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PRIMARY TERNARY Γ-IDEALS IN TERNARY Γ-SEMIGROUPS

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ABSTRACT

In this paper the terms left primary Γ-ideal, lateral primary Γ-ideal ,right primary Γ-ideal ,primary Γ-ideal, left primary ternary Γ-semigroup , lateral primary ternary Γ-semigroup , right primary ternary Γ-semigroup ,primary ternary Γ-semigroup are introduced.It is proved that A be an Γ-ideal in a ternary Γ-semigroup T and if X,Y,Z are three Γ-ideals of T such that 1) $X\Gamma Y\Gamma Z \subseteq A$ and $Y \not\subseteq A$, $Z \not\subseteq A$ then $X \subseteq \sqrt{A}$ iff $x,y,z \in T$, $\langle x\rangle \Gamma \langle y\rangle \Gamma \langle z\rangle \subseteq A$ and $y \notin A$, $z \notin A$, $x \in \sqrt{A}$. 2) $X\Gamma Y\Gamma Z \subseteq A$ and $X \not\subseteq A$, $Z \not\subseteq A$ then $Y \subseteq \sqrt{A}$ iff $x,y,z \in T$, $\langle x\rangle \Gamma \langle y\rangle \Gamma \langle z\rangle \subseteq A$ and $x \notin A$, $z \notin A$, $y \in \sqrt{A}$. 3) $X\Gamma Y\Gamma Z \subseteq A$ and $X \not\subseteq A$, $Y \not\subseteq A$ then $Z \subseteq \sqrt{A}$ iff $x,y,z \in T$, $\langle x\rangle \Gamma \langle y\rangle \Gamma \langle z\rangle \subseteq A$ and $x \notin A$, $y \notin A$, $z \in \sqrt{A}$. Further it is proved that if T be a commutative ternary Γ-semigroup and A be a Γ-ideal of T then the conditions 1) A is left primary ternary Γ-ideal . 2) $X\Gamma Y\Gamma Z \subseteq A$ and $Y \not\subseteq A$, $Z \not\subseteq A$ then $X \subseteq \sqrt{A}$. 3) $x,y,z \in T$, $\langle x\rangle \Gamma \langle y\rangle \Gamma \langle z\rangle \subseteq A$ and $y \notin A$, $z \notin A$, $x \in \sqrt{A}$ are equivalent. It is also proved that if T be a commutative ternary Γ-semigroup and A be a Γ-ideal of T then the conditions (1)A is lateral primary ternary Γ-ideal. 2) $X\Gamma Y\Gamma Z \subseteq A$ and $X \not\subseteq A$, $Z \not\subseteq A$ then $Y \subseteq \sqrt{A}$ .3) $x, y, z \in T$, $\langle x\rangle \Gamma \langle y\rangle \Gamma \langle z\rangle \subseteq A$ and $x \notin A$, $z \notin A$, $y \in \sqrt{A}$. Further the conditions for an Γ-ideal in a commutative ternary Γ-semigroup T , 1) A is right primary Γ-ideal 2) $X\Gamma Y\Gamma Z \subseteq A$ and $X \not\subseteq A$, $Y \not\subseteq A$ then $Z \subseteq \sqrt{A}$ 3) $x,y,z \in T$, $\langle x\rangle \Gamma \langle y\rangle \Gamma \langle z\rangle \subseteq A$ and $x \notin A$, $y \notin A$, $z \in \sqrt{A}$ are equivalent . It is proved that every Γ-ideal A in a ternary Γ-semigroup T , 1) T is a left primary iff every Γ-ideal A satisfies X,Y,Z are three Γ-ideals of T such that $X\Gamma Y\Gamma Z \subseteq A$ and $Y \not\subseteq A$, $Z \not\subseteq A$ then $X \subseteq \sqrt{A}$. 2) T is a lateral primary iff every Γ-ideal A satisfies $X\Gamma Y\Gamma Z \subseteq A$ and $X \not\subseteq A$, $Z \not\subseteq A$ then $Y \subseteq \sqrt{A}$ .3) T is a right primary iff every Γ-ideal satisfies X,Y,Z are three Γ-ideals of T such that $X\Gamma Y\Gamma Z \subseteq A$ and $X \not\subseteq A$, $Y \not\subseteq A$ then $Z \subseteq \sqrt{A}$. It is proved that T be a ternary Γ-semigroup with identity and M be the unique maximal Γ-ideal in T. If $\sqrt{A} = M$ for some Γ-ideal A in T then A is a primary Γ-ideal. Further it is proved that if identity and M is the *unique maximal Γ-ideal of T then for any odd natural number n , $(m\Gamma)^{n-1}m$ is a primary Γ-ideal of T . It is proved that if A is a Γ-ideal of quasi commutative ternary Γ-semigroup T then 1) A is primary .2) A is left primary 3) A is lateral primary 4) A is right primary are equivalent.*

