

FREE VIBRATION ANALYSIS OF STEPPED BEAMS WITH CIRCULAR AND SQUARE CROSS SECTIONS

Prashant S.Warke¹, Deepak A. Warke²

¹M.E.Scholar J.T.M.C.O.E Faizpur, ²Professor J.T.M.C.O.E Faizpur

ABSTRACT

This paper presents a innovative mathematical technique applicable to analyses the free vibration analysis of stepped beams with circular cross section and Square stepped section. In this approach in which the finite element method is used. To make known the accuracy and effectiveness of the offered method, a number of numerical examples are given for free vibration analysis of beams. Numerical results showing good agreement with the results of other available studies, address the effects of the location and depth of the steps on the natural frequencies and mode shapes of the step beams. The outcomes of the study verified that presented method is appropriate for the vibration analysis of stepped beams with circular cross section and square cross section. The study compares various materials natural frequencies such as mild steel, copper, aluminum. In this study the cross section and length of the beam is kept constant and natural frequencies are calculated with Ansys software and compared with experimental values.

I. INTRODUCTION

In case of a free vibration study of a structure the main objective is to determine the natural frequencies corresponding to different modes of vibration of the system. Several different techniques and methodologies have been adopted for this purpose by different researchers. Vibration analysis of a beam is an important and peculiar subject of study in mechanical engineering. All real physical structures, when subjected to loads or displacements, behave dynamically. Engineering structures are designed to withstand the loads they are expected to be subject to while in service. Among them Beams are a standout amongst the most usually utilized structural components within various structural elements in numerous engineering applications and experience a wide mixed bag of static and element loads. Beams are widely used as structural components in engineering applications and also provide a fundamental model for many engineering applications. Aircraft wings, helicopter rotor blades, spacecraft antennae, and robot arms are all examples of structures that may be modeled with beam-like elements. Beam sort structures are being generally utilized in steel shaped structure and manufacturing of machines.

Beams with variable cross-section and/or material properties are frequently used in aeronautical engineering (e.g., rotor shafts and functionally graded beams), mechanical engineering (e.g., robot arms and crane booms), and civil engineering (e.g., beams, columns, and steel composite floor slabs in the single direction loading case). Stepped beam-like structures are widely used in various engineering fields, such as robot arm and tall building, etc. Therefore there is a necessary that construction should securely work during its service period. But, wreck initiates a failure span on structure. The instant changes introduced into a structure, either intentionally or

unintentionally which leads to adverse effect the current or future performance of that structure is defined as damage. Damage is one of the important aspects in structural analysis because of safety reason as well as economic growth of the industries

II. LITERATURE REVIEW

[1] **Kapurja et al.** presented a theoretical model and its experimental validation for the free vibration of a layered FG beam. Some research works are also available on the effect of nonlinear elastic foundations on free vibration behavior of FG beams. The governing equations were based on Euler-Bernoulli beam theory and solved using Galerkin's method and He's variational iteration method. A few researchers have concentrated on the free vibration of FG beams where the material property variation is along the length of the beam.

[2] **Simsek et al.** derived the equation of motion by using Lagrange's equations and Newmark method was employed to find the dynamic responses of AFG beam.

[3e6] **Shahba et al.** and [4,5] **Shahba and Rajasekaran** studied the free vibration and stability analysis of Euler-Bernoulli and Timoshenko beams through finite element approach and various numerical analysis methods.

[7] **Alshorbagy et al.** employed numerical FEM and Euler-Bernoulli beam theory to investigate the dynamic characteristics of FG beams. [8] **Huang et al.** presented a new approach for investigating the vibration behaviors of non-uniform AFG Timoshenko beams by changing the coupled governing equations to a single governing equation by introducing an auxiliary function. [9,10] **Huang and Li** studied the dynamic and buckling behavior of AFG tapered beams by reducing the corresponding governing differential equation to Fredholm integral equations. [11] **Aydogdu**, [12] **Elishakoff et al.** and [13] **Wu et al.** investigated the free vibrations of AFG tapered beams using the semi inverse method. [14] **Mazzei and Scott** studied stability and vibration of AFG tapered shafts loaded by axial compressive forces.

