

DESIGN OF LOW PASS DIGITAL FIR FILTER USING DIFFERENT MUTATION STRATEGIES OF DIFFERENTIAL EVOLUTION

Sukhdeep Kaur Sohi¹, Balraj Singh Sidhu²

^{1,2}Department of Electronics and Engineering,

Giani Zail Singh Punjab Technical University Campus Bathinda(India)

ABSTRACT

This paper builds up a technique for the design of low pass FIR filter by exploring different mutation strategies of Differential Evolution (DE) technique. DE algorithm is population based just like Genetic Algorithm (GA), but differs in crossover and mutation. Five different mutation strategies have been explored, out of these five mutation strategies the best mutation strategy has been used for the design of Low Pass FIR digital filter. DE parameters have been varied to check the robustness of the designed filter. Finally, standard deviation has been calculated to check the robustness of the designed filter.

Index Term-Digital FIR filters, Differential Evolution Algorithm, Objective Function.

I. INTRODUCTION

Anything that transmits information is known as signal. Numerical manipulation of signals in discrete time is known as “Digital Signal Processing (DSP)”. Features like programmable operations, high speed and flexibility leads to increase in attractiveness of DSP. Filters are described as frequency choosy components permitting a particular frequency range to pass [19] [21] Filters are classified as Analog filters and Digital filters. Filters serve the following purposes:

- (a) Partition of signals that have been combined.
- (b) Restoration of imprecise signals.

Analog filters can also perform the above mentioned tasks. But digital filter furnishes more accuracy and give superior results [17]. Digital Filters based on their impulse response are classified as Finite Impulse Response (FIR) and Infinite Impulse Response (IIR). FIR filters have following advantages over IIR:

- (a) FIR does not require feedback from output, hence known as non-recursive filters. Where in IIR filter consists of feedback, thus given the name of recursive filters.
- (b) FIR filters have linear phase, so are easily controlled by user. On the other hand IIR filters have no particular phase, hence are hard to control.
- (c) FIR filters are stable by nature. IIR filters are unstable.
- (d) FIR filter output depends only on input and IIR filter output is combination of both input and output.

FIR filters are useful in applications where linear phase is needed [9]. FIR filters could be designed using window method, frequency sampling method and optimal filter design method.

Window method is a simple method which includes well defined equations for computing window coefficients. But this method does not provide sufficient flexibility. Frequency Sampling method allow the user to design FIR filter by choosing any magnitude response. However the obtained frequency response matches the desire frequency response only at sampled values [20].

Optimization algorithms are meant to minimize or maximize a particular task [18]. Optimization techniques are categorized as direct search method, gradient based method and nature inspired method [15].

Gradient methods suffer from the limitation that it gets easily trapped in local minima of error surface. To emerge from these limitations focus came on nature inspired methods. Nature inspired methods include Genetic Algorithm (GA), Particle Swarm Optimization (PSO), evolutionary methods, Tabu Search, Predator Prey Optimization (PPO), Differential Evolution (DE)[12]. Although GA achieve better results but it encounters problems of slow convergence and may sometimes trap in local optima[5].

PSO is simple, easy to implement and computationally fast. But if initial parameters of PSO are not chosen correctly it results into local minima. PPO has an additional predator to improve the performance [13]. DE is a simple, powerful, population based technique [12].

This paper purposes an efficient, "Differential Evolution algorithm" for the design of low-pass FIR digital filter. This paper is organized as follows. Section II focuses on the FIR filter design problem statement. Differential Evolution algorithm has been elaborated in Section III. Performance of the proposed algorithm has been illustrated in Section IV. Conclusion has been drawn in Section V.