Subject Classification: 16Y30, 16Y99

Keywords: Left primary Γ-ideal lateral primaryΓ-ideal , right primary Γ-ideal , primary Γ-ideal , left primary ternary Γ-semigroup , lateral ternary Γ-semigroup , right ternary Γ-semigroup , primary ternary Γ-semigroup .

I INTRODUCTION

The literature of ternary algebraic system was introduced by D. H. Lehmer [8] in 1932. He investigated certain ternary algebraic systems called triplexes which turn out to be ternary groups. Dutta and Kar [7] have introduced the notion of ternary semi rings and characterized many results in terms of regular ternary semiring. In the year 1980, A. Anajneyulu made a study of primary ideals in semigroups. Later, in the year 2011, D. Madhusudhana Rao extended those results to Γ-semigroups. Further, D. Madhusudhana Rao and Ch. Manikya Rao applied those notions to ternary semigroups. In this paper we mainly introduce the notion of primary ternary Γ-ideals of ternary Γ-semigroup and characterize those primary ternary Γ-ideals.

II PRELIMINARIES

Definition 2.1: Let T and Γ be any two non-empty sets. T is called **a ternary Γ-semigroup** if there exists a mapping from $T\times\Gamma\times T\times\Gamma\times T$ to T which maps $(a,b,c,\alpha,\beta)\rightarrow a\alpha b\beta c$ satisfying the condition $(a\alpha b\beta c)\gamma d\delta e = a\alpha(b\beta c\gamma d)\delta e = a\alpha b\beta(c\gamma d\delta e)\ \forall\ a,b,c,d,e \in T,\ \alpha,\beta,\gamma,\delta \in \Gamma$.

Note 2.2: Let T be a ternary Γ –semigroup. If A,B,C are subsets of T we shall denote the set $\{a\alpha b\beta c : a \in A, b \in B, c \in C, \alpha,\beta \in \Gamma\}$ by $A\Gamma B\Gamma C$.

Definition 2.3: An element a of a ternary Γ –semigroup T is said to be **left identity** of T provided $a\alpha a\beta t = t\ \forall t \in T$, $\alpha,\beta \in \Gamma$.

Definition 2.4: An element a of a ternary Γ-semigroup T is said to be **right identity** of T provided $t\alpha a\beta a = t\ \forall t \in T$, $\alpha,\beta \in \Gamma$.

Definition 2.5: An element a of a ternary Γ –semigroup T is said to **be two sided identity or an identity** provided it is both left identity and right identity.

Note 2.6: Let T be a ternary Γ –semigroup. If T has an identity, let $T^1 = T$ and if T does not have an identity , let T^1 be the ternary Γ –semigroup T with an identity adjoined usually denoted by symbol 1.

Definition 2.7: A ternary Γ –semigroup T is said to be **commutative** provided $a\Gamma b\Gamma c = b\Gamma a\Gamma c = c\Gamma a\Gamma b = c\Gamma b\Gamma a = b\Gamma c\Gamma a \ \forall a,b,c \in T$.

Definition 2.8: A ternary Γ –semigroup T is said to be **quasi commutative** provided for all $a,b,c \in$ T there exists an odd natural number n such that $a\alpha b\alpha c = (b\alpha)^n a\alpha c = b\alpha c\alpha a = (c\alpha)^n b\alpha a = c\alpha a\alpha b = (a\alpha)^n c\alpha a \ \forall a,b,c \in T, \ \alpha \in \Gamma$.

Definition 2.9: A ternary Γ–semigroup T is said to be **globally idempotent** ternary Γ-semigroup provided $T\Gamma T\Gamma T = T$.