III. OBJECTIVE OF THE STUDY

To study the free vibrational analysis of cantilever beams of varying cross sections such as rectangular and circular, stepped beams and study the result of position of steps, Step depth and number of step present in the beams..

The objective of this research is to study and simulate the vibration characteristics of a vibration of a simply supported beam without and with attached multiple absorbers. Based on the research, there are several objectives that need to achieve.

- i. To determine the vibration reduction of a single vibration absorber attach to a beam
- ii. To investigate the effect of mass and damping on the absorber performance.
- iii. To determine the effect of attaching vibration absorber to reduce vibration level of a cantilever beam.

IV. VARIOUS METHOS FOR FINDING NATURAL FREQUENCIES

4.1 Finite Element Method

Among the numerical tools finite element method (FEM) is a competent one for the dynamic analysis of structures. Frequency-independent polynomial shape functions are used in the formulation of conventional FEM models. These can work for dynamic problems with lower frequencies wave modes but solutions become increasingly inaccurate with higher modes, where FEM model needs very large number of elements for better accuracy.

4.2 Dynamic Stiffness Method

It is an exact solution method. Here exact wave solutions to the governing differential equations is obtained to derive exact dynamic shape function leading to formulation of exact dynamic stiffness matrix in the frequency domain. In Dynamic Stiffness Method (DSM), governing differential equations adopted in the formulation of exact dynamic stiffness matrix decide the accuracy level.

4.3 Spectral Analysis Method

Among the frequency-domain methods the spectral analysis method (SAM) is one corresponding to the solutions by continuous Fourier transformation. Instead of continuous Fourier Transform, Discrete Fourier Transform (DFT) is widely practiced. This approach involves determining an infinite set of spectral components (or Fourier coefficients) in the frequency domain and performing the inverse Fourier transform to reconstruct the time histories of the solutions transform.

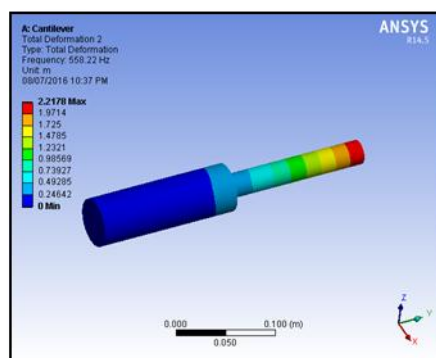
4.4 Spectral Element Method

Assembly and meshing of finite elements, exactness of the dynamic stiffness matrix with minimum number of DOFs from DSM and superposition of wave modes via DFT theory and FFT algorithm from SAM is found in Spectral element method (SEM).

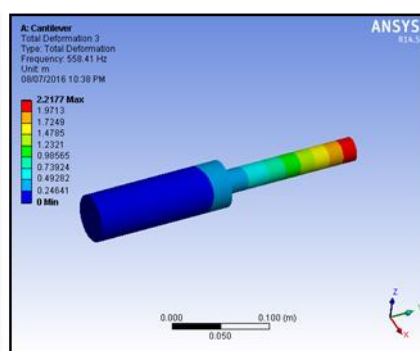
Case Study For Mils Steel

Cantilever Stepped Beam:-Model Dimensions (Big Dia. 50mm , Small dia 25 mm and Length 150 mm each)

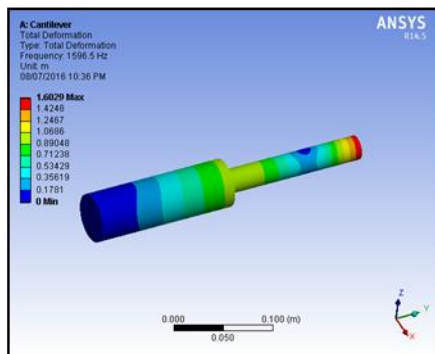
Mechanical Properties:- Young's modulus (E) of mild steel is 210 Gpa, Yield strength (Sy) is 205 Mpa, Ultimate tensile strength (Sut) is 515 Mpa , density is 7850 kg/m^3 , young's Modulus is 210 Gpa



