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# FIXED POINTS OF WEAK COMPATIBILITY IN MENGER SPACE

**V. H. Badshah[1], Suman Jain[2], Arihant Jain[3], Subhash Mandloi[4]**

*[1,4]School of Studies in Mathematics, Vikram University, Ujjain (M.P.) (India)*

*[2]Department of Mathematics, Govt. College, Kalapipal (M.P.) (India)*

*[3]Department of Applied Mathematics, Shri Guru Sandipani Institute of Technology and Science, Ujjain M.P. (India)*

**ABSTRACT**

*The present paper deals with a common fixed point theorem for six self maps which generalizes the result of Pant and Chauhan [10], using the concept of weak compatibility in Menger space.*

***Keywords and Phrases.*** ***Menger space, Common fixed points, Compatible maps, Weak Compatible maps.***

**AMS Subject Classification (2000).** ***Primary 47H10, Secondary 54H25.***

## I. INTRODUCTION

Jungck and Rhoades [6] termed a pair of self maps to be coincidentally commuting or equivalently weakly compatible if they commute at their coincidence points. Sessa [14] initiated the tradition of improving commutativity in fixed-point theorems by introducing the notion of weak commuting maps in metric spaces. Jungck [5] soon enlarged this concept to compatible maps. Menger [7] introduced the notion of probabilistic metric space which is a generalization of metric space. It is also of fundamental importance in probabilistic functional analysis. The development of fixed point theory in PM-spaces was due to Schweizer and Sklar [12]. Sehgal and Bharucha-Reid [13] obtained a generalization of Banach Contraction Principle on a complete Menger space which is a milestone in developing fixed-point theory in Menger space.

The notion of compatible mapping in a Menger space has been introduced by Mishra [8]. Cho, Murthy and Stojakovik [1] proposed the concept of compatible maps of type (A) in Menger space and gave some fixed point theorems. Recently, using the concept of compatible mappings of type (A), semi-compatibility and occasionally weak compatibility in Menger space, Jain et. al. [2, 3, 4] proved some interesting fixed point theorems in Menger space. In the sequel, Patel and Patel [10] proved a common fixed point theorem for four compatible maps of type (A) in Menger space by taking a new inequality.

In this paper a fixed point theorem for six self maps has been proved using the concept of weak compatible mappings. We also cited an example.

## II. PRELIMINARIES

**Definition 2.1.**[8] A mapping $\mathcal{F} : \mathrm{R} \to \mathrm{R}^+$ is called a *distribution* if it is non-decreasing left continuous with

$$\inf \{ \mathcal{F}(t) \mid t \in \mathrm{R} \} = 0 \qquad \text{and} \qquad \sup \{ \mathcal{F}(t) \mid t \in \mathrm{R} \} = 1.$$

We shall denote by L the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t)=\begin{cases}0 & , \quad t\le 0\\ 1 & , \quad t>0\end{cases}.$$

**Definition 2.2.** [2] A mapping t :[0, 1] × [0, 1] → [0, 1] is called a *t-norm* if it satisfies the following conditions :

(t-1) $t(a, 1) = a$, $t(0, 0) = 0$ ;

(t-2) $t(a, b) = t(b, a)$ ;

(t-3) $t(c, d) \ge t(a, b)$ ; for $c \ge a$, $d \ge b$,

(t-4) $t(t(a, b), c) = t(a, t(b, c))$ for all $a, b, c, d \in [0, 1]$.

**Definition 2.3.** [2] A *probabilistic metric space (PM-space)* is an ordered pair $(X, \mathcal{F})$ consisting of a non empty set X and a function $\mathcal{F}: X \times X \to L$, where L is the collection of all distribution functions and the value of $\mathcal{F}$ at $(u, v) \in X \times X$ is represented by $F_{u, v}$. The function $F_{u,v}$ assumed to satisfy the following conditions:

(PM-1 ) $F_{u,v}(x) = 1$, for all $x > 0$, if and only if $u = v$;

(PM-2) $F_{u,v}(0) = 0$;

(PM-3) $F_{u,v} = F_{v,u}$;

(PM-4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ then $F_{u,w}(x + y) = 1$,

for all $u,v,w \in X$ and $x, y > 0$.