Definition 2.10: A ternary Γ –semigroup T is said to be **left duo ternary Γ-semigroup** provided every left ternary Γ-ideal of T is a two sided ternary Γ-ideal of T.

Definition 2.11: A ternary Γ –semigroup T is said to be **right duo ternary Γ-semigroup** provided every right ternary Γ-ideal of T is a two sided ternary Γ-ideal of T.

Definition 2.12: A ternary Γ –semigroup T is said to be **duo ternary Γ-semigroup** provided it is both left duo ternary Γ –semigroup and right ternary Γ –semigroup.

Definition 2.13: A nonempty subset A of a ternary Γ-semigroup T is said to be **right ternary Γ-ideal** provided $A\Gamma T\Gamma T \subseteq A$.

Definition 2.14: A nonempty subset A of a ternary Γ -semigroup T is said to be **two sided ternary Γ-ideal** provided it is both left and right ternary Γ-ideals of T.

Definition 2.15: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be **principal ternary Γ-ideal** provided A is a ternary Γ-ideal generated by single element a . It is denoted by $J[a] = <a>$.

Note 2.16: If T is a ternary Γ-semigroup and a∈ T then $<a> = J[a] = a \cup a\Gamma T\Gamma T \cup T\Gamma T\Gamma a \cup T\Gamma a\Gamma T = T\Gamma T^1\Gamma a\Gamma T^1\Gamma T^1$.

Definition 2.17: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be a **completely prime ternary Γ-ideal** provided $x\Gamma y\Gamma z \subseteq A \ \forall x, y, z \in T$ implies either $x \in A$ or $y \in A$ or $z \in A$.

Definition 2.18: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be a **prime ternary** Γ- ideal provided $X\Gamma Y\Gamma Z \subseteq A$ where X,Y,Z are ternary Γ-ideals then either $X \subseteq A$ or $Y \subseteq A$ or $Z \subseteq A$.

Definition 2.19: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be a **completely semiprime ternary Γ- ideal** provided $x\Gamma x\Gamma x \subseteq A; x \in T$ implies either $x \in A$.

Definition 2.20: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be a **semiprime ternary Γ- ideal** provided $x\Gamma x\Gamma T\Gamma x\Gamma x \subseteq A; x \in T$ implies either $x \in A$.

Theorem 2.21: Every prime ternary Γ- ideal of a ternary Γ–semigroup T is a semiprime ternary Γ- ideal of T.

Definition 2.22: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be **semipseudo symmetric ternary Γ-ideal** provided for any odd natural number n , $x \in T$, $(x\Gamma)^{n-1} x \subseteq A \Rightarrow (<x> \Gamma)^{n-1}<x> \subseteq A$.

III PRIMARY TERNARY Γ –IDEALS

Definition 3.1: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be **left primary ternary Γ-ideal** provided

1. If X, Y, Z are three ternary Γ- ideals of T such that $X\Gamma Y\Gamma Z \subseteq A$ and $Y \nsubseteq A$, $Z \nsubseteq A$ then $X \subseteq \sqrt{A}$.
2. $\sqrt{A}$ is a prime ternary Γ- ideal of T.

Definition 3.2: A ternary Γ- ideal A of a ternary Γ–semigroup T is said to be **lateral primary ternary Γ- ideal** provided

1. If X, Y, Z are three ternary Γ- ideals of T such that $X\Gamma Y\Gamma Z \subseteq A$ and $X \nsubseteq A$, $Z \nsubseteq A$ then $Y \subseteq \sqrt{A}$.
2. $\sqrt{A}$ is a prime ternary Γ- ideal of T.

Definition 3.3: A ternary Γ- ideal A of a ternary Γ –semigroup T is said to be **right primary ternary Γ-ideal** provided

1. If X, Y, Z are three ternary Γ- ideals of T such that $X\Gamma Y\Gamma Z \subseteq A$ and $X \nsubseteq A$, $Y \nsubseteq A$ then $Z \subseteq \sqrt{A}$.
2. $\sqrt{A}$ is a prime ternary Γ- ideal of T.

Definition 3.4: A ternary Γ- ideal A of a ternary Γ–semigroup T is said to be **primary ternary Γ-ideal** provided A is left primary ternary Γ- ideal , lateral primary ternary Γ- ideal , right primary ternary Γ- ideal.

