A MATHEMATICAL MODEL FOR FINGERO-IMBIBITION PHENOMENON IN A CRACKED POROUS MEDIA WITH MAGNETIC FLUID

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ABSTRACT

In this paper, we have discussed finger-imbibition phenomenon in double phase flow through porous media. The phenomenon arises on account of simultaneous occurrence of two phenomenon known as imbibitions and fingering. We assumed that injection of preferentially wetting, less viscous fluid into porous medium saturated with resident fluid. The mathematical formulation leads to non linear partial differential equation governing the phenomenon in a cracked porous medium with magnetic fluid. The mathematical solution has been obtained by finite element method with appropriate initial and boundary condition. Finite element method is a numerical method for finding an approximation solution of differential equation in finite region or domain. We obtained graphical representation of the solution using a Matlab coding.

Keywords: Cracked porous medium, Fingero - imbibition, Finite element method, Magnetic fluid.

I. INTRODUCTION

It is well known that when a porous medium filled with some resident fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to difference in wetting abilities is called counter-current imbibition. Similarly, when a fluid contained in a porous medium is displaced by another fluid of lesser viscosity, instead of regular displacement of whole front, protuberances (fingers) may occur which shoot through the porous medium at relatively great speeds. This phenomenon is called fingering or instabilities. The phenomena of fingering and imbibition occurring simultaneously in displacement process, have gained much current importance due to their frequent occurrence in the problem of petroleum technology and many authors have discussed them from different point of view.

II. STATEMENT OF THE PROBLEM

We consider here a finite cylindrical mass of porous matrix of length L (=1) saturated with native liquid (o), is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibition face (x =0) and this end is exposed to an adjacent formation of 'injected' liquid (w) which involves a thin layer of suitable magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous.

This arrangement gives rise to a displacement process in which the injection of the fluid (w) is initiated by imbibition and the consequent displacement of native liquid (o) produces protuberances (fingers).This arrangement describes a one-dimensional phenomenon of fingero-imbibition.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

In cracked porous media, the problem becomes more intricate due to additional non-linear term called as impregnation function $\phi(T-\tau(u))$. The equation of continuity for native and injected liquid, (neglecting variation of phase density), are

$$P\left(\frac{\partial S_w}{\partial t}\right)+\left(\frac{\partial V_w}{\partial x}\right)+\phi(T-\tau(u))=0$$ …………………………..

(1) where $\phi(T-\tau(u))$ is the impregnation function and $P\left(\frac{\partial S_o}{\partial t}\right)+\left(\frac{\partial V_o}{\partial x}\right)=0$ where P = porosity of the medium. S_w, S_o = Saturation of the injected and native liquid respectively. From the definition of phase saturation, it is obvious that $S_w + S_o = 1$ and also for imbibition phenomenon $V_o = - V_w$ ……………………….. (2)

Now using equation (1)

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial x}\left(\left(\frac{K_o/\delta_o.K_w/\delta_w}{K_o/\delta_o+K_w/\delta_w}\right)K\left[\frac{\partial P_c}{\partial x}-\gamma H\frac{\partial H}{\partial x}\right]\right)+\phi(T-\tau(u))=0$$ .... (3)

Substituting the value of $\phi(T-\tau(u))$ into equation (3) we

get, $$\varepsilon P\left(\frac{\partial S_w}{\partial T}\right)+\frac{\partial}{\partial x}\left(\left(\frac{K_o/\delta_o.K_w/\delta_w}{K_o/\delta_o+K_w/\delta_w}\right)K\left[\frac{\partial P_c}{\partial x}-\gamma H\frac{\partial H}{\partial x}\right]\right)=-\frac{D}{\sqrt{T-Rx^2}}$$ …………………. (4)

Assuming $\frac{K_oK_w}{K_o\delta_w+K_w\delta_o}\approx\frac{K_o}{\delta_o}$ and considering the magnetic fluid H in the x-direction only, we may write $H=\frac{\Lambda}{x^n}$ where Λ is a constant parameter and n is an integer. Using the value of H for n = -1 in equation (4), we get,

$$\varepsilon P\left(\frac{\partial S_w}{\partial T}\right)+\frac{K}{\delta_o}\frac{\partial}{\partial x}\left((1-S_w)\left[-\beta_o\frac{\partial S_w}{\partial x}-\gamma\Lambda^2 x\right]\right)=-\frac{D}{\sqrt{T-Rx^2}}$$ ……………… (5)

This is the desired non linear differential equation of motion which describes the linear counter current imbibition phenomena in a cracked porous medium with effect of magnetic fluid.