II. FIR FILTER DESIGN PROBLEM

FIR filters has an impulse response that is zero outside some finite interval [19]. The structure of FIR filters is made up of delay elements, adders and multipliers [10]. FIR filters are expressed by a difference equation:

$$x(n) = \sum_{k=0}^{M-1} a_k y(n-k) \quad (1)$$

where $x(n)$ is output sequence, a_k are input coefficients, $y(n)$ is output sequence, M is the order of filter [19] Transfer function of FIR filter is specified as by equation (2): [5]

$$H(z) = \sum_{k=0}^{M-1} a_k z^{-k} \quad (2)$$

The unit sample response of FIR system is equivalent to the coefficients a_k as described in equation (3):

$$h(n) = \begin{cases} a_n & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Output sequence $x(n)$, can be expressed as convolution of unit impulse response $h(n)$ of the system with its input signal:

$$x(n) = \sum_{k=0}^{M-1} h(k) y(n-k) \quad (4)$$

FIR filters have symmetric and anti-symmetric properties as illustrated in equation (5a), (5b)

$$h(n) = h(N-1-M) \quad \text{for even} \quad (5.a)$$

$$h(n) = -h(N-1-n) \quad \text{for odd} \quad (5.b)$$

For such a system number of multiplications is decreased from M to $M/2$ for M even and to $(M-1)/2$ for odd

Magnitude errors $e_1(x)$ represents the absolute error of L_1 - norm of magnitude response is illustrated in equation (7) and $e_2(x)$ the squared error L_2 - norm of magnitude response is given in equation (8)

$$e_1(x) = \sum_{i=0}^K |H_d(\omega_i) - |H(\omega_i, x)|| \quad (7)$$

$$e_2(x) = \sum_{i=0}^K (|H_d(\omega_i) - |H(\omega_i, x)||) \quad (8)$$

FIR filter's desired magnitude response is given by equation (9) and equation (10):

$$\delta_1(x) = \max\{|H(\omega_i, x)|\} - \min\{|H(\omega_i, x)|\} \quad (9)$$

$$\delta_2(x) = \max\{|H(\omega_i, x)|\} \quad (10)$$

ω_i in equation 8 belongs to pass-band, whereas in equation (9) it belongs to stop-band. The multivariable constraints optimization problem is stated as follows:

$$\text{Minimize } O_1(x) = e_1(x) \quad (11.a)$$

$$\text{Minimize } O_2(x) = e_2(x) \quad (11.b)$$

$$\text{Minimize } O_3(x) = \delta_1(x) \quad (11.c)$$

$$\text{Minimize } O_4(x) = \delta_2(x) \quad (11.d)$$

where O stands for Objective Function[14] . Equation (12) describes the multi-criterion optimization problem :

$$\text{Minimize } O_x = \sum_{i=1}^4 \omega_i O_i(x) \quad (12)$$

Prescribed conditions for Low Pass Filters are depicted in TABLE I.

TABLE I: Design Conditions for Low Pass Filters

Filter Type	Pass-Band	Stop-Band	Maximum value of $ H(\omega, x) $
Low-Pass	$0 \leq \omega \leq 0.2\pi$	$0.3\pi \leq \omega \leq \pi$	1

III. DIFFERENTIAL EVOLUTION

DE is population based optimization technique introduced by Storn and Price [12]. It involves operations like Mutation, Cross-Over (CR) and Selection [1]. Fig.1 shows the basic algorithm for DE

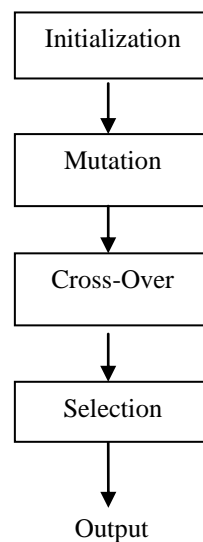


Fig.1: Basic Algorithm of DE

The characteristics of DE such as it doesn't get easily trapped in the problem of local minima, Convergence speed is fast, DE is trouble-free and is easy to put into practice. Involves few control parameters algorithm make it well-liked among other optimization techniques:

DE is almost similar to GA but be at variance in Crossover and Selection procedure [5]. Key parameters selected by user in DE algorithm are size of population (N), Mutation Factor (f_m), Crossover Rate (CR) and stopping decisive factor maximum number of iterations (I_{max}) [5,12]. Let the size of population is N. The parameters vector has the form

$$x_{i,G} = [x_{1,i,G}, x_{2,i,G}, \dots, x_{D,i,G}] \quad (13)$$

$i = 1, 2, 3, \dots, N$, G is generation number [10]. The primary population is selected randomly which covers the entire parameter space [1]