Theorem 3.5: Let A be a ternary Γ-ideal of a ternary Γ-semigroup T. If X, Y, Z are three ternary Γ-ideals of T such that $X\Gamma Y \Gamma Z \subseteq A$ and $Y \nsubseteq A$, $Z \nsubseteq A$ then $X \subseteq \sqrt{A}$ iff $<x> \Gamma <y> \Gamma <z> \subseteq A$ and $y \notin A$, $z \notin A$, $x \in \sqrt{A}$.

Proof: Suppose that A is a ternary Γ – ideal of a ternary Γ – semigroup T and if X, Y, Z are three ternary Γ–ideals of T such that X ΓY ΓZ ⊆A and y ⊈ A , z ⊈ A then x ⊆ $\sqrt{A}$.

Let x, y, z ∈ T , y ∉ A , z ∉ A . Then <x>Γ<y>Γ<z> ⊆ XΓYΓZ ⊆ A and <y> ⊈ A , <z> ⊈ A.

∴ by supposition <x>Γ<y>Γ<z> ⊆ A and < y >⊈A, < z >⊈A ⟹ < x >⊆ $\sqrt{A}$. ∴ x ∈ $\sqrt{A}$.

Conversely suppose that x,y,z ∈ T , <x> Γ<y> Γ<z> ⊆ A and y∉A , z∉A then x ∈ $\sqrt{A}$.

Let X, Y, Z be three ternary Γ – ideals of T such that XΓY ΓZ ⊆ A and y⊈A , z⊈A .

Suppose if possible X ⊈ $\sqrt{A}$. then there exists x ∈ X such that x ∉ $\sqrt{A}$.

Since Y ⊈ A , let y ∈ Y so that y ∉ $\sqrt{A}$ .Since Z ⊈ A , let z ∈ Z so that z ∉ $\sqrt{A}$.

Now $\langle x\rangle\ \Gamma\langle y\rangle\ \Gamma\langle z\rangle \subseteq X\ \Gamma\ Y\ \Gamma\ Z \subseteq A$ and $y\notin A$, $z\notin A \Rightarrow x \in \sqrt{A}$. It is a contradiction. Therefore, $x \subseteq \sqrt{A}$.

Therefore if X, Y, Z are three ternary Γ – ideals of T such that $X\ \Gamma Y\ \Gamma Z \subseteq A$ and $y\not\subseteq A$, $z\not\subseteq A$ then $X \subseteq \sqrt{A}$. Hence the theorem.

Theorem 3.6: Let A be a ternary Γ–ideal of a ternary Γ-semigroup T. If X, Y, Z are three ternary Γ-ideals of T such that $X\Gamma Y\ \Gamma Z \subseteq A$ and $X \not\subseteq A$, $Z \not\subseteq A$ then $Y \subseteq \sqrt{A}$ *iff* $\langle x\rangle\ \Gamma\langle y\rangle\ \Gamma\langle z\rangle \subseteq A$ *and* $x\notin A$, $z\notin A$, $y \in\sqrt{A}$.

Proof: The proof is similar to the theorem 3.5.

Theorem 3.7: **Let A be a ternary Γ – ideal of a ternary Γ-semigroup T. If X, Y, Z are three ternary Γ-ideals of T such that $X\Gamma Y\ \Gamma Z \subseteq A$ and $X \not\subseteq A$, $Y \not\subseteq A$ then $Z \subseteq \sqrt{A}$ *iff* $\langle x\rangle\ \Gamma\langle y\rangle\ \Gamma\langle z\rangle \subseteq A$ *and* $x\notin A$, $y\notin A$, $z \in \sqrt{A}$.**

Proof: The proof is similar to the theorem 3.5.

Theorem 3.8: Let T be a commutative ternary Γ – semigroup and A is a ternary Γ-ideal of T. Then the following conditions are equivalent.

1. A is a primary ternary Γ- ideal

2. X, Y, Z are three ternary Γ-ideals of T , $X\Gamma Y\ \Gamma Z \subseteq A$ and $Y \not\subseteq A$, $Z \not\subseteq A$ then $X \subseteq \sqrt{A}$

3. x, y, z ∈T, $x\ \Gamma y\ \Gamma z \subseteq A$, $y\notin A$,$z\notin A$ then $x \in \sqrt{A}$.

