ANALYSIS TO OBSERVE THE EFFECTS OF HEAT TRANSFER ON OSCILLATORY BLOOD FLOW IN A CONSTRICTED TUBE

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ABSTRACT

The effect of heat transfer on the motion of blood, in a constricted artery has been modeled along with the assumption of an optically thin fluid. The analytical results are obtained for the oscillatory flow of blood which behaves as a Newtonian fluid. It is assumed here that the surface of roughness is cosine-shaped and the maximum height of the roughness is very small compared with the radius of the unconstructed tube. The study shows that, in addition to the constriction of the blood vessel and the effect of the magnetic field, the heat transfer also affects the blood flow in the cardiovascular system. This effect is noticeable in the flow velocity and the temperature distributions. Finally the effects of heat transfer on the instantaneous flow rate, temperature distribution and wall shear stress is computed for the values of radiation parameter, magnetic parameter and peclet numbers.

I. INTRODUCTION

On the leading causes of deaths in the world, is due to heart diseases and the most commonly heard names among the same are ischemia, atherosclerosis and angina pectoris. Ischemia is the deficiency of the oxygen in a part of the body, sally temporary. Which is due to the constriction (stenosis) or obstruction in the blood vessel supplying that part. Atherosclerosis is a type of arteriosclerosis. It comes from the Greek words athero (meaning grue or paste) and sclerosis (hardness). It involves deposits of fatty substances, cholesterol, cellular waste products, calcium and fibrin (a clothing material in the blood) in the inner lining of an artery. The build up that results is called plague. Plague may partially or totally block the blood flow through an artery. Two things that can happen where plague occurs are (i) bleeding (hemorrhage) into the plague and (ii) formation of the blood clot (thrombus) on the plague’s surface. If either of these occurs and blocks the entire artery, a heart attack, or stroke may result. Atherosclerosis affects large and medium sized arteries. The type of artery where the plague develops varies with each person. A system complex of ischemic heart disease characterized by paroxysmal attacks of chest pain, usually sub sternal or pre-cordial is referred as angina pectoris. Coronary usually high-grade stenosis with acute changes results in sudden cardiac arrest (or death) which strikes 300000-4000000 persons annually around the globe. Owing to its serious concerns, a major research work is being done over all parts of the world for early detection and prevention from being affected by the cardiac attack and the art therapies for the diagnosis of the heart diseases. The study of blood flow in the cardiovascular system has generated a lot of interest hence the huge amount of literature of the subject. The mathematical model of blood flow in arteries with stenosis was presented by Misra and Chakravarty 1986. They modeled the artery as an initially stressed orthotropic elastic tube filled with a viscous incompressible fluid incorporating the effect of the surrounding connective tissues on the motion. Another model of blood flow was discussed by Haldar (1987). Where we considered the oscillatory but laminar flow of blood, which the study assumed to be Newtonian in character, through an artery with a mild constriction (stenosis). The study showed that, for a constant value of the oscillation frequency parameter, the wall shear stress increased with increasing stenosis height. Haldar and Ghosh (1994) studied the effect of a magnetic field and the presence of electrolytes on the flow field in an artery with a mild constriction. Blood circulation is considered to play an important role in heat transfer between living tissues, particularly, in peripheral, vessels where the temperature is, generally, closely related with blood flow rate. Blood flow characteristics through an artery in the presence of multi-stenosis is developed by Chakravarty and Sannigrahi (1998). The study has been carried out to propose the unsteady flow characteristics of blood as Newtonian fluid in a multi-stenosesd distensible artery constrained with pulsatile pressure gradient and variable viscosity when subjected to body acceleration. The result showed that the flow resistance due to stenosis was also reduced by increasing the permeability of the blood vessel. Anderson used computational fluid dynamics to simulate the blood flow in human vein with the effect of surface irregularities on flow conditions. The previous work has made comparisons of three different stenosis models which are cosine shape, irregular shape and smooth shape geometry of the vein wall. A computational model of blood flow in curved arteries with varying angles and different degrees of occlusion has been investigated by Yao and Ang. Several other workers, Takuji (1998) and Elshahed (2003) to mention but a few, have in one way or the other modeled and studied the flow of blood through a rigid tube under the influence of a pulsatile pressure gradient. In this study, we present a simple model for the flow of blood in an artery with a mild stenosis in the presence of a uniform magnetic field incorporating radiative heat transfer and a pulsatile pressure gradient. Amos and Oguly 2003 conducted a similar study in which they obtained numerical results for the stream function and the vorticity using the Galerkin technique of the finite element method, and found that at high values of the magnetic field there was considerable flattening of the shear stress profile.

