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A PECULIAR CASE OF UNLIMITED RESOURCES IN ECOLOGICAL AMMENSALISM

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ABSTRACT

The paper aims to discuss a peculiar case of Ecological Ammensalsim. The model is constructed by a coupled system of first order non-linear ordinary differential equations. Both the species have unlimited resources. A series solution of this Ammensalal model is derived.

Keywords: Ammensalism, Homotopy Analysis.

I. INTRODUCTION

Abbasbandy,S [1] used this perturbation technique and invented some innovative results in the concept of asymptotic techniques . Liao[5-8] develped Homotopy Perturbation Method (HPM) in 1992. Few other methods with independent physical parameters were introduced by eminent Mathematicians [2,4]. HPM methodology has been used in many fields of Engineering and Modern Sciences[3,9,10].

II. BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

Step (1): Let us consider nonlinear differential equation:

$$A(u)-f(r)=0,\ \ r\in\Omega \qquad \text{(I)}$$

With the boundary condition

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\qquad r\in\Gamma$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω .

Step (2): In general the operator A , is divided into two parts: linear part *L* and nonlinear part *N*. Therefore above differential equation(I) is expressed in the form of

$$L(u)-N(u)-f(r)=0 \qquad \text{(II)}$$

Step (3):

With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy $v(r,p):\Omega\times[0,1]\to R$ which satisfies

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0,\ \ p\in[0,1],r\in\Omega \qquad \text{(III)}$$

It is nothing but

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[A(v)-f(r)\right]=0 \qquad \text{(IV)}$$

where $p\in[0,1]$ is named as an embedding parameter ,and u_0 is an initial approximation of equation(1),which satisfies the boundary conditions.

Step (4): Then equations (III), (IV) follow that

$$H(v,0)=L(v)-L(u_0)=0$$

and $H(v,1)=A(v)-f(r)=0$

Thus the changing process of p from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$.

Step (5): According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (III) and (IV) can be written as a power series in p :

$$v=v_0+pv_1+p^2v_2+p^3v_3+p^4v_4+----------$$

The approximate solution of equation (I) can be obtained as

$$u=\underset{p\to 1}{Lt}\, v=v_0+v_1+v_2+v_3+v_4+-----------$$

III. NOTATIONS ADOPTED:

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) :The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{12} :The inhibition coefficient of S_1 due to S_2 i.e The Commensal coefficient.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α , h_1, h_2 are assumed to be non-negative constants.

IV. BASIC EQUATIONS:

$$\frac{dN_1}{dt}=a_1N_1-a_{12}N_1N_2 \qquad (1)$$

$$\frac{dN_2}{dt}=a_2N_2 \quad \text{with initial conditions } N_1(0)=c_1 \text{ and } N_2(0)=c_2 \qquad (2)$$

The following system can be constructed by the concept of homotopy as follows

$$v_1'-N_{10}'+p(N_{10}'-a_1v_1-a_{12}v_1v_2)=0 \qquad (3)$$

$$v_2'-N_{20}'+p(N_{20}'-a_2v_2)=0 \qquad (4)$$

The initial approximations are considered as

$$v_{1,0}(t)=N_{10}(t)=v_1(0)=c_1 \qquad (5)$$

$$v_{2,0}(t)=N_{20}(t)=v_2(0)=c_2 \qquad (6)$$

and $v_1(t)=v_{1,0}(t)+pv_{1,1}(t)+p^2v_{1,2}(t)+p^3v_{1,3}(t)+p^4v_{1,4}(t)+p^5v_{1,5}(t)+\cdots$ (7)

$$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots \quad (8)$$

Where $v_{i,J}(i = 1,2, J = 1,2,3 \ldots)$ *are to be computed by substituting* (5), (6), (7), (8) *in* (3), (4)

We get

$$v'_{1,0}(t) + pv'_{1,1}(t) + p^2v'_{1,2}(t) + p^3v'_{1,3}(t) + p^4v'_{1,4}(t) + p^5v'_{1,5}(t) + \cdots - N'_{10} +$$
$$p[N'_{10} - a_1(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots)$$
$$+a_{12}(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + p^5v_{1,5}(t) + \cdots)(v_{2,0}(t) +$$
$$pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots)] = 0 \quad (9)$$

From equation (4)

$$v'_{2,0}(t) + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - N'_{20}$$
$$+p[N'_{20} - a_2(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + p^5v_{2,5}(t) + \cdots] = 0 \quad (10)$$