3.1. Mutation

Initially chosen population focuses on a particular segment of entire search space. To find the best possible solution of a problem, this search space has to be expanded. Mutation fulfills the duty of expanding search space. In this new parameter vectors are produced by adding the weighted difference between two population vectors to a third vector. For each target a mutant vector is generated according to, for a given parameter vector $x_{i,G}$ randomly select three vectors $x_{r1,G}$, $x_{r2,G}$ and $x_{r3,G}$ such that indices i , r_1 , r_2 and r_3 are different. Then the weighted difference of two vectors is added to the third as follows:

$$v_{i,G+1} = x_{r1,G} + f_m \cdot (x_{r2,G} - x_{r3,G}) \quad (14)$$

where f_m is mutation factor chosen from $[0, 2]$. $v_{i,G+1}$ is called the donor vector [1]. Different mutation strategies considered for study are enlightened below in Eq. 15 to Eq. 19:

$$MS - 1 \quad v_{ij}^t = x_{r1}^t + f_m (x_{r2}^t - x_{r3}^t) \quad (15)$$

$$MS - 2 \quad v_{ij}^t = x_{Bj}^t + f_m (x_{r1j}^t - x_{r2j}^t) \quad (16)$$

$$MS - 3 \quad v_{ij}^t = x_{ij}^t + f_B (x_{Bj}^t - x_{ij}^t) + f_m (x_{r1j}^t - x_{r2j}^t) \quad (17)$$

$$MS - 4 \quad v_{ij}^t = x_{Bj}^t + f_m (x_{r1j}^t + x_{r2j}^t - x_{r3j}^t - x_{r4j}^t) \quad (18)$$

$$MS - 5 \quad v_{ij}^t = x_{r5j}^t + f_m (x_{r1j}^t + x_{r2j}^t - x_{r3j}^t - x_{r4j}^t) \quad (19)$$

where $(i = 1, 2, \dots, N; j = 1, 2, \dots, D)$

3.2. Crossover

Crossover is done to generate a trial vector. Parameters of donor vector are mixed with parameters of target vector to produce trial vector [1]. Crossover can be mathematically expressed as:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (\text{randb}(j) \leq CR) \text{ or } j = \text{rnbr}(i) \\ x_{ji,G} & \text{if } (\text{randb}(j) > CR) \text{ and } j \neq \text{rnbr}(i) \end{cases} \quad (20)$$

Where $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, D$

$\text{randb}(j)$ is the j th evaluation of a uniform random generator with outcome $\in [0, 1]$. CR is the crossover constant $\in [0, 1]$ which is selected by the user, $\text{rnbr}(i)$ is a randomly chosen index $\in 1, 2, \dots, D$ just to ensure that $u_{i,G+1}$ gets minimum of one parameter from $v_{i,G+1}$. $\text{rnbr}(i)$ ensures that $v_{i,G+1} \neq x_{i,G}$.

3.3. Selection

Finally the decision has to be made that parameters of target vector should be replaced by trial vector or not. This final step of choosing the parameters is done on the basis of minimum value of cost function (could be objective function, magnitude error or any other chosen parameters) and is known as Selection [12].

$$x_{i,G+1} = \begin{cases} u_{i,G+1} & \text{if } f(u_{i,G+1}) < f(x_{i,G}) \\ x_{i,G} & \text{otherwise} \end{cases} \quad (21)$$

Where $i = 1, 2, \dots, N$

3.4. DE algorithm:

1. Specify population size (N), mutation factor (f_m), crossover rate (CR)
2. Specify maximum no. of iterations (Stopping criteria)

3. Generate an array of (N×D) size of uniform random number
4. Calculated objective function is arranged in ascending order and select first half of the population members.
4. Set iteration counter
5. Increment iteration counter
6. Select best member whose objective function is optimum
7. Apply mutation, Crossover and selection procedure as described in equation 15- equation 21
8. If (IT< MAXIT) then go back to step 5
9. If not go back to step 5.
10. Note down the Global Best.
11. Check the maximum no. of runs (MAXRUN), if not go back to step 2.
12. Stop.