In this analysis a quantitative discussion of the consequences of the numerical results obtained along with the, heat transfer and fluid flow characteristics of blood in multi-stenosed arteries with the effect of magnetic field are studied. It is assumed that the arterial segment to be a rigid cylindrical tube with stenosis and blood flowing through it to be Newtonian with constant viscosity. The effect of Hartmann number on shear wall as wall as Nusselt Number is investigated. In addition, the effect of degree of stenosis on flow and heat characteristics is studied.

II. PROBLEM FORMULATION

In this model we consider an oscillatory but laminar and axially symmetric flow of blood in a heated uniform cylindrical blood vessel in the presence of a uniformly applied magnetic field M and a mild constriction. The blood is modeled as a Newtonian fluid and the blood vessel is heated externally with the condition that the wall of the blood vessel is maintained at a temperature T^*. Here ρ and μ are the density and viscosity of the fluid which are assumed to be constant. It is also assumed that the constriction develops symmetrically due to some abnormal growth in the lumen of the artery. The idealized geometry of stenosis is given by:

$$\frac{R(x)}{R_o}=1-\frac{\delta}{2R_o}\left(1+\cos\frac{\pi x}{d}\right) \quad (1)$$

where *R(x)* is the radius of the artery in the stenotic region, R_o is the radius of the normal artery, *2d* the length of the stenosis and δ is the maximum height of the stenosis. Here we assuming a Boussinesq incompressible fluid model, the appropriate equations governing the motion following Bird al.et. are :

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\sigma B^2 u+g\alpha\left(T-T^*\right) \quad (2)$$

$$\rho C_p\frac{\partial T}{\partial t}=K\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)-\frac{\partial w}{\partial r} \quad (3)$$

$$\frac{\partial^2 w}{\partial r^2}-3\beta^2 w-16\beta S T^2\frac{\partial T}{\partial r}=0 \quad (4)$$

where *u* is the velocity in the axial direction, *p* is the fluid pressure, C_p is the specific heat at constant pressure, *K* is the thermal conductivities, *S* is steffan- Boltzmann constant, *g* is the gravitational field vector, *w* the radiative heat transfer vector. Simplifies equation (1-4) according to the appropriate boundary conditions which are as;

$u = 0$, $T=T_w$ on $r = R$ no slip at the wall (5a)

$\frac{\partial u}{\partial r}=0$, $T=T_\infty$ on $r = 0$ (5b)

III. SOLUTION OF THE PROBLEM

The solution of the oscillatory motion of a viscous fluid will be obtained under a pressure gradient which varies with time. In this section we assume blood is an optically thin fluid, its density and $\beta \le 1$ then we can simplified equation (4) thus we get,

$$\frac{\partial w}{\partial r}=4\beta^2\left(T-T_\infty\right) \quad (6)$$

where

$$\beta^2=\int_0^\infty \varepsilon\lambda\frac{\partial B}{\partial T} \quad (7)$$

where *B* is planks constant; introducing the following non- dimensional parameters:

$$u=\frac{w}{w_0}\qquad \lambda=\frac{R_0}{w_0}\qquad \beta^2=\frac{\rho R_0}{\mu}\qquad y=\frac{r}{R_0}\qquad t^*=\frac{t}{w_0}$$

$$N^2=\frac{R_0^2\beta}{K_0}\qquad M^2=\frac{R_0^2\sigma B_0^2}{\mu}\qquad p^*=\frac{p-p_\infty}{\mu}\qquad \theta=\frac{T-T_\infty}{T_w-T_\infty}$$

$$P_e=\frac{\rho R_0^2 C_p}{K_0}\qquad \varpi=\frac{\omega R_0^2}{\omega_0}\qquad G_r=g\alpha\frac{T_\infty R_0^2}{\mu w_0}\qquad x^*=\frac{x}{R_0}$$

Using above non-dimensional parameters and substituting equation (6) and equation (7) into equation (2) and equation (3), we get;

$$\beta^2\frac{\partial u}{\partial t}=-\lambda\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{y}\frac{\partial u}{\partial y}-M^2 u+G_r\theta \quad (8)$$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{y}\frac{\partial \theta}{\partial y} + N^2 \theta \quad (9)$$

and the boundary conditions:

$$u = 0 \qquad \theta = \theta_w \quad on \quad y = \frac{R}{R_o} \quad (10.a)$$

$$\frac{\partial u}{\partial y} = 0 \qquad \theta = 1 \quad on \quad y = 0 \quad (10.b)$$