A set of suitable boundary conditions associated to problem (5) are

$S_w(x,0) = S_{wc}$ for all $x > 0$; ……………….. (6)

$S_w(0,t) = S_{w0}$; $S_w(L,t) = S_{w1}$ for all $t \geq 0$ ……………………. (7)

Equation (5) is reduced to dimensionless form by setting

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$$x^* = x/L\,,\quad T^* = \frac{K\beta T}{L^2\delta_o\varepsilon P},\qquad \left(1-S_w(x,T)\right) = S_w^*(x^*,T^*)$$

So that $\frac{\partial S_w}{\partial T} - \frac{\partial}{\partial X}\left(S_w\frac{\partial S_w}{\partial X}\right) - C_o\frac{\partial}{\partial X}\left(S_w X\right) = \frac{DL}{C_2\sqrt{R}}\frac{1}{\sqrt{\frac{C_1T}{C_2R} - X^2}}$ …………………… (8)

Where, $C_o = \frac{-\gamma\Lambda^2L^2}{\beta}$, $\quad C_1 = P$, $\quad C_2 = \frac{K\beta}{\delta_o}$

The initial and boundary conditions (6) & (7) now becomes,

$S_w(x,0) = 1 - S_{wc}$ $\quad \forall \quad x > 0$ ……………. (9)

$S_w(0,t) = 1 - S_{w0}$; $\quad S_w(L,t) = 1 - S_{w1}$ $\quad$ for all $t \geq 0$ .... (10)

Equation (8) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in cracked porous medium with effect of magnetic fluid.

A Matlab Code is prepared and executed with $C_o = 3.645 \times 10^{-11}$, c1(= $C_2\sqrt{R}/D$) = 0.3×10^{11} ,c2(= C_1/C_2R) = 90, L = 1,h =1/15 ,k = 0.002223 for 225 time levels, $S_{w0} = 0.5$ and $S_{w1} = S_{wc} = 0$. Curves indicating the behavior of Saturation of injected fluid with respect to various time period.

IV. FINITE ELEMENT METHOD

The domain of the problem consists of all points between $x = 0$ and $x = 1$and the domain is divided into a set of linear elements. Now, the variational form of given partial differential equation (8) is

$$J(S_w) = \frac{1}{2}\int_R\left[S_w\left(\frac{\partial S_w}{\partial x}\right)^2 + 2S_w\frac{\partial S_w}{\partial T} + C_o x S_w\frac{\partial S_w}{\partial x} + \frac{1}{c1\sqrt{c2\,T - x^2}}S_w\right]dx \qquad \text{..................... (11)}$$

Choose an arbitrary linear element $R^{(e)} = [S_1^{(e)}, S_2^{(e)}]$ & obtain interpolation function for $R^{(e)}$ using Lagrange interpolation Method such as

$$S^{(e)}(x) = \sum_{j=1}^{2} N_j(x)S_j^{(e)} = N^{(e)}\phi^{(e)} = \phi^{(e)T}N^{(e)T} \qquad \text{……………. (12)}$$

$$where \qquad N^{(e)} = [N_1\ N_2] \quad \& \quad \phi^{(e)} = [S_1\ \ S_2]^T$$

Now, apply Variational Method to $R^{(e)}$,therefore eq. (5.13.1) becomes

$$J(S^{(e)}) = \frac{1}{2}\int_{R^{(e)}}\left[S^{(e)}\left(\frac{\partial S^{(e)}}{\partial x}\right)^2 + 2S^{(e)}\frac{\partial S^{(e)}}{\partial t} + C_o x S^{(e)}\frac{\partial S^{(e)}}{\partial x} + \frac{1}{c1\sqrt{c2\,T - x^2}}S^{(e)}\right]dx \qquad \text{…………… (13)}$$

$$as,\quad S^{(e)}(x) = N^{(e)}\phi^{(e)} = \phi^{(e)T}N^{(e)T}$$

$$\therefore\ \frac{\partial S^{(e)}}{\partial x} = \frac{\partial N^{(e)}}{\partial x}\phi^{(e)} = \phi^{(e)T}\frac{\partial N^{(e)T}}{\partial x} \quad and \quad \left(\frac{\partial S^{(e)}}{\partial x}\right)^2 = \phi^{(e)T}\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\phi^{(e)}$$

