2005, pp. 593-600.
- [31] W. Y. Ma and B. S. Manjunath, "Netra: A toolbox for navigating large image databases," *Multimedia Systems*, vol. 7, no. 3, pp. 184-198, 1999.
- [32] J. R. Smith and S.-F. Chang, "VisualSEEK: A fully automated content-based image query system," in *Proc. of ACM Multimedia*, 1996.
- [33] Y. Song, W. Wang, and A. Zhang, "Automatic Annotation and Retrieval of Images." vol. 6, no.2: *World Wide Web: Internet and Web Information Systems*, 2003, pp. 209-231.
- [34] Z. Lu, S. Li, and H. Burkhardt, "A Content-Based Image Retrieval Scheme in JPEG Compressed Domain," *International Journal of Innovative Computing, Information and Control*, vol. 2(4), pp. 831-839, 2006.
- [35] Y. Rui, T. S. Huang, and S. F. Chang, "Image Retrieval: Current Techniques, Promising Directions, and Open Issues," *Journal of Visual Communication and Image Representation*, vol. 10(4), pp. 39-62, 1999.
- [36] P. S. Hiremath and J. Pujari, "Content Based Image Retrieval using Color, Texture and Shape features," in *15th International Conference on Advanced Computing and Communications (ADCOM)*, 2007, pp. 780-784.
- [37] Z. Xue, S. Antani, L. R. Long, J. Jeronimo, and G. R. Thoma, "Investigating CBIR Techniques for Cervicographic Images," in *Proceedings of the 2007 Annual Symposium of the American Medical Information Association (AMIA)*, Chicago, IL, 2007, pp. 826-830.
- [38] Z. Katol, T.-C. Pong, and S. G. Qiang, "Unsupervised Segmentation of Color Textured Images Using a Multi-layer MRF Model," in *Proceedings of International Conference on Image Processing (ICIP)*, September 2003, pp. 961-964.
- [39] J. D. Foley, A. V. Dam, S. K. Feiner, and J. F. Hughes, "Computer graphics: principles and practice," 2nd ed, M. Reading, Addison-Wesley, Ed., 1990.
- [40] A. K. Jain, "Fundamentals of Digital Image Processing," E. Cliffs, Ed.: prenticeHall, 1989.
- [41] H. Kauppinen, T. Seppanen, and M. Pietikainen, "An Experimental Comparison of Autoregressive and Fourier-Based Descriptors in 2D Shape Classification," *IEEE Transaction on Pattern Analysis and Machine Intelligence*, vol. 17, no. 2,, pp. 201-207, 1995
- [42] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 11, no. 7, pp. 674-693, July 1989.
- [43] A. K. Jain and F. Farroknia, "Unsupervised texture segmentation using Gabor filters," *Pattern Recognition*, vol. 24, No.12, pp. 1167-1186, 1991.
- [44] J. Mao and A. K. Jain, "Texture Classification and Segmentation Using Multiresolution Simultaneous Autoregressive Models," *Pattern Recognition*, vol. 25, No. 2, pp. 173-188, 1992.