IV. RESULTS AND DISCUSSIONS

Low pass FIR digital filter has been designed using the optimization technique, “Differential Evolution (DE)”. Initially DE has been implemented from order 20 to 40 on MS- 2, after selection of the order different mutation strategies has been implemented. To observe the performance of DE algorithm, parameters of best mutant strategy of DE have been varied. The magnitude response and phase response of designed digital FIR filters are also plotted. The results are described below:

4.1. Selection of Order- Out of 5 mutation strategies, MS-2 has been selected randomly. Differential evolution algorithm has been implemented from order 20 to order 40 on MS-2. Objective function value keeps on decreasing but as value of stop-band ripple and pass-band ripple had also to be optimum. Selection of order has been done by observing trend of all three parameters. Fig.2 shows the trend of pass-band ripple and stop-band ripple.

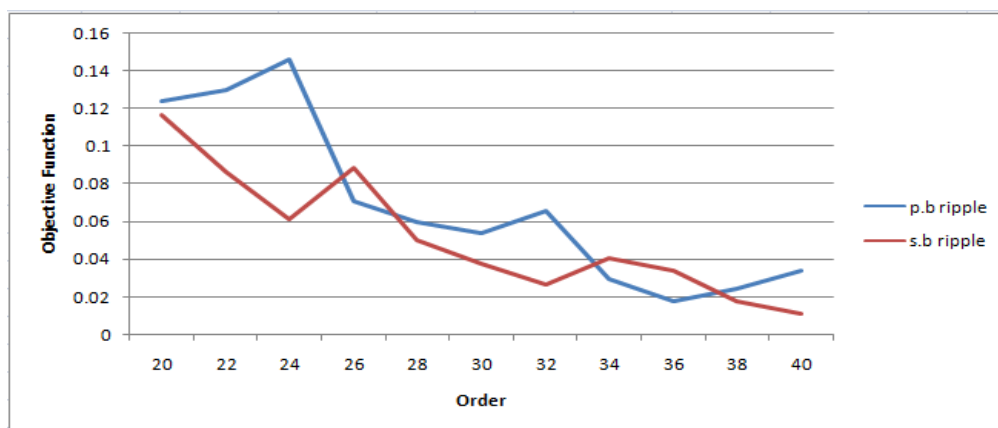


Fig.2: Pass-Band Ripple and Stop-Band Ripple Trend

As depicted in Fig.2 it is evident that at order 38 the values of pass-band ripple and stop-band ripple and objective function value are optimum. So order 38 has been selected for the design of low pass digital FIR filter.

For filter order 38 selected above, all 5 mutation strategies given in Eq. 15 to Eq. 19 have been implemented for the design of Low Pass FIR digital filter and achieved value of objective function have been shown in TABLE II.

TABLE II: Objective Function achieved for different mutation strategies at filter order 38

Sr. No.	Mutation Strategies	Objective Function
1	Mutation Strategy 1	1.026515
2	Mutation Strategy 2	1.026329
3	Mutation Strategy 3	1.026370
4	Mutation Strategy 4	1.026392
5	Mutation Strategy 5	1.027159

From the results depicted in Table II, it is clear that mutation strategy 2 gives the minimum objective function value. Keeping the filter order 38, different parameters of DE algorithm have been varied by implementing MS-2 for filter order 38-Population has been varied from 60 to 180. The objective function obtained for different values of population has been depicted in Table III and Fig. 3

TABLE III: Objective Function Obtained for different values of Population implemented on mutation strategy 2 for filter order 38

Sr. No	Population value	Objective function
1	60	1.026363
2	80	1.026154
3	100	1.026329
4	120	1.026217
5	140	1.026198
6	160	1.026215
7	180	1.026227