Therefore equation (13) becomes,

$$J\left(S^{(e)}\right)=\frac{1}{2}\int_{R^{(e)}}\varphi^{(e)T}\left[N^{(e)}\varphi^{(e)}\left(\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\right)\varphi^{(e)}+2\left(N^{(e)T}N^{(e)}\right)\frac{\partial\varphi^{(e)}}{\partial t}+C_0xN^{(e)T}\frac{\partial N^{(e)}}{\partial x}\varphi^{(e)}+\frac{1}{c1\sqrt{c2\,T-x^2}}N^{(e)T}\right]dx \quad \ldots\ldots\ldots\ldots\ldots (14)$$

For minimization, first differentiate equation (14) with respect to $\phi^{(e)}$

$$\frac{\partial J^{(e)}}{\partial\phi^{(e)}}=\int_{R^{(e)}}\left[N^{(e)}\phi^{(e)}\left(\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\right)\phi^{(e)}+2\left(N^{(e)T}N^{(e)}\right)\frac{\partial\phi^{(e)}}{\partial t}+C_0xN^{(e)T}\frac{\partial N^{(e)}}{\partial x}\phi^{(e)}+\frac{1}{c1\sqrt{c2\,T-x^2}}N^{(e)T}\right]dx$$

Now, $\frac{\partial J^{(e)}}{\partial\phi^{(e)}}=0$ Therefore the Element equation is

$$A^{(e)}\frac{\partial\phi^{(e)}}{\partial T}+B^{(e)}(\phi^{(e)})\phi^{(e)}+C^{(e)}\phi^{(e)}+F^{(e)}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

where $A^{(e)}=\int_{S_1^{(e)}}^{S_2^{(e)}}\left(N^{(e)T}N^{(e)}\right)dx,$ $\quad B^{(e)}(\phi^{(e)})=\int_{S_1^{(e)}}^{S_2^{(e)}}N^{(e)}\phi^{(e)}\left(\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\right)dx$

$$C^{(e)}=C_o\int_{S_1^{(e)}}^{S_2^{(e)}}x\left(N^{(e)T}\frac{\partial N^{(e)}}{\partial x}\right)dx,\qquad F^{(e)}=\int_{S_1^{(e)}}^{S_2^{(e)}}\frac{1}{c1\sqrt{c2\,T-x^2}}N^{(e)T}\,dx$$

By Gauss Legendre Quadrature Method, Interpolation function becomes

, $N_1(z)=\frac{1}{2}(1-z)$; $N_2(z)=\frac{1}{2}(1+z)$. Thus, element matrix transform to

$$A^{(e)}=\int_{-1}^{1}\left(N^{(e)T}N^{(e)}\right)Jdz\approx\sum_{I=1}^{r}A^{(e)}(z_I)W_I$$

$$B^{(e)}(\phi^{(e)})=\int_{-1}^{1}N^{(e)}\phi^{(e)}\left(\frac{1}{J}\frac{\partial N^{(e)T}}{\partial X}\frac{1}{J}\frac{\partial N^{(e)}}{\partial X}\right)Jdz\approx\sum_{I=1}^{r}B^{(e)}(z_I)W_I$$

$$C^{(e)}=C_0\int_{-1}^{1}\left(\frac{h}{2}(1+z_I)\right)\left(N^{(e)T}\frac{1}{J}\frac{\partial N^{(e)}}{\partial z}\right)Jdz\approx\sum_{I=1}^{r}C^{(e)}(z_I)W_I$$

$$F^{(e)}=\frac{1}{c1}\int_{-1}^{1}\left(\sqrt{c2\,T^{(e)}-\left(\frac{h}{2}(1+z_I)\right)^2}\right)\left(N^{(e)T}\right)Jdz\approx\sum_{I=1}^{r}F^{(e)}(z_I)W_I$$