Proof: (1) ⇒ (2) : Suppose A is a primary ternary Γ– ideal of T . Then A is a left primary ternary Γ– ideal of T. So, by definition 3.1, we get X, Y, Z are three ternary Γ– ideals of T , $X\Gamma Y\Gamma Z \subseteq A$, $Y\not\subseteq A, Z\not\subseteq A \Rightarrow X \subseteq \sqrt{A}$

(2) ⇒ (3): Suppose that X,Y,Z are three ternary Γ – ideals of T , $X\Gamma Y\Gamma Z \subseteq A$ and $Y\not\subseteq A$, $Z\not\subseteq A$ then $X \subseteq \sqrt{A}$. Let x,y,z ∈ T , $x\Gamma y\Gamma z \subseteq A$, $y\notin A$, $z\notin A$, $x\Gamma y\Gamma z \subseteq A \Rightarrow \langle x\rangle\Gamma\langle y\rangle\Gamma\langle z\rangle \subseteq A$. Also, $y\notin A$, $z\notin A$. Now $\langle x\rangle\Gamma\langle y\rangle\Gamma\langle y\rangle \subseteq A$ and $\langle y\rangle\not\subseteq A$, $\langle z\rangle\not\subseteq A$. ∴ by assumption $\langle x \rangle \subseteq \sqrt{A} \Rightarrow x \in \sqrt{A}$.

(3) ⇒ (1): suppose $x,y,z \in T$, $x\ \Gamma y\ \Gamma z \subseteq A$, $y\notin A$, $z\notin A$ *then* $x \in \sqrt{A}$. Let X, Y, Z are three ternary Γ– ideals of T , $X\Gamma Y\Gamma Z \subseteq A$ and $Y\not\subseteq A$, $Z\not\subseteq A$ ⇒ there exists $y\in Y$, $z\in Z$ such that $y\notin A$, $z\notin A$.

Suppose if possible $X\not\subseteq\sqrt{A}$. Then there exists x∈X such that $x\notin\sqrt{A}$.

Now $x\Gamma y\Gamma z \subseteq X\Gamma Y\Gamma Z \subseteq A$. ∴ $x\Gamma y\Gamma z \subseteq A$ and $y\notin A$, $z\notin A$ *and* $x\notin A$. It is a contradiction.

∴ $X \subseteq \sqrt{A}$. Let $x,y,z \in T$, $x\Gamma y\Gamma z \subseteq \sqrt{A}$. Suppose $y\notin\sqrt{A}$, $z\notin\sqrt{A}$.

Now $x\ \Gamma y\Gamma z \subseteq \sqrt{A} \Rightarrow (x\Gamma y\Gamma z\Gamma)^{m-1}$ for some odd natural number $m \Rightarrow (x\Gamma)^{m-1}x\Gamma(y\Gamma)^{m-1}y\Gamma(z\Gamma)^{m-1}z \subseteq A \Rightarrow (y\Gamma)^{m-1}y\Gamma$ $\not\subseteq A$, $(z\Gamma)^{m-1}z \not\subseteq A \Rightarrow (x\Gamma)^{m-1}x \subseteq \sqrt{A} \Rightarrow x \in \sqrt{(\sqrt{A})} = \sqrt{A}$.

$\sqrt{A}$ is a completely prime ternary Γ-ideal and hence $\sqrt{A}$ is a prime ternary Γ– ideal.

∴ A is a left primary ternary Γ-ideal. Similarly, A is a right primary ternary Γ-ideal and A is a lateral primary ternary Γ-ideal. Hence A is a primary ternary Γ-ideal.

Note 3.9: In an arbitrary ternary Γ-semigroup a left primary ternary Γ – ideal is not necessarily a right primary ternary Γ – ideal.

Theorem 3.10: Let T be a commutative ternary Γ – semigroup and A is a ternary Γ – ideal of T . Then the following conditions are equivalent.

1. A is a primary ternary Γ – ideal.

2. X, Y, Z are three ternary Γ – ideals of T, X ΓY ΓZ ⊆ A and X⊈A,Z⊈A then Y ⊆ $\sqrt{A}$

3. x, y, z ∈ T, xΓyΓz ⊆ A, x∉A, z∉A then x ∈ $\sqrt{A}$.