Using equation (10.a, 10.b) and the solution for *u* and *p* be set in the forms:

$$u(y,t) = u_o(y)\, e^{i\omega t} \quad (11)$$

$$-\frac{\partial p}{\partial x} = p e^{i\omega t} \quad (12)$$

$$\theta(y,t) = \theta_o(y)\, e^{i\omega t} \quad (13)$$

Solving equation (.8) and equation (9) by using equation (10) and equation (11) then we have;

$$u_0 = \frac{\alpha_1 G_r}{\Omega^2 - \alpha_1^2}\left\{\frac{J_o(\alpha_1 y)}{J_o\left(\alpha_1 \frac{R}{R_0}\right)} - \frac{J_o(\Omega y)}{J_o\left(\Omega \frac{R}{R_o}\right)}\right\} + \frac{\lambda p_o}{\alpha_1}\left\{\frac{J_o(\alpha_1 y)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)}\right\} \quad (14)$$

$$\theta = \theta_w \frac{J_o(\Omega y)}{J_o\left(\Omega \frac{R}{R_o}\right)} \quad (15)$$

Where $\Omega^2 = \left(N^2 - i\omega P_e\right), \quad \alpha_1^2 = \left(M^2 - i\omega\beta^2\right)$

$$u(y,t) = \left\{\frac{\alpha_1 G_r}{\left(\Omega^2 - \alpha_1^2\right)}\left(\frac{J_o(\alpha_1 y)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)} - \frac{J_o(\Omega y)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)}\right) + \frac{\lambda p_o}{\alpha_1^2}\left(\frac{J_o(\alpha_1 y)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)}\right)\right\} e^{i\omega t}$$

(16)

J_o is the Bessel function of order zero with complex argument. Then the resulting expression for the axial velocity in the tube is given by (16) and the temperature is given by

$$\theta = \theta_w \frac{J_o(\Omega y)}{J_o\left(\Omega \frac{R}{R_o}\right)} e^{i\omega t} \quad (17)$$

The volumetric flow rate Q is given by :

$$Q = 2\pi \int_0^{\frac{R}{R_0}} u\, y\, dy \quad (18)$$

Substitute equation (14) into equation (16) then we get;

$$Q = \left\{ \frac{2\pi\alpha_1 G_r}{(\Omega^2 - \alpha_1^2)} \left(\frac{R}{R_o}\right) \left(\left(\frac{R_o}{R}\right) \frac{J_1\left(\alpha_1 \frac{R}{R_o}\right)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)} - \frac{J_1\left(\Omega \frac{R}{R_o}\right)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)} \right) + \frac{\lambda p_o (\Omega^2 - \alpha_1^2)}{G_r \alpha_1^2} \left(\frac{J_o(\alpha_1 y)}{J_o\left(\alpha_1 \frac{R}{R_o}\right)} \right) \right\} e^{i\omega t} \tag{19}$$

The rate of heat transfer at the wall of the blood vessel as;

$$\frac{\partial \theta}{\partial t} = \theta_w \Omega \frac{J_1\left(\Omega \frac{R}{R_o}\right)}{J_o\left(\Omega \frac{R}{R_o}\right)} e^{i\omega t} \qquad when\ y = \frac{R}{R_o} \tag{20}$$

The shear stress at the wall $y = \frac{R}{R_o}$ is defined by;