From (9),

$$0 + pv'_{1,1}(t) + p^2v'_{1,2}(t) + p^3v'_{1,3}(t) + p^4v'_{1,4}(t) + p^5v'_{1,5}(t) + \cdots - 0$$
$$+p[0 - a_1v_{1,0}(t) - a_1pv_{1,1}(t) - a_1p^2v_{1,2}(t) - a_1p^3v_{1,3}(t) - a_1p^4v_{1,4}(t) - a_1p^5v_{1,5}(t) - \ldots$$
$$+a_{12}p^3v_{1,0}(t)v_{2,3}(t) + a_{12}p^4v_{10}(t)v_{2,4}(t)\ldots\ldots + a_{12}\,pv_{1,1}(t)v_{2,0}(t) + a_{12}p^2v_{1,1}(t)v_{2,1}(t)$$
$$+a_{12}p^3v_{1,1}(t)v_{2,2}(t) + a_{12}p^4v_{1,1}(t)v_{2,3}(t).\ldots + a_{12}p^2v_{2,0}(t)v_{1,2}(t) + a_{12}p^3v_{1,2}(t)v_{2,1}(t)$$
$$+a_{12}p^4v_{1,2}(t)v_{2,2}(t)\ldots + a_{12}p^3v_{1,3}(t)v_{2,0}(t) + a_{12}p^4v_{1,3}(t)v_{2,1}(t).\ldots + a_{12}p^4v_{1,4}(t)v_{2,0}(t)$$
$$\ldots.] = 0 \quad (11)$$

From (10),

$$0 + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - 0 + p[0 - a_2v_{2,0}(t)$$
$$-a_2pv_{2,1}(t) - a_2p^2v_{2,2}(t) - a_2p^3v_{2,3}(t) - a_2p^4v_{2,4}(t) - a_2p^5v_{2,5}(t)\ldots] = 0 \quad (12)$$

Now comparing the coefficient of various powers of p in (11)&(12),we obtain

The co efficient of P^1:

$$v'_{1,1}(t) - a_1v_{1,0}(t) + a_{12}v_{1,0}(t)v_{2,0}(t) = 0$$
$$v'_{2,1}(t) - a_2v_{2,0}(t) = 0$$

The co efficient of P^2:

$$v'_{1,2}(t) - a_1v_{1,1}(t) + a_{12}v_{1,0}(t)v_{2,1}(t) + a_{12}v_{1,1}(t)v_{2,0}(t) = 0$$
$$v'_{2,2}(t) - a_2v_{2,1}(t) = 0$$

The co efficient of P^3:

$$v'_{1,3}(t) - a_1v_{1,2}(t) + a_{12}v_{1,0}(t)v_{2,2}(t) - a_{12}v_{1,1}(t)v_{2,1}(t) + a_{12}v_{2,0}(t)v_{1,2}(t) = 0$$
$$v'_{2,3}(t) - a_2v_{2,2}(t) = 0$$

The co efficient of P^4:

$$v'_{1,4}(t) - a_1v_{1,3}(t) + a_{12}v_{1,0}(t)v_{2,3}(t) + a_{12}v_{1,1}(t)v_{2,2}(t) + a_{12}v_{2,1}(t)v_{1,2}(t)$$

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$+ a_{12}v_{2,0}(t)v_{1,3}(t) = 0$

$v_{2,4}'(t) - a_2 v_{2,3}(t) = 0$

Now $v_1(0) = c_1, v_2(0) = c_2$

$$v_{1,1}(t) = a_1 \int_0^t v_{1,0}(t)dt - a_{12}\int_0^t v_{1,0}(t)v_{2,0}(t)dt$$

$$= c_1 a_1 t + a_{12}c_1 c_2 t$$

$$\therefore v_{1,1}(t) = (a_1 - a_{12}c_2)c_1 t$$

$$v_{2,1}(t) = a_2 \int_0^t v_{2,0}(t)dt = a_2 c_2 t$$

$$\therefore v_{2,1}(t) = a_2 c_2 t$$

$$v_{1,2}(t) = a_1 \int_0^t v_{1,1}(t)dt - a_{12}\int_0^t v_{1,0}(t)v_{2,1}(t)dt - a_{12}\int_0^t v_{1,1}(t)v_{2,0}(t)dt$$

$$= a_1(a_1 - a_{12}c_2)c_1\frac{t^2}{2} - a_{12}c_1(a_2 - a_{22}c_2)c_2\frac{t^2}{2} - a_{12}c_2(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2}$$