For $A^{(e)}$, $C^{(e)}$ and $F^{(e)}$,degree of polynomial p =2 then r = 2 and For $B^{(e)}$, p = 1 then r =1. z_I and W_I are corresponding gauss points and gauss weights with respect to 'r'. Then, the element matrix becomes,

$$\mathsf{A}^{(e)}=\frac{h^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix},\quad \mathsf{B}^{(e)}(\phi^{(e)})=\frac{1}{2h^{(e)}}\begin{bmatrix}S_1+S_2 & -S_1-S_2\\ -S_1-S_2 & S_1+S_2\end{bmatrix}, C^{(e)}=\frac{C_o h^{(e)}}{6}\begin{bmatrix}-1 & -2\\ 1 & 2\end{bmatrix},\quad F(e)=\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

where

$$F_1=\frac{1}{c1}\sum_{I=1}^{r}\left(\sqrt{c2\,T-\left(\frac{h}{2}(1+z_I)\right)^2}\right)\left(\frac{h}{2}(1-z_I)\right)\ W_I,$$

$$F_2=\frac{1}{c1}\sum_{I=1}^{r}\left(\sqrt{c2\,T-\left(\frac{h}{2}(1+z_I)\right)^2}\right)\left(\frac{h}{2}(1+z_I)\right)\ W_I$$

…………………… (16)

4.1 Assembling of Elements

For a uniform mesh of N elements, by equation (15) & equation (16), the assembled equation becomes

$$A\frac{\partial\varphi}{\partial T}-B(\varphi)\varphi+C\ \ \varphi+F1=0 \qquad \text{..............(17)}$$

where,

$$\mathsf{A}=\frac{h}{6}\begin{bmatrix}2 & 1 & 0 & 0 & \cdots & 0 & 0\\ 1 & (2+2) & 1 & 0 & \cdots & 0 & 0\\ 0 & 1 & (2+2) & 0 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & 0 & \ldots & (2+2) & 1\\ 0 & 0 & 0 & 0 & \cdots & 1 & 2\end{bmatrix}$$

$$\mathsf{B}(\phi)=\frac{1}{2h}\begin{bmatrix}S_1+S_2 & -S_1-S_2 & 0 & 0 & \cdots & 0 & 0\\ -S_1-S_2 & S_1+2S_2+S_3 & -S_2-S_3 & 0 & \cdots & 0 & 0\\ 0 & -S_2-S_3 & S_2+2S_3+S_4 & -S_3-S_4 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & 0 & \ldots & S_{14}+2S_{15}+S_{16} & -S_{15}-S_{16}\\ 0 & 0 & 0 & 0 & \cdots & -S_{15}-S_{16} & S_{15}+S_{16}\end{bmatrix}$$

$$\mathsf{C}=\frac{C_o h}{6}\begin{bmatrix}-1 & -2 & 0 & 0 & \cdots & 0 & 0\\ 1 & 1 & -2 & 0 & \cdots & 0 & 0\\ 0 & 1 & 1 & 0 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & 0 & \ldots & 1 & -2\\ 0 & 0 & 0 & 0 & \cdots & 1 & 2\end{bmatrix} \qquad F1=\begin{bmatrix}F_1^1\\ F_2^1+F_1^2\\ F_2^2+F_1^3\\ \vdots\\ \vdots\\ F_2^{N-1}+F_1^N\\ F_2^N\end{bmatrix} \qquad \phi=[S_1\ \ S_2\ \ \cdots\ \ \cdots\ \ S_{N+1}]^T$$

………………. (18)

Equation (17) represents the assembled equation.

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4.2 Time Approximation

We have obtained the finite element equation in the global form, which represent a system of simultaneous ordinary differential equation by introducing approximation family of weighted average of a dependent variable of two consecutive time steps by linear interpolation of the values of the variable at the two time. Therefore the equation (15) written as,

$[\ A + \delta\ k(B(\phi^{(n+1)}\ ) + C)\ ]\phi^{(n+1)} = [\ A - (1-\delta)\ k(B(\phi^{(n)}) + C)\ ]\phi^{(n)} + F1^{(n)}$ …….. (19) where **δ =1/2** and n = 0,1,2,……