**Definition 2.4.** [2] A *Menger space* is a triplet $(X, \mathcal{F}, t)$ where $(X, \mathcal{F})$ is a PM-space and t is a t-norm such that the inequality

(PM-5) $F_{u,w}(x + y) \ge t\{F_{u,v}(x), F_{v,w}(y)\}$, for all $u, v, w \in X$, $x, y \ge 0$.

**Definition 2.5.** [12] A sequence $\{x_n\}$ in a Menger space $(X, \mathcal{F}, t)$ is said to be *convergent* and *converges to a point* x in X if and only if for each $\varepsilon > 0$ and $\lambda > 0$, there is an integer $M(\varepsilon, \lambda)$ such that

$$F_{x_n, x}(\varepsilon) > 1 - \lambda \text{ for all } n \ge M(\varepsilon, \lambda).$$

Further the sequence $\{x_n\}$ is said to be *Cauchy sequence* if for $\varepsilon > 0$ and $\lambda > 0$, there is an integer $M(\varepsilon, \lambda)$ such that

$$F_{x_n, x_m}(\varepsilon) > 1- \lambda \qquad \text{for all } m, n \ge M(\varepsilon, \lambda).$$

A Menger PM-space $(X, \mathcal{F}, t)$ is said to be *complete* if every Cauchy sequence in X converges to a point in X.

A complete metric space can be treated as a complete Menger space in the following way :

**Proposition 2.1.** [3] If (X, d) is a metric space then the metric d induces mappings $\mathcal{F}: X \times X \to L$, defined by $F_{p,q}(x) = H(x - d(p, q))$, $p, q \in X$, where

$$H(k) = 0, \quad \text{for } k \le 0 \quad \text{and} \quad H(k) = 1, \quad \text{for } k > 0.$$

Further if, $t : [0,1] \times [0,1] \to [0,1]$ is defined by $t(a,b) = \min\{a, b\}$. Then $(X, \mathcal{F}, t)$ is a Menger space. It is complete if (X, d) is complete.

The space $(X, \mathcal{F}, t)$ so obtained is called the *induced Menger space.*

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**Proposition 2.2.** [8] In a Menger space (X, $\mathcal{F}$, t) if t(x, x) ≥ x, for all x ∈ [0, 1] then t(a, b) = min{a, b}, for all a, b ∈ [0, 1].

**Definition 2.6.** [7] Self mappings A and S of a Menger space (X, $\mathcal{F}$, t) are said to be weak compatible if they commute at their coincidence points i.e. Ax = Sx for x ∈ X implies ASx = SAx.

**Definition 2.7.** [9] Self mappings A and S of a Menger space (X, $\mathcal{F}$, t) are said to be *compatible* if $F_{ASx_n, SAx_n}(x) \to 1$ for all x > 0, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Sx_n \to u$ for some u in X, as $n \to \infty$.

**Definition 2.8.** [10] Self maps S and T of a Menger space (X, $\mathcal{F}$, t) are said to be *semi-compatible* if $F_{STx_n, Tu}(x) \to 1$ for all x > 0, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Tx_n \to u$ for some u in X, as $n \to \infty$.

Now, we give an example of pair of self maps (A, S) which are weak compatible but not semi-compatible.

**Example 2.1.** Let (X, d) be a metric space where X = [0, 4] and (X, $\mathcal{F}$, t) be the induced Menger space with $F_{p,q}(\varepsilon) = H(\varepsilon - d(p, q))$, ∀ p, q ∈ X and ε > 0.