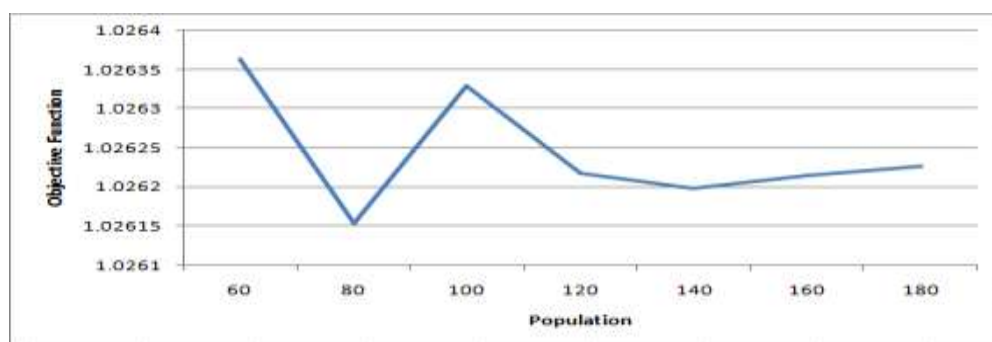


Fig.3: Objective Function versus Population Size

In Fig.3, decrease in objective function for population range 60 to 80 is quite large, whereas objective function value increases for population range 80 to 100 and once again value of objective function decreases from population range 100 to 180. It is concluded that minimum value of objective function obtained was 1.026154 with population value 80 for mutation strategy 2 at filter order 38.

For filter order 38 with Mutation Strategy-2, population size as 80, the value of mutation factor (f_m) has been varied from 0.4 to 1.0 in steps of 0.2. Obtained objective function for different values of f_m has been shown in TABLE IV and the performance has been depicted in Fig.4.

TABLE IV: Objective function vs. Mutation Factor values

Sr.No	Mutation factor	Objective function
1	0.4	1.026390
2	0.6	1.026391
3	0.8	1.026154
4	1.0	1.059753

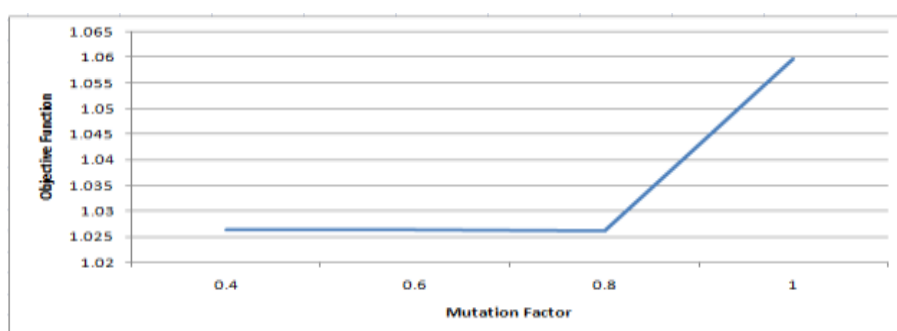


Fig.4: Objective Function vs. Mutation Factor

In Fig. 4 it is observed that value of objective function remains almost linear for f_m range 0.4 to 0.8 and it increases sharply from 0.8 to 1. Minimum value of objective function has been obtained with f_m value 0.8.

So keeping the mutation factor as 0.8, the values of Crossover rate (CR) has been changed from 0.1 to 0.4. The achieved values of objective function during variation of CR from 0.1 to 0.4 have been given in TABLE V and Fig.5.

TABLE V: Objective Function achieved for different CR values

Sr. No	CR Value	Objective function
1	0.1	1.026801
2	0.2	1.026154
3	0.3	1.026149
4	0.4	1.026130

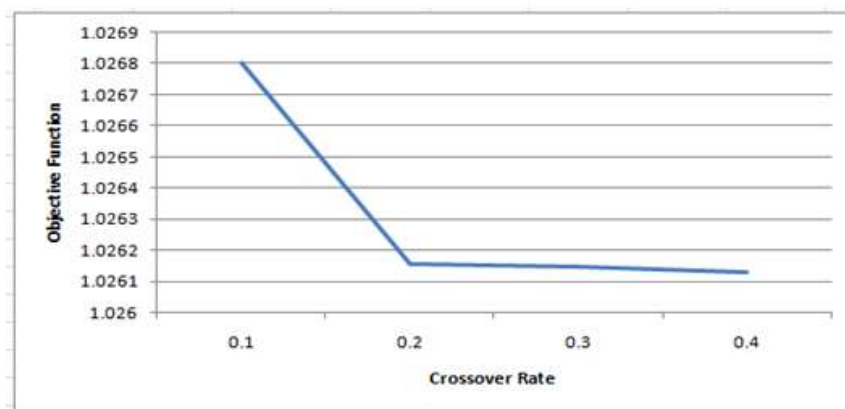


Fig.5: Objective Function vs. CR values

From Fig.5 it is concluded that Differential Evolution algorithm with a crossover rate 0.4 gives the best value of objective function.