For a uniform mesh of N elements, by equation (18), the global equation (19) takes the form,

$$\left[K(\phi^{(n+1)})\right]\phi^{(n+1)} = \left[F_1(\phi^{(n)})\right]\phi^{(n)} + F1^{(n)} \quad = F(\phi^{(n)}) + F1^{(n)} = F \quad \text{………… (20)}$$

Where,

$$K(\phi^{(n+1)}) =$$

$$\begin{bmatrix}
\frac{h}{3}+\delta k\left(\frac{1}{2h}\left(S_1^{(n+1)}+S_2^{(n+1)}\right)+\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_1^{(n+1)}-S_2^{(n+1)})+\frac{C_0 h}{3}\right) & 0 & \cdots\cdots & 0 & 0 \\
\frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_1^{(n+1)}-S_2^{(n+1)})-\frac{C_0 h}{6}\right) & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_1^{(n+1)}+2S_2^{(n+1)}+S_3^{(n+1)})-\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_2^{(n+1)}-S_3^{(n+1)})+\frac{C_0 h}{3}\right) & 0\ \cdots & 0 & 0 \\
0 & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_2^{(n+1)}-S_3^{(n+1)})-\frac{C_0 h}{6}\right) & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_2^{(n+1)}+2S_3^{(n+1)}+S_4^{(n+1)})-\frac{C_0 h}{6}\right) & \cdots\cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots \\
0 & 0 & 0 & 0\ \cdots & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_{14}^{(n+1)}+2S_{15}^{(n+1)}+S_{16}^{(n+1)})-\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_{15}^{(n+1)}-S_{16}^{(n+1)})+\frac{C_0 h}{3}\right) \\
0 & 0 & 0 & 0\ \cdots & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_{15}^{(n+1)}-S_{16}^{(n+1)})-\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(+S_{15}^{(n+1)}+S_{16}^{(n+1)})-\frac{C_0 h}{3}\right)
\end{bmatrix}$$

$$F_1\left(\phi^{(n)}\right) =$$

$$\begin{bmatrix}
\frac{h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+S_2^{(n)})+\frac{C_0 h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})+\frac{C_0 h}{3}\right) & 0 & 0\ \cdots & 0 & 0 \\
\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})-\frac{C_0 h}{6}\right) & \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)})-\frac{C_0 h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})+\frac{C_0 h}{3}\right) & 0\ \cdots & 0 & 0 \\
0 & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_2^{(n)}-S_3^{(n)})-\frac{C_0 h}{6}\right) & \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_2^{(n)}+2S_3^{(n)}+S_4^{(n)})-\frac{C_0 h}{6}\right) & \cdots\ \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0\ \cdots & \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_{14}^{(n)}+2S_{15}^{(n)}+S_{16}^{(n)})-\frac{C_0 h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})+\frac{C_0 h}{3}\right) \\
0 & 0 & 0 & 0\ \cdots & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})-\frac{C_0 h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(S_{15}^{(n)}+S_{16}^{(n)})-\frac{C_0 h}{3}\right)
\end{bmatrix}$$

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$$\phi^{(n)} = \begin{bmatrix} S_1^{(n)} \\ S_2^{(n)} \\ S_3^{(n)} \\ \vdots \\ \vdots \\ S_{15}^{(n)} \\ S_{16}^{(n)} \end{bmatrix}, \qquad F1^{(n)} = \begin{bmatrix} F1^{(1)} \\ F1^{(2)} \\ F1^{(3)} \\ \vdots \\ \vdots \\ F1^{(15)} \\ F1^{(16)} \end{bmatrix}$$