Proof: The proof is similar to theorem 3.8.

Theorem 3.11: Let T be a commutative ternary Γ – semigroup and A is a ternary Γ – ideal of T. Then the following conditions are equivalent .

1. A is a primary ternary Γ – ideal.

2. X, Y, Z are three ternary Γ – ideals of T, XΓYΓZ ⊆ A , x⊈A,Y⊈A then Z ⊆ $\sqrt{A}$.

3. x, y, z ∈ T , xΓyΓz ⊆ A , x∉A, y∉A then z ∈ $\sqrt{A}$.

Proof: The proof is similar to theorem 3.8 .

Theorem 3.12: Every ternary Γ-ideal A in a ternary Γ– semigroup T is left primary iff every ternary Γ-ideal A satisfies that X,Y,Z are three ternary Γ-ideals of T such that X ΓY ΓZ ⊆ A and Y ⊈ A , Z⊈ A ⇒ X ⊆ $\sqrt{A}$.

Proof : If every ternary Γ – ideal A in T is left primary then clearly every ternary Γ – ideal satisfies if X,Y,Z are three ternary Γ – ideals of T such that XΓYΓZ ⊆ A and y⊈A , z ⊈ A ⇒ X ⊆ $\sqrt{A}$ .Conversly suppose that for every ternary Γ – ideal A of T satisfies that XΓYΓZ ⊆ A and y⊈A , z⊈A ⇒ X ⊆ $\sqrt{A}$. Let A be any ternary Γ – ideal in T. Suppose that < X >Γ< Y >Γ< Z > ⊆ $\sqrt{A}$. If < Y > ⊈ $\sqrt{A}$, < Z > ⊈ $\sqrt{A}$ then by our assumption $x \in \sqrt{\sqrt{A}} = \sqrt{A}$. Therefore $\sqrt{A}$ is a prime ternary Γ-ideal . Hence A is left ternary.

Theorem 3.13: Every ternary Γ-ideal A in a ternary Γ-semigroup T is lateral primary iff every ternary Γ-ideal A satisfies that X, Y, Z are three ternary Γ-ideals of T such that XΓYΓZ ⊆ A and X ⊈ A, Z ⊈ A ⇒ Y ⊆ $\sqrt{A}$.

Proof: The proof is similar to theorem 3.12.

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Theorem 3.14: Every ternary Γ-ideal A in a ternary Γ-semigroup T is right primary iff every ternary Γ-ideal A satisfies that X, Y, Z are three ternary Γ-ideals of T such that $X\Gamma Y\Gamma Z \subseteq A$ and $X \nsubseteq A$, $Y \nsubseteq A$ $\Rightarrow Z \subseteq \sqrt{A}$.

Proof : The proof is similar to theorem 3.12.

Definition 3.15: A ternary Γ-semigroup T is said to be **left primary** provided every ternary Γ-ideal in T is a left primary ternary Γ-ideal.

Definition 3.16: A ternary Γ-semigroup T is said to be **lateral primary** provided every ternary Γ-ideal in T is a lateral primary ternary Γ-ideal.

Definition 3.17: A ternary Γ-semigroup T is said to be **right primary** provided every ternary Γ-ideal in T is a right primary ternary Γ-ideal.

Definition 3.18: A ternary Γ-semigroup T is said to be **primary** provided every ternary Γ-ideal in T is a primary ternary Γ-ideal .

Theorem 3.19: Let T be a ternary Γ-semigroup with identity and let M be the unique Maximal ternary Γ-ideal of T . If $\sqrt{A}$ = M for some ternary Γ-ideal of T then A is a primary ternary Γ-ideal.