Combining the results it is concluded that DE algorithm with mutation strategy 2 and filter order 38, having population size 80, mutation factor 0.8 and crossover rate 0.4 gives the optimum results for designing a low pass FIR digital filter. With these selected parameters the DE algorithm for the mutation strategy 2 with filter order 38 has been executed 100 times. Out of this the best results have been obtained at 22nd run.

TABLE VI: Design Results for Low-Pass FIR Digital Filter

Sr. No.	Magnitude Error	Pass-Band Performance	Stop-Band Performance
1	0.534747	$1.011473 \leq H(\omega) \leq 0.986658$ (0.024815)	$ H(\omega) \leq 0.017748$ (0.017748)

4.2. Magnitude response- Fig.6 has been plotted between the magnitude value in decibels and normalized frequency. In low pass filter magnitude decreases as frequency increases.

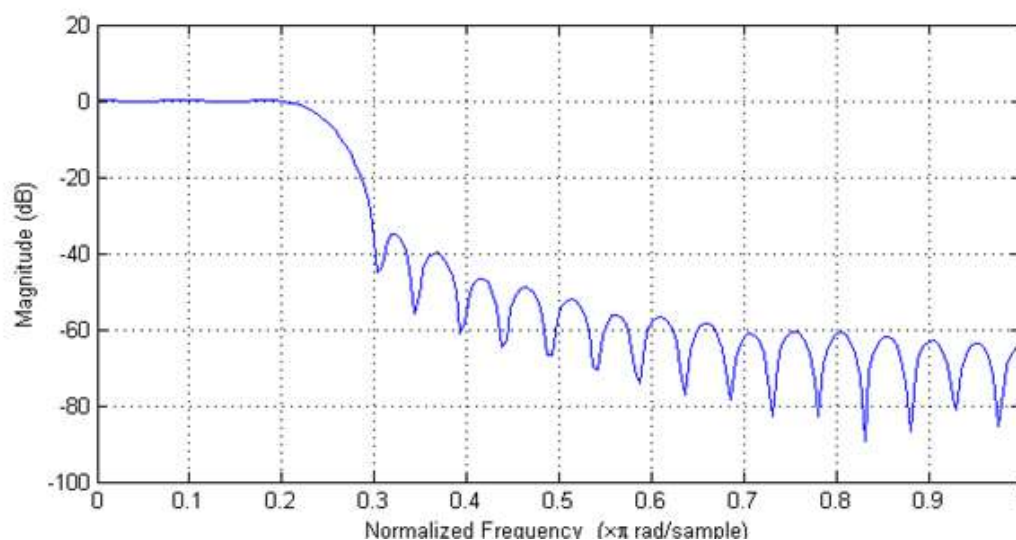


Fig.6: Magnitude Response for Low Pass FIR filter with Mutation Strategy -2 and Filter Order 38

The above graph satisfies the requirement of Low Pass Filter. The graph shown in Fig. 7 has been plotted between magnitude response and normalized frequency.