$$F = \begin{bmatrix} \frac{h}{3} - (1-\delta)\,k\left(\frac{1}{2h}(S_1^{(n)} + S_2^{(n)}) + \frac{C_o h}{6}\right)S_1^{(n)} + \frac{h}{6} - (1-\delta)\,k\left(\frac{1}{2h}(-S_1^{(n)} - S_2^{(n)}) + \frac{C_o h}{3}\right)S_2^{(n)} + F1^{(1)} \\ \frac{h}{6} - (1-\delta)\,k\left(\frac{1}{2h}(-S_1^{(n)} - S_2^{(n)}) - \frac{C_o h}{6}\right)S_1^{(n)} + \frac{2h}{3} - (1-\delta)\,k\left(\frac{1}{2h}(S_1^{(n)} + 2S_2^{(n)} + S_3^{(n)}) - \frac{C_o h}{6}\right)S_2^{(n)} + \frac{h}{6} - (1-\delta)\,k\left(\frac{1}{2h}(-S_2^{(n)} - S_3^{(n)}) + \frac{C_o h}{3}\right)S_3^{(n)} + F1^{(2)} \\ \vdots \\ \vdots \\ \vdots \\ \frac{2h}{3} - (1-\delta)\,k\left(\frac{1}{2h}(S_{14}^{(n)} + 2S_{15}^{(n)} + S_{16}^{(n)}) - \frac{C_o h}{6}\right)S_{15}^{(n)} + \frac{h}{6} - (1-\delta)\,k\left(\frac{1}{2h}(-S_{15}^{(n)} - S_{16}^{(n)}) + \frac{C_o h}{3}\right)S_{16}^{(n)} + F1^{(15)} \\ \frac{h}{6} - (1-\delta)\,k\left(\frac{1}{2h}(-S_{15}^{(n)} - S_{16}^{(n)} - \frac{C_o h}{6}\right)S_{15}^{(n)} + \frac{h}{3} - (1-\delta)\,k\left(\frac{1}{2h}(S_{15}^{(n)} + S_{16}^{(n)}) - \frac{C_o h}{3}\right)S_{16}^{(n)} + F1^{(16)} \end{bmatrix}$$

where K is called global stiffness matrix and F is called global generalized force vector and N+1 is total number of global nodes .

4.3 Imposing Boundary Conditions

We now apply the boundary condition (10) to the global equation (8) of the problem and simplifying, we get

$$[K(\phi^{(n+1)})]\phi^{(n+1)} = F \qquad \text{.........................} \ (21)$$

Where,

$$K(\phi^{(n+1)}) = \begin{bmatrix} 1 & 0 & 0 & \cdots\ \cdots & 0 & 0 \\ \frac{h}{6} + \delta k\left(\frac{1}{2h}(-S_1^{(n+1)} - S_2^{(n+1)}) - \frac{C_O h}{6}\right) & \frac{2h}{3} + \delta k\left(\frac{1}{2h}(S_1^{(n+1)} + 2S_2^{(n+1)} + S_3^{(n+1)}) - \frac{C_O h}{6}\right) & \frac{h}{6} + \delta k\left(\frac{1}{2h}(-S_2^{(n+1)} - S_3^{(n+1)}) + \frac{C_O h}{3}\right) & 0\ \cdots & 0 & 0 \\ 0 & \frac{h}{6} + \delta k\left(\frac{1}{2h}(-S_2^{(n+1)} - S_3^{(n+1)}) - \frac{C_O h}{6}\right) & \frac{2h}{3} + \delta k\left(\frac{1}{2h}(S_2^{(n+1)} + 2S_3^{(n+1)} + S_4^{(n+1)}) - \frac{C_O h}{6}\right) & \cdots\ \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0\ \cdots & \frac{2h}{3} + \delta k\left(\frac{1}{2h}(S_{14}^{(n+1)} + 2S_{15}^{(n+1)} + S_{16}^{(n+1)}) - \frac{C_O h}{6}\right) & \frac{h}{6} + \delta k\left(\frac{1}{2h}(-S_{15}^{(n+1)} - S_{16}^{(n+1)}) + \frac{C_O h}{3}\right) \\ 0 & 0 & 0 & 0\ \cdots & 0 & 1 \end{bmatrix}$$

$$F(\phi^{(n)})=\begin{bmatrix} 0.5 \\ \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})-\frac{C_o h}{6}\right)S_1^{(n)}+\frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)})-\frac{C_o h}{6}\right)S_2^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_2^{(n)}-S_3^{(n)})+\frac{C_o h}{3}\right)S_3^{(n)}+F1^{(2)} \\ \vdots \\ \vdots \\ \vdots \\ \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_{14}^{(n)}+2S_{15}^{(n)}+S_{16}^{(n)})-\frac{C_o h}{6}\right)S_{15}^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})+\frac{C_o h}{3}\right)S_{16}^{(n)}+F1^{(N-1)} \\ 0 \end{bmatrix}$$

Thus, equation (21) is the resulting system of nonlinear algebraic equation.