Natural Frequency 1



Natural Frequency 2



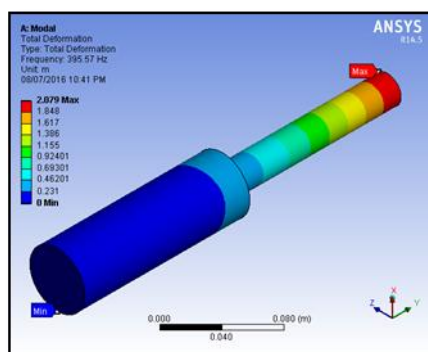
Sr. No.	Natural frequency at different nodes
Node 1	558.22 Hz
Node 2	558.41 Hz
Node 3	1596.5 Hz

Natural Frequency 3

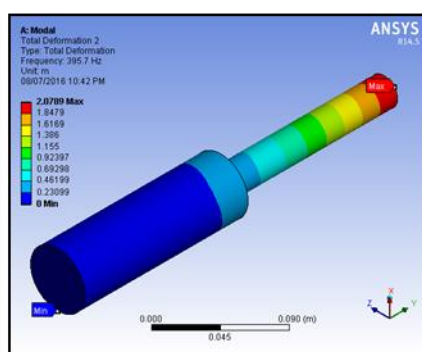
All Natural Values of mild steel Cantilever Beam

CANTILEVER COPPER BEAM WITHOUT DAMPING:- Model specifications are kept same.

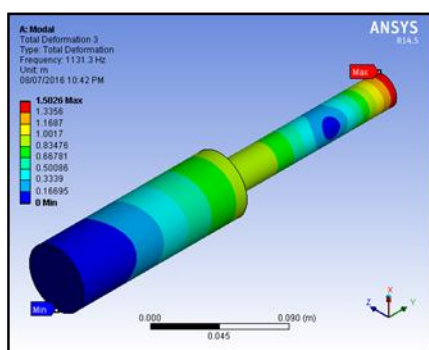
Young's modulus: - 1.2 Gpa. Density :- 8900 kg/m³



Natural Frequency 1



Natural Frequency 2



Natural Frequency 3

Sr. No.	Natural frequency at different nodes
Node 1	395.57 Hz
Node 2	395.7 Hz
Node 3	1131.3 Hz

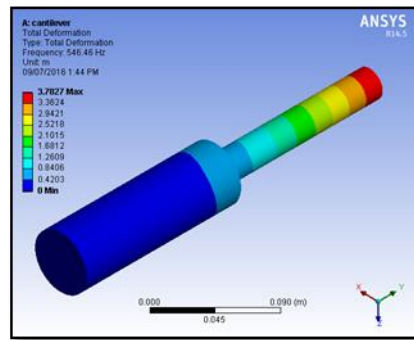
Beam

All Natural Values of Copper Cantilever

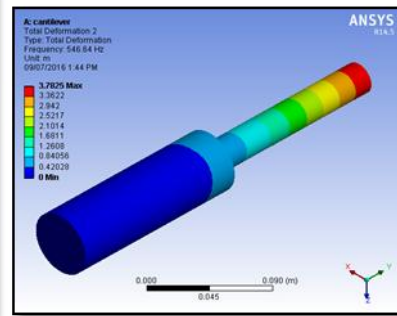
CANTILEVER ALUMINUM BEAM WITHOUT DAMPING:-

Young's modulus: - 0.69 Gpa

Density :- 2700 kg/m³



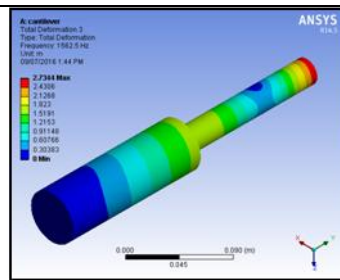
Natural Frequency 1



Natural Frequency 2

The values of natural frequency for Aluminum without damper are listed in table

Sr. No.	Natural frequency at different nodes
Node 1	546.46 Hz
Node 2	546.64 Hz
Node 3	1562.5 Hz