$$\tau = \mu \frac{\partial u}{\partial y} \tag{21}$$

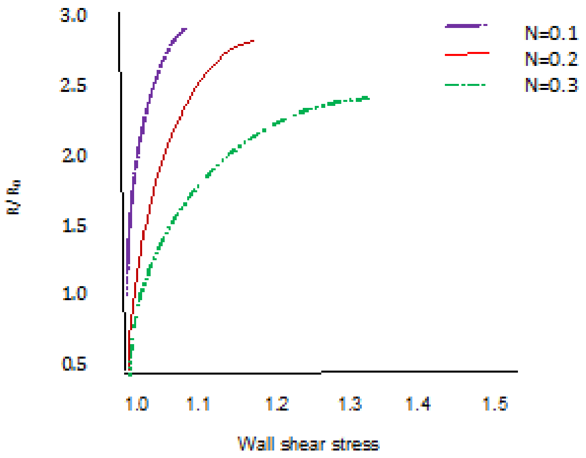

Fig. (a) Effect of Radiation Parameter on the Wall Shear Stress

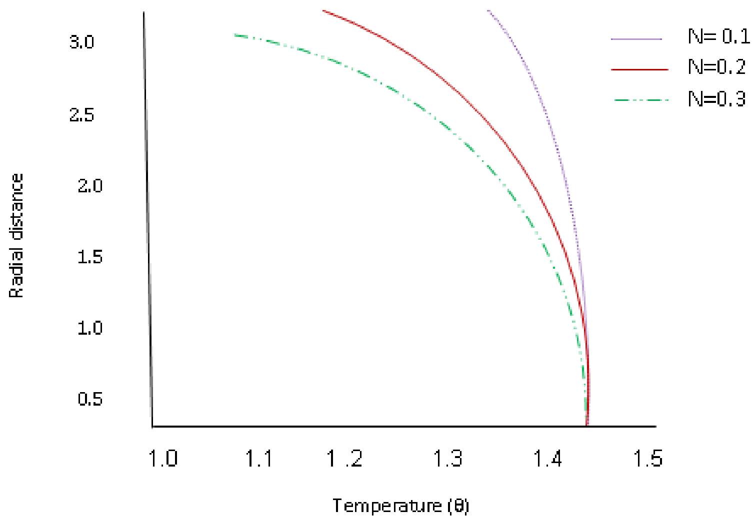

Fig. (b) Variation of Radial Distance on the Temperature Distribution with Radiation Parameter