4.4 Solution Of Nonlinear Algebraic Equation

we obtained the assembled equation which is nonlinear. The assembled nonlinear equations after imposing boundary conditions is given by equation (21).We seek an approximate solution by the linearization which based on scheme

$$\left[K(\phi^{(n)})\right]\phi^{(n+1)} = F \qquad \text{............ (22)}$$

Where $\phi^{(n)}$ denotes the solution of the n iteration. Thus, the coefficient K_{ij} are evaluated using the solution $\phi^{(n)}$ from the previous iteration and the solution at the $(n+1)^{th}$ iteration can be obtained by solving equation (22) using Gauss Elimination Method. At the beginning of the iteration (i.e. n=0), we assume the solution $\phi^{(0)}$ from initial condition (9) which requires to have $S_1^{(0)} = S_2^{(0)} = \ldots\ldots\ldots\ldots\ldots = S_{N+1}^{(0)} = 0$.

V. GRAPHICAL REPRESENTATION AND CONCLUSION

A Matlab Code is prepared for 15 elements model and resulting equation (22) for N = 15 is solved by Gauss Elimination method.

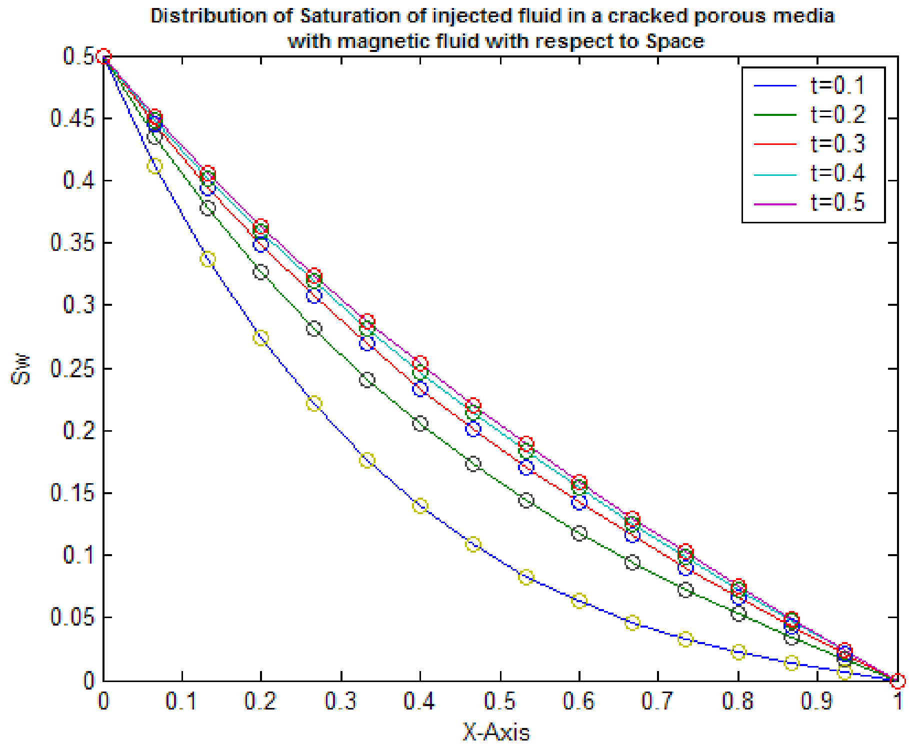

Figure 1.

In above graphs, X-axis represents the saturation of injected liquid involving magnetic fluid (S_w) in cracked porous media of length one. Y- axis represents the time 't' in seconds. Solution obtained with $\mathbf{C_o}$ = **3.645 $\times$ 10^{-11} , c1(= $C_2\sqrt{R}/D$) = 0.3 $\times$ 10^{11} ,c2(= C_1/C_2R) = 90, L = 1,h =1/15 , k = 0.002223** for **225** time levels. It is clear from graph that $S_w = S_0(=0.5)$ at layer x =0 and there is no saturation of injected liquid at other end (x =1) irrespective of time. At particular time, saturation of injected liquid involving magnetic fluid decrease with increase in value of x (or as we move ahead) and at x = 1, saturation is decreased to zero and at particular point x of observed region, saturation of injected fluid increases with increase in time but rate of increase of the saturation lessen at each point as time increases.

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