Define self maps A and S as follows :

$$A(x) = \begin{cases} 4-x, & if \quad 0 \le x < 2 \\ 4, & if \quad 2 \le x \le 4, \end{cases} \qquad S(x) = \begin{cases} x, & if \quad 0 \le x < 2 \\ 4, & if \quad 2 \le x \le 4 \end{cases}.$$

Taking $x_n = 2 - \frac{1}{n}$, we get $F_{Ax_n,2}(\varepsilon) = H\left(\varepsilon - \frac{2}{n}\right)$.

Hence, $\lim_{n\to\infty} F_{Ax_n,2}(\varepsilon) = 1$.

Thus, $Ax_n \to 2$. Similarly, $Sx_n \to 2$ as $n \to \infty$.

Again,

$$F_{ASx_n,S(2)}(\varepsilon) = H\left(\varepsilon - \left(2 - \frac{1}{n}\right)\right).$$

$$\lim_{n\to\infty} F_{ASx_n,S(2)}(\varepsilon) = H(\varepsilon - 2) \ne 1, \ \forall\ \varepsilon > 0.$$

Hence, (A, S) is not semi-compatible. Also, set of coincidence points of A and S is [2, 4]. Now, for any x ∈ [2, 4], Ax = Sx = 4 and AS(x) = A(4) = 4 = S(4) = SA(x).

Hence, the pair (A, S) is weak compatible.

**Remark 2.2.** In view of above example, it follows that the concept of weak compatible maps is more general than that of semi-compatible maps.

**Proposition 2.3.** Let $\{x_n\}$ be a Cauchy sequence in a Menger space (X, $\mathcal{F}$, t) with continuous t-norm t. If the subsequence $\{x_{2n}\}$ converges to x in X, then $\{x_n\}$ also converges to x.

**Proof.** As $\{x_{2n}\}$ converges to x, we have

$$F_{x_n,x}(\varepsilon) \ge t\left(F_{x_n,x_{2n}}\left(\frac{\varepsilon}{2}\right), F_{x_{2n},x}\left(\frac{\varepsilon}{2}\right)\right).$$

Then

$$\lim_{n\to\infty} F_{x_n,x}(\varepsilon) \ge t(1,1), \text{ which gives } \lim_{n\to\infty} F_{x_n,x}(\varepsilon) = 1, \ \forall\ \varepsilon > 0 \text{ and the result follows.}$$

**Lemma 2.1.** [16] Let $\{x_n\}$ be a sequence in a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t and $t(a, a) \geq a$. If there exists a constant $k \in (0, 1)$ such that $F_{x_n, x_{n+1}}(kt) \geq F_{x_{n-1}, x_n}(t)$ for all $t \geq 0$ and n = 1, 2, 3, ..., then $\{x_n\}$ is a Cauchy sequence in X.

**Lemma 2.2.** [15] Let $(X, \mathcal{F}, t)$ be a Menger space. If there exists a constant $k \in (0, 1)$ such that

$F_{x, y}(kt) \geq F_{x, y}(t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.

**A class of implicit relation.** Let $\Phi$ be the set of all real continuous functions $\phi : (R^+)^4 \to R$, non-decreasing in the first argument with the property :

a. For $u, v \geq 0$, $\phi(u, v, v, u) \geq 0$ or $\phi(u,v,u,v) \geq 0$ implies that $u \geq v$.

b. $\phi(u, u, 1, 1) \geq 0$ implies that $u \geq 1$.

**Example 2.2.** Define $\phi(t_1,t_2,t_3,t_4) = 18t_1 - 16t_2 + 8t_3 - 10t_4$. Then $\phi \in \Phi$.

## III. MAIN RESULT

**Theorem 3.1.** Let A, B, L, M, S and T be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.1.1) $L(X) \subseteq ST(X)$, $M(X) \subseteq AB(X)$;

(3.1.2) AB = BA, ST = TS, LB = BL, MT = TM;

(3.1.3) One of ST(X), M(X), AB(X) or L(X) is complete;

(3.1.4) The pairs (L, AB) and (M, ST) are weak compatible;

(3.1.5) for some $\phi \in \Phi$, there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t>0$,

$$\phi(F_{Lx, My}(kt), F_{ABx, STy}(t), F_{Lx, ABx}(t), F_{My, STy}(kt)) \geq 0$$

then A, B, L, M, S and T have a unique common fixed point in X.