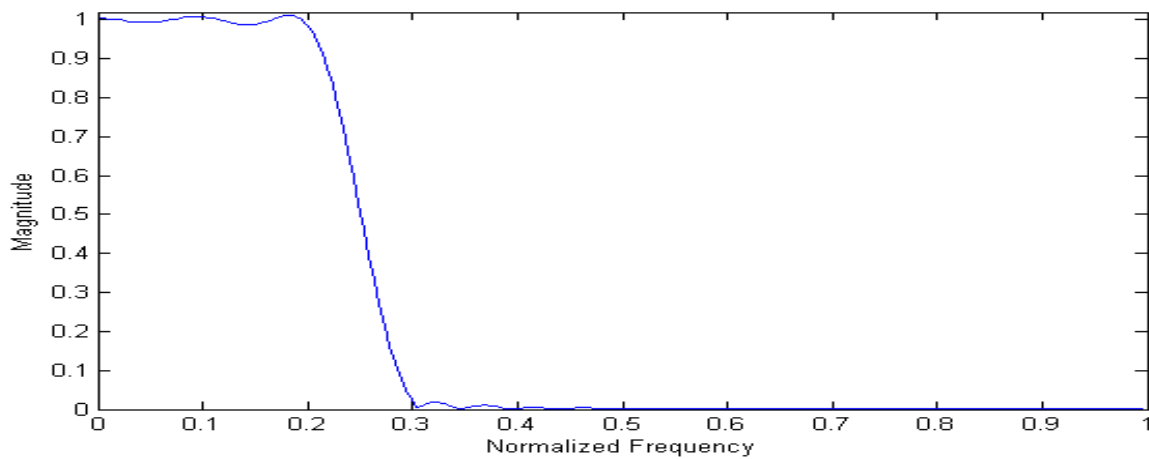


Fig.7: Magnitude response for Low-Pass FIR Digital filter with Mutation Strategy-2 and Filter Order 38

4.3. Phase response

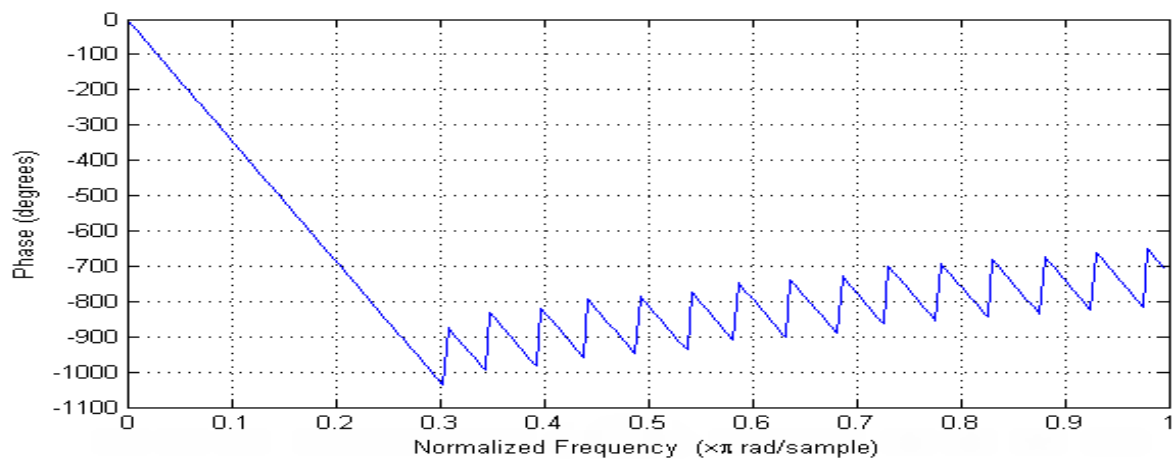


Fig.8: Phase response of Low pass FIR Digital filter with Mutation Strategy-2 and Filter Order 38

TABLE VII: Maximum, Minimum, Average Value of Objective Function along with Standard Deviation

Sr. No.	Maximum value of Objective Function	Minimum value of Objective Function	Average Value of Objective Function	Standard Deviation
1	1.030263	1.026130	1.026996	0.000723

From TABLE VII, it is clear that standard deviation is only 0.000723 which is less than 1, which shows the robustness of the designed filter.

V.CONCULSION

A digital FIR low pass filter has been designed using Differential Evolution. Order of the filter has been selected as 38 for designing low pass FIR digital filter as it gives the optimum value for objective function, pass-band ripple and stop-band ripple. Different mutation strategies were implemented on order 38. Mutation strategy - 2 gives the best Objective Function out of all 5 strategies. Then by keeping the filter order 38 and by selecting mutation strategy 2, the DE parameters values were changed to check the performance of algorithm. After parameters values were varied, it was observed that best results for designed low pass FIR digital filter have been obtained with population 80, mutation factor 0.8 and CR 0.4. Magnitude response and phase response of designed low pass FIR digital filter have been plotted.

The achieved standard deviation of the objective function for mutation strategy 2 with filter order 38 is less than 1 which authenticates the robustness of the designed filter.

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