Proof: Let $<x> \Gamma <y> \Gamma <z> \subseteq A$ and $y \notin A$, $z \notin A$. *If* $x \notin \sqrt{A}$ *then* $<x> \nsubseteq \sqrt{A} = M$. Since M is union of all proper ternary Γ – ideals of T , we have $<x> = T$ *and hence* $<y> = <z> = <x> \Gamma <y> \Gamma <z> \subseteq \sqrt{A}$. It is a contradiction. Therefore, $x \in \sqrt{A}$. Let $<x> \Gamma <y> \Gamma <z> \subseteq \sqrt{A}$ and $<y> \nsubseteq \sqrt{A}$, $<z> \nsubseteq \sqrt{A}$. Since M is the maximal ternaryΓ-ideal we have $<x> = T$. hence $<y> = <z> = <x> \Gamma <y> \Gamma <z> \subseteq \sqrt{A}$. It is a contradiction. Therefore $<x> \subseteq \sqrt{A}$. Similarly, if $<x> \nsubseteq \sqrt{A}$ then $<y> \subseteq \sqrt{A}$, $<z> \subseteq \sqrt{A}$ and hence $\sqrt{A}$ = M is a prime ternary Γ – ideal . Thus A is left primary. By symmetry it follows that A is right primary, lateral primary. Therefore, A is a primary ternary Γ – ideal.

Note 3.20: If a ternary Γ semigroup T has no identity then the theorem 3.19 is not true even if the ternary Γ–semigroup T has a unique maximal ternary Γ-ideal.

Theorem 3.21: If T is a ternary Γ-semigroup with identity then for any odd natural number n, $(m\Gamma)^{n-1} m$ is primary ternary Γ-ideal of T where m is unique maximal ternary Γ-ideal of T.

Proof: Since M is the only prime ternary Γ-ideal containing $(m\,\Gamma)^{n-1}m$, we have $\sqrt{(m\,\Gamma)^{n-1}m} = m$ and hence by theorem 3.19 , $(m\,\Gamma)^{n-1}m$ is prime ternary Γ-ideal .

Note 3.22: If T has no identity then theorem 3.21 is not true.

Theorem 3.23: In Quasi commutative ternary Γ-semigroup T a ternary Γ-ideal A of T is left primary iff right primary.

Proof: Suppose A is a left primary ternary Γ-ideal. Let $x\,\Gamma y\,\Gamma z\in A$. Since T is a quasi-commutative ternary Γ-semigroup we have $x\,\Gamma y\,\Gamma z = y\,\Gamma z\,\Gamma x = (z\,\Gamma)^{n-1} z\,\Gamma y\,\Gamma x = z\,\Gamma x\,\Gamma y = (x\,\Gamma)^{n-1} x\,\Gamma z\,\Gamma y$ for some odd natural number n. So $(z\,\Gamma)^{n-1} z\,\Gamma y\,\Gamma x \in A$ and $x\notin A$, $y\notin A$. Since A is left primary we have $(z\,\Gamma)^{n-1} z \in\sqrt{A}$ and since $\sqrt{A}$ is prime ideal $z\in\sqrt{A}$. Therefore A is a right primary ternary Γ-ideal.

Similarly, we can prove that if A is a right primary ternary Γ-ideal then A is a left ternary Γ-ideal.

Theorem 3.24: In a quasi-commutative ternary Γ-semigroup T a ternary Γ-ideal A of T is left primary iff A is lateral primary.

Proof: Suppose that A is a left primary ternary Γ-ideal. Let $x\,\Gamma y\,\Gamma z \in A$ *and* $x\in A$, $z\notin A$. Since T is quasi commutative ternary Γ-semigroup we have $x\,\Gamma y\,\Gamma z = y\,\Gamma z\,\Gamma x = (z\,\Gamma)^{n-1} z\,\Gamma y\,\Gamma x = z\,\Gamma x\,\Gamma y = (x\,\Gamma)^{n-1} x\,\Gamma z\,\Gamma y$ for some odd natural number n. *So* $y\,\Gamma z\,\Gamma x\in A$ *and* $x\notin A$, $z\notin A$. Since A is left primary we have $y\in\sqrt{A}$ and since $\sqrt{A}$ is prime ternary Γ-ideal, $y\in\sqrt{A}$. Therefore, A is lateral primary ternary Γ-ideal.

Similarly we can prove that if A is a lateral ternary Γ-ideal then A is left primary ternary Γ-ideal .

Corollary 3.25: If A is a ternary Γ-ideal of quasi commutative ternary Γ-semigroup T then the following are equivalent

1. A is primary

2. A is left primary

3. A is lateral primary

4. A is right primary.

IV CONCLUSION

In this paper, efforts are made to introduce the notion of primary ternary Γ-ideals in ternary Γ-semigroups and characterize them. This literature of primary ternary Γ-ideals can use many other algebraic strictures.

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