**Proof.** Let $x_0 \in X$. From condition (3.1.1) $\exists$ $x_1, x_2 \in X$ such that

$$Lx_0 = STx_1 = y_0 \quad \text{and} \quad Mx_1 = ABx_2 = y_1.$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Lx_{2n} = STx_{2n+1} = y_{2n} \quad \text{and} \quad Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$$

for n = 0, 1, 2, ... .

**Step 1.** Putting $x = x_{2n}$ and $y = x_{2n+1}$ in (3.1.5), we get

$$\phi(F_{Lx_{2n}, Mx_{2n+1}}(kt), F_{ABx_{2n}, STx_{2n+1}}(t), F_{Lx_{2n}, ABx_{2n}}(t), F_{Mx_{2n+1}, STx_{2n+1}}(kt)) \geq 0.$$

Letting $n \to \infty$, we get

$$\phi(F_{y_{2n}, y_{2n+1}}(kt), F_{y_{2n-1}, y_{2n}}(t), F_{y_{2n}, y_{2n-1}}(t), F_{y_{2n+1}, y_{2n}}(kt)) \geq 0.$$

Using (a), we get

$$F_{y_{2n}, y_{2n+1}}(kt) \geq F_{y_{2n-1}, y_{2n}}(t).$$

Therefore, for all n even or odd, we have

$$F_{y_n, y_{n+1}}(kt) \geq F_{y_{n-1}, y_n}(t).$$

Therefore, by lemma 2.1, $\{y_n\}$ is a Cauchy sequence in X.

**Case I. ST(X) is complete.** In this case $\{y_{2n}\} = \{STx_{2n+1}\}$ is a Cauchy sequence in ST(X), which is complete. Thus $\{y_{2n+1}\}$ converges to some $z \in ST(X)$. By proposition 2.3, we have

$$\{Mx_{2n+1}\} \to z \text{ and } \{STx_{2n+1}\} \to z, \quad (3.8)$$

$$\{Lx_{2n}\} \to z \text{ and } \{ABx_{2n}\} \to z. \quad (3.9)$$

As $z \in ST(X)$ there exists $v \in X$ such that $z = STv$.

**Step I.** Putting $x = x_{2n}$ and $y = v$ in (3.1.5), we get

$$\phi(F_{Lx_{2n}, Mv}(kt), F_{ABx_{2n}, STv}(t), F_{Lx_{2n}, ABx_{2n}}(t), F_{Mv, STv}(kt)) \geq 0.$$

Letting $n \to \infty$, we get

$$\phi(F_{z, Mv}(kt), F_{z, STv}(t), F_{z, z}(t), F_{Mv, z}(kt)) \geq 0$$

$$\phi(F_{z, Mv}(kt), 1, 1, F_{z, Mv}(kt)) \geq 0$$

Using (a), we have

$$F_{z, Mv}(kt) \geq 1, \text{ for all } t > 0.$$

Hence, $F_{z, Mv}(t) = 1$.

Thus, $z = Mv$.

Therefore, $z = Mv = STv$.

As (M, ST) is weakly compatible, we have

$STMv = MSTv$. Thus, $STz = Mz$.