Natural Frequency 3

Natural frequency Result Table For circularsection using Mild Steel, Aluminum, and Copper

Sr. No.	Natural frequency at different nodes for Aluminum	Natural frequency at different nodes for Copper	Natural frequency at different nodes for Mild Steel
Node 1	546.46 Hz	395.57 Hz	558.22 Hz
Node 2	546.64 Hz	395.7 Hz	558.41 Hz
Node 3	1562.5 Hz	1131.3 Hz	1596.5 Hz

Damping frequency Result Table for circular section using Mild Steel, Aluminum, and Copper

Sr. No.	Damped frequency at different nodes for Aluminum	Damped frequency at different nodes for Copper	Damped frequency at different nodes for Mild Steel
Node 1	545.6 Hz	365.8 Hz	540.99 Hz
Node 2	545.79 Hz	365.92 Hz	541.17 Hz
Node 3	1560.4 Hz	1045.8 Hz	1547.2 Hz

Natural frequency Result Table for square section using Mild Steel, Aluminum, and Copper

Sr. No.	Natural frequency at different nodes for Aluminum	Natural frequency at different nodes for Copper	Natural frequency at different nodes for Mild Steel
Node 1	626.51 Hz	419.44 Hz	626.07 Hz
Node 2	626.79 Hz	419.55 Hz	626.26 Hz
Node 3	1772.9 Hz	1186.6 Hz	1772.3 Hz

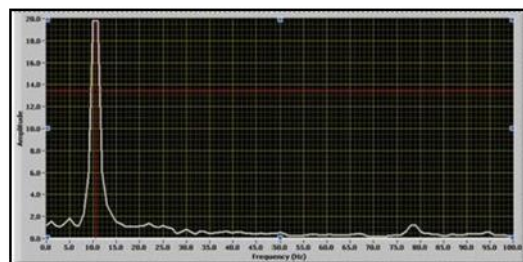
Sr. No.	Damped frequency at different nodes for Aluminum	Damped frequency at different nodes for Copper	Damped frequency at different nodes for Mild Steel
Node 1	626.51 Hz	424.19 Hz	620.17 Hz
Node 2	626.79 Hz	424.43 Hz	620.36 Hz
Node 3	1772.9 Hz	1195.3 Hz	1755.6 Hz

Damped frequency Result Table for square section using Mild Steel, Aluminum, and Copper

V. EXPERIMENTAL VALIDATION:-

Calculation of experimental natural frequency

To observe the natural frequencies of the cantilever beam subjected to small initial disturbance experimentally up to third mode, the experiment was conducted with the specified cantilever beam specimen. The data of time history (Displacement-Time), and FFT plot was recorded. The natural frequencies of the system can be obtained directly by observing the FFT plot. The location of peak values relates to the natural frequencies of the system. Fig. below shows a typical FFT plot



- A beam of a particular material (steel, aluminum or copper), dimensions (L, w, d) and transducer (i.e., measuring device, e.g. strain gauge, accelerometer, vibrato meter) was chosen.
- One end of the beam was clamped as the cantilever beam support.
- An accelerometer (with magnetic base) was placed at the free end of the cantilever beam ,to observe the free vibration response (acceleration).
- An initial deflection was given to the cantilever beam and allowed to oscillate on its own. To get the higher frequency it is recommended to give initial displacement at an arbitrary position apart from the free end of the beam (e.g. at the mid span).
- The reading is observed

Good agreement between the theoretically calculated natural frequency and the experimental one is found. The correction for the mass of the sensor will improve the correlation better. The present theoretical calculation is based on the assumption that one end of the cantilever beam is properly fixed. However, in actual practice it may not be always the case because of flexibility in support. The experimental values obtained are 559.5 Hz for first mode. The Various results are listed below.

SQUARE SCETION

- Mild Steel Square cross section the natural frequency is 559.5 Hz
- Copper with square cross section having step the natural frequency is 385.88 Hz
- Aluminum with square cross section step the natural frequency is 551.3.