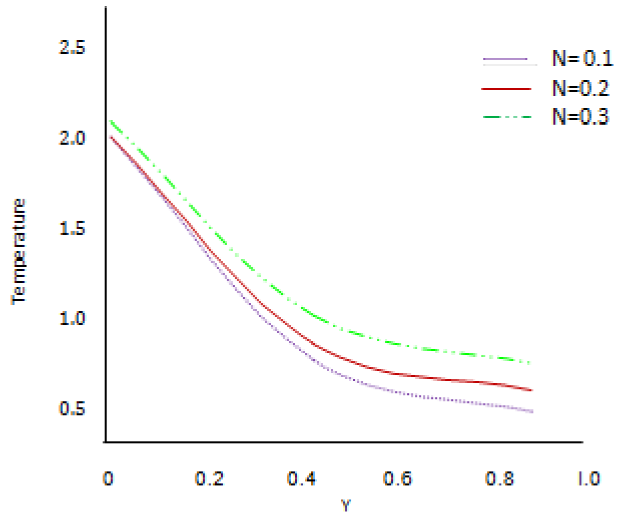

Fig. (C) Variation of Radiation Parameter on the Temperature Distribution

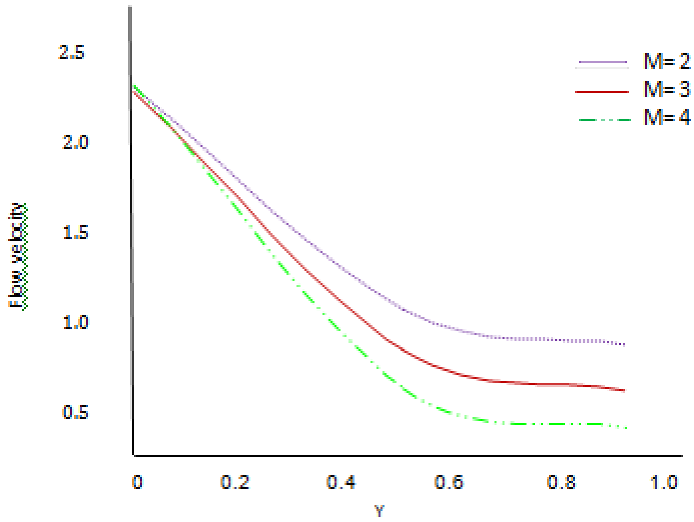

Fig. (D) Variation of Velocity Distribution with Magnetic Effect

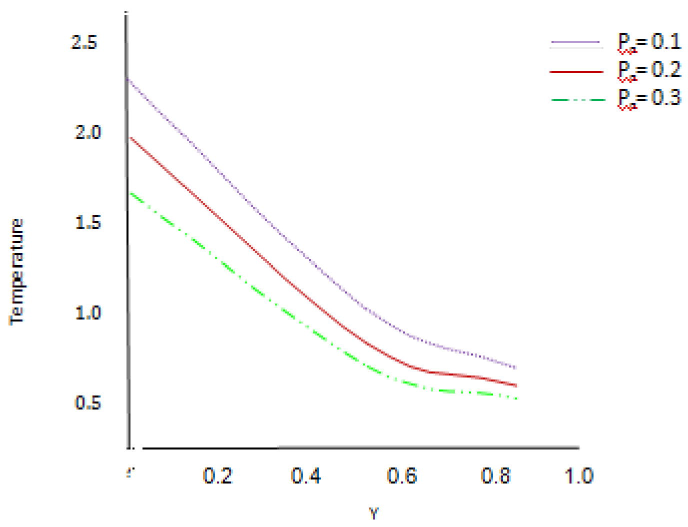

Fig. (E) Variation of Temperature Distribution with Different Peclet Number

IV. RESULTS AND DISCUSSION

This is much circumstantial evidence suggesting, that heat transport plays an important role in atherogensis. In this study we have analyzed the effect of heat transfer on the blood flow in an artery with mild constriction. Our analysis shows that the flow velocity and temperature distribution are modified by the radiation parameter and magnetic parameter. Fig.(a) shows the effect of radiation parameter on the wall shear stress. We observed that the shear stress at vessel wall is higher than that along the axis of the vessel. Fig. (b) shows the temperature distribution along the radiation parameter. We analyzed that the temperature decreases as the radiation parameter increases. We observed also that for any value of the radiation parameter the temperature at the vessel wall is lower than that along the axis. Fig. (c) shows the temperature distribution where we observe that the temperature decreases as the radiation parameter increases. Here we analyzed that for any value of the radiation parameter the temperature at the vessel is lower than that along the axis. Fig.(b) and Fig.(c) depicts how the shear stress τ is affected by the radiation parameter N,. In both cases we observe that the shear stress at the wall is higher than that along the axis of the vessel. Fig(d) shows the variation of flow velocity with y and for different values of magnetic parameter. It shows that the flow velocity increases as magnetic parameter decreases. Fig.(e) shows the variation of temperature distribution for different values of Peclet number. It shows that for a fixed value of y, the temperature decreases as the value of Peclet number increases. We analyzed the heat transfer in a model of blood flow in the cardiovascular system. Analytical solutions are obtained for the volumetric flow rate, rate of heat transfer and shear stress at the vessel wall. The study shows that in addition to the constriction of the blood vessel and the effect of the magnetic field, the heat transfer also affects the blood flow in the cardiovascular system.

REFERENCES

[1]. Mishra, J.C. and Chakravarty, S. (1986): “Flow in arteries in the presence of stenosis”, Journal of Biomechanics, vol.19, pp. 907-918.

[2]. Chakravarty, S. (1987): “Effects of stenosis on the flow behaviour of blood in an artery”, International Journal Engineering Science, vol.25, pp.1003-1016.

[3]. Haldar, K. (1987): “Oscillatory flow of blood in a stenosed artery”, Bulletin of Mathematical Biology, vol.49 (13), pp. 279-287.

[4]. Haldar, K. and Ghosh, S.N. (1994): “Effect of a magnetic field on blood flow through an indented tube in the presence of erythrocytes”, Indian Journal of Pure and Applied Mathematics, vol.25 (3), pp.345-352.

[5]. Amos, E. and Ogulu, A. (2003): “Magnetic effect on pulsatile flow in a constricted axis-symmetric”, International Journal of pure and applied mathematics, vol.34, pp.1315-1326.

[6]. Elshahed, M. (2004): “Blood flow in capillary under starling hypothesis”, Journal of Applied Mathematics and Computation, vol.149, pp. 431-439.

[7]. Ogulu, A. and Abbey, T.M. (2005): “Simulation of heat transfer on an oscillatory blood flow in an indented porous artery”, Journal of International Communication in Heat and Mass Transfer, vol.32, pp.983-989.

[8]. Chakravarty, S. and Sen, S. (2005): "Dynamic response of heat and mass transfer in blood flow through stenosed bifurcated arteries", Journal of Korea-Australia Rheology, vol. 17, no. 2, pp. 47-62.

[9]. Bali, R. and Awasthi, U. (2007): "Effect of a magnetic field on the resistance to blood flow through stenotic artery", Journal of Applied Mathematics and Computation, vol.188, pp.1635-1641.

[10]. Bourhan, T. and Ahmad, M. (2008): "Magnetic field on heat transfer and fluid flow characteristics of blood flow in multi-stenosis arteries", International Journal of Heat and Mass Transfer, vol.44, pp.297-304.

[11]. Sanjeev Kumar, Subhash Chandra (2009): Heat Transfer and Fluid Flow Characteristic of Blood Flow in Multi-Stenosis Artery with Effect of Magnetic Field. Indian Journal of Biomechanics (NCBN-Special issue.)

[12]. Alana R. Sankar1,(2013): "Two-layered Blood Flow through a Composite Stenosis in the Presence of a Magnetic Field" International Journal of Application or Innovation in Engineering & Management,vol.2, pp.30-41.