**Step II.** Putting $x = x_{2n}$ and $y = z$ in (3.1.5), we get

$$\phi(F_{Lx_{2n}, Mz}(kt), F_{ABx_{2n}, STz}(t), F_{Lx_{2n}, ABx_{2n}}(t), F_{Mz, STz}(kt)) \geq 0$$

Letting $n \to \infty$, we get

$$\phi(F_{z, Mz}(kt), F_{z, Mz}(t), 1, 1) \geq 0.$$

As $\phi$ is non-decreasing in the first argument, we have

$$\phi(F_{z, Mz}(t), F_{z, Mz}(t), 1, 1) \geq 0.$$

Using (b), we have

$$F_{z, Mz}(t) \geq 1, \text{ for all } t > 0.$$

Thus, $F_{z, Mz}(t) = 1$, we have

$z = Mz = STz$.

**Step III.** Putting $x = x_{2n}$ and $y = Tz$ in (3.1.5), we get

$$\phi(F_{Lx_{2n}, MTz}(kt), F_{ABx_{2n}, STTz}(t), F_{Lx_{2n}, ABx_{2n}}(t), F_{MTz, STTz}(kt)) \geq 0.$$

Letting $n \to \infty$, we get

$$\phi(F_{Lz, Tz}(kt), F_{z, Tz}(t), F_{z, z}(t), F_{Tz, Tz}(kt)) \geq 0$$

$$\phi(F_{z, Tz}(kt), F_{z, Tz}(t), 1, 1) \geq 0.$$

As $\phi$ is non-decreasing in the first argument, we have

$$\phi(F_{z, Tz}(t), F_{z, Tz}(t), 1, 1) \geq 0.$$

Using (b), we have

$F_{z,\,Tz}(t) \geq 1$, for all $t > 0$.

Thus, $F_{z,\,Tz}(t) = 1$, we have

$z = Tz$.

Since $Tz = STz$, we also have $z = Sz$.

Hence $Sz = Tz = Mz = z$.

**Step IV.** As $M(X) \subseteq AB(X)$, there exists $w \in X$ such that

$$z = Mz = ABw.$$

Putting $x = w$ and $y = x_{2n+1}$ in (3.1.5), we get

$$\phi(F_{Lw,\,Mx_{2n+1}}(kt), F_{ABw,\,STx_{2n+1}}(t), F_{Lw,\,ABw}(t), F_{Mx_{2n+1},\,STx_{2n+1}}(kt)) \geq 0.$$

Letting $n \to \infty$, we get

$$\phi(F_{Lw,\,z}(kt), F_{Lw,\,z}(t), F_{Lw,\,Lz}(t), F_{z,\,z}(kt)) \geq 0$$

$$\phi(F_{Lz,\,z}(kt), F_{Lz,\,z}(t), 1, 1) \geq 0.$$

As $\phi$ is non-decreasing in the first argument, we have

$$\phi(F_{Lw,\,z}(t), F_{Lw,\,z}(t), 1, 1) \geq 0.$$

Using (b), we have

$F_{z,\,Lw}(t) \geq 1$, for all $t > 0$.

Thus, $F_{z,\,Lw}(t) = 1$

$\Rightarrow$ $z = Lw$.

Therefore,

$Lz = ABz$.

**Step V.** Putting $x = z$ and $y = x_{2n+1}$ in (3.1.5), we get

$$\phi(F_{Lz,\,Mx_{2n+1}}(kt), F_{ABz,\,STx_{2n+1}}(t), F_{Lz,\,ABz}(t), F_{Mx_{2n+1},\,STx_{2n+1}}(kt)) \geq 0.$$

Letting $n \to \infty$, we get

$$\phi(F_{Lz,\,z}(kt), F_{ABz,\,z}(t), F_{Lz,\,ABz}(t), F_{z,\,z}(kt)) \geq 0$$

$$\phi(F_{Lz,\,z}(kt), 1, F_{Lz,\,z}(t), 1) \geq 0.$$

As $\phi$ is non-decreasing in the first argument, we have

$$\phi(F_{Lz,\,z}(kt), 1, F_{Lz,\,z}(t), 1) \geq 0.$$

Using (a), we get

$F_{z,\,Lz}(kt) = 1$, for all $t > 0$,

i.e. $z = Lz$.