CIRCULAR SECTION

- Mild Steel the natural frequency is 491.22 Hz
- Copper the natural frequency is 340.19 Hz
- Aluminum the natural frequency is 502.8 Hz

VI. CONCLUSION

From this study we have concluded following results Square section of Mild steel have higher values of natural frequencies than circular section.

The frequencies of vibration of beams are more affected by the position of the depths. Near fixed end of a cantilever beam reduces free vibration frequencies more than a relatively bigger at free end.

The frequencies of circular section are smaller than the square section it means that the shape of the section plays important role for find the natural frequency of the beam. The natural frequency tables are listed below for both section which is self explemetry.

For Circular Section Stepped Beam

Sr. No	Natural frequency 1 Experimental Result	Natural frequency 1 Ansys Result	% Variation
Mild Steel	491.22	558.22	12.00%
Copper	340.19	359.57	5.3%
Aluminum	502.8	546.64	8.01%

For Square Section Stepped Beam

Sr. No	Natural frequency 1 Experimental Result	Natural frequency 1 Ansys Result	% Variation
Mild Steel	559.5	626.07	10.6%
Copper	385.88	419.44	8.00%
Aluminum	551.3	626.51	12.00%

REFERENCES

- [1] S. Kapuria, M. Bhattacharyya, A.N. Kumar, Bending and free vibration response of layered functionally graded beams: a theoretical model and its experimental validation, *Compos. Struct.*82 (2008) 390e402.
- [2] M. Simsek, T. Kocaturk, S.D. Akbas, Dynamic behavior of an axially functionally graded beam under action of a moving harmonic load, *Compos. Struct.*94 (2012) 2358e2364.
- [3] A. Shahba, S. Rajasekaran, Free vibration and stability of tapered EulerBernoulli beams made of axially functionally graded materials, *Appl. Math. Model* 36 (2012) 3094e3111.
- [4] A. Shahba, R. Attarnejad, S. Hajilar, Free vibration and stability of axially functionally graded tapered EulerBernoulli beams, *Shock Vib.* 18 (2011) 683e696.
- [5] A. Shahba, R. Attarnejad, S. Hajilar, A mechanical-based solution for axially functionally graded tapered EulerBernoulli beams, *Mech. Adv. Mater. Struct.*20 (2013) 696e707.
- [6] A. Shahba, R. Attarnejad, M. Marvi, S. Hajilar, Free vibration and stability analysis of axially functionally graded tapered Timoshenko beams with classical and non-classical boundary conditions, *Compos. Part B* 42 (2011) 801e808.
- [7] A.E. Alshorbagy, M.A. Eltaher, F.F. Mahmoud, Free vibration characteristics of a functionally graded beam by finite element method, *Appl. Math. Model* 35 (2011) 412e425.
- [8] Y. Huang, L.-E. Yang, Q.-Z. Luo, Free vibration of axially functionally graded Timoshenko beams with non-uniform cross-section, *Compos. Part B* 45 (2013) 1493e1498.
- [9] Y. Huang, X.-F. Li, A new approach for free vibration of axially functionally graded beams with non-uniform cross-section, *J. Sound. Vib.*329 (2010) 2291e2303.
- [10] Y. Huang, X.-F. Li, Buckling analysis of nonuniform and axially graded columns with varying flexural rigidity, *J. Eng. Mech.* 137 (2011) 73e81.
- [11] M. Aydogdu, Semi-inverse method for vibration and buckling of axially functionally graded beams, *J. Reinf. Plast. Compos.*27 (7) (2008) 683e691.
- [12] I. Elishakoff, Z. Guede, Analytical polynomial solutions for vibrating axially graded beams, *Mech. Adv. Mater. Struct.*11 (2004) 517e533.
- [13] L. Wu, Q. Wang, I. Elishakoff, Semi-inverse method for axially functionally graded beams with an Anti-symmetric vibration mode, *J. Sound. Vib.*284 (2005) 1190e1202.
- [14] A.J. Mazzei Jr., R.A. Scott, On the effects of non-homogeneous materials on the vibrations and static stability of tapered shafts, *J. Vib. Control* 19 (5) (2012) 771e786.