Thus, we have $z = Lz = ABz$.

**Step VI.** Putting $x = Bz$ and $y = x_{2n+1}$ in (3.1.5), we get

$$\phi(F_{LBz,\,Mx_{2n+1}}(kt), F_{ABBz,\,STx_{2n+1}}(t), F_{LBz,\,ABBz}(t), F_{Mx_{2n+1},\,STx_{2n}}(kt)) \geq 0.$$

Letting $n \to \infty$, we get

$$\phi(F_{Bz,\,z}(kt), F_{Bz,\,z}(t), F_{Bz,\,Bz}(t), F_{z,\,z}(kt)) \geq 0$$

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$\phi(F_{Bz,\,z}(kt), F_{Bz,\,z}(t), 1, 1) \geq 0.$

As $\phi$ is non-decreasing in the first argument, we have

$\phi(F_{Bz,\,z}(t), F_{Bz,\,z}(t), 1, 1) \geq 0.$

Using (b), we have

$F_{Bz,\,z}(t) = 1,$ for all $t > 0$,

i.e. $z = Bz$.

Since $z = ABz$, we also have

$z = Az$.

Therefore, $z = Az = Bz = Lz$.

Combining the results from different steps, we get

$$Az = Bz = Lz = Mz = Tz = Sz = z.$$

Hence, the six self maps have a common fixed point in this case.

Case when $L(X)$ is complete follows from above case as $L(X) \subseteq ST(X)$.

**Case II. AB(X) is complete.** This case follows by symmetry. As $M(X) \subseteq AB(X)$, therefore the result also holds when $M(X)$ is complete.

**Uniqueness.** Let w be another common fixed point of A, B, L, M, S and T; then $w = Aw = Bw = Lw = Mw = Sw = Tw$.

Putting $x = z$ and $y = w$ in (3.1.5), we get

$\phi(F_{Lz,\,Mw}(kt), F_{ABz,\,STw}(t), F_{Lz,\,ABz}(t), F_{Mw,\,STw}(kt)) \geq 0$

$\phi(F_{z,\,w}(kt), F_{z,\,w}(t), F_{z,\,z}(t), F_{w,\,w}(kt)) \geq 0$

$\phi(F_{z,\,w}(kt), F_{z,\,w}(t), 1, 1) \geq 0.$

As $\phi$ is non-decreasing in the first argument, we have

$\phi(F_{z,\,w}(t), F_{z,\,w}(t), 1, 1) \geq 0.$

Using (b), we have

$F_{z,\,w}(t) \geq 1$, for all $t > 0$.

Thus, $F_{z,\,w}(t) = 1$,

i.e., $z = w$.

Therefore, z is a unique common fixed point of A, B, L, M, S and T.

This completes the proof.

**Remark 3.1.** If we take $B = T = I$, the identity map on X in theorem 3.1, then the condition (3.1.2) is satisfied trivially and we get

**Corollary 3.1.** Let A, L, M and S be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.1.6) $L(X) \subseteq S(X)$, $M(X) \subseteq A(X)$;

(3.1.7) One of $S(X)$, $M(X)$, $A(X)$ or $L(X)$ is complete;

(3.1.8) The pairs (L, A) and (M, S) are weak compatible;

(3.1.9) for some $\phi \in \Phi$, there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t>0$,

$$\phi(F_{Lx,\, My}(kt), F_{Ax,\, Sy}(t), F_{Lx,\, Ax}(t), F_{My,\, Sy}(kt)) \geq 0$$

then A, L, M and S have a unique common fixed point in X.

**Remark 3.2.** In view of proposition 2.2, t(a, b) = min{a, b}. Thus, corollary 3.1 generalizes the result of Pant et. al. [10] by reducing the semi-compatibility of the pair to its weak-compatibility and dropping the condition of continuity in a Menger space with continuous t-norm.

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