$$\therefore v_{1,2}(t) = [(a_1 - a_{12}c_2)(a_1 - a_{12}c_2)c_1 - a_{12}c_1 a_2\, c_2]\frac{t^2}{2}$$

$$v_{2,2}(t) = a_2 \int_0^t v_{2,1}(t)dt$$

$$= [a_2{}^2 c_2]\frac{t^2}{2}$$

$$\therefore\ v_{2,2}(t) = a_2{}^2 c_2 \frac{t^2}{2}$$

$$v_{1,3}(t) = a_1 \int_0^t v_{1,2}(t)dt - a_{12}c_1\int_0^t v_{2,2}(t)dt - a_{12}c_2\int_0^t v_{1,2}(t)dt - a_{12}\int_0^t v_{1,1}(t)v_{2,1}(t)dt$$

$$= (a_1 - a_{12}c_2)\{(a_1 - a_{12}c_2)(a_1 - a_{12}c_2)c_1 - a_{12}c_1c_2a_2\}\frac{t^3}{6} - a_{12}c_1\{a_2{}^2c_2\}\frac{t^3}{6} - a_{12}c_2(a_2a_1$$

$$-a_{12}c_2)c_1\frac{t^3}{3}$$

$$\therefore v_{1,3}(t) = (a_1 - a_{12}c_2)\{(a_1 - a_{12}c_2)(a_1 - a_{12}c_2)c_1 - a_{12}c_1c_2a_2\}\frac{t^3}{6} - a_{12}c_1\{a_2{}^2c_2\}\frac{t^3}{6}$$

$$-a_{12}c_2(a_2a_1 - a_{12}c_2)c_1\frac{t^3}{3}$$

$$v_{2,3}(t) = a_2\int_0^t v_{2,2}(t)dt$$

$$= \{a_2{}^2c_2\}\frac{t^3}{6}$$

$$\therefore v_{2,3}(t) = a_2{}^2c_2\frac{t^3}{6}$$

$$v_{1,4}(t) = (a_1 - a_{12}c_2)\int_0^t v_{1,3}(t)dt - a_{12}\int_0^t v_{1,1}(t)v_{2,2}(t)dt$$
$$-a_{12}\int_0^t v_{1,2}(t)v_{2,1}(t)dt - a_{12}c_1\int_0^t v_{2,3}(t)dt$$

$$\therefore v_{1,4}(t) == [(a_1 - a_{12}c_2)\{(a_1 - c_2)\{(a_1 - a_{12}c_2)(a_1 - a_{12}c_2)c_1$$
$$-a_2a_{12}c_1c_2\ \} + (a_1 - a_{12}c_2) - a_{12}c_1c_2a_2{}^2]\frac{t^4}{24} - a_{12}c_{1_2}a_2c_2\frac{t^4}{8}$$
$$\{-a_{12}c_1a_2{}^3c_2{}^2 -\}\frac{t^4}{24} - [c_1(a_1 - a_{12}c_2)a_2{}^2c_2]\frac{t^4}{8} + a_{12}a_2c_2[(a_1 - a_{12}c_2)^2c_1 - a_{12}c_1c_2a_2\ \ ]\frac{t^4}{8}$$

$$v_{2,4}(t) = a_2\int_0^t v_{2,3}(t)dt$$

$$\therefore v_{2,4}(t) == a_2{}^3c_2\frac{t^4}{24}$$

Up to the terms which contain maximum the power of four, we obtain

$$N_1(t) = \lim_{p\to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + v_{1,1}(t) + v_{1,2}(t) + v_{1,3}(t) + v_{1,4}(t)$$

$$N_1(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + v_{2,1}(t) + v_{2,2}(t) + v_{2,3}(t) + v_{2,4}(t)$$

The solutions by Homotopy Perturbation Method are derived as

$$N_1(t) = c_1 + [(a_1 - a_{12}c_2)c_1]t + [(a_1 - a_{12}c_2)^2c_1 - a_{12}a_2c_1c_2]\frac{t^2}{2} + \{(a_1 - a_{12}c_2)$$
$$[(a_1 - a_{12}c_2)^2c_1 - a_2a_{12}c_1c_2](a_1 - a_{12}c_2)c_1[2 - a_2a_{12}c_2] - a_2a_{12}c_1c_2\}\frac{t^3}{6} + \{[(a_1 - a_{12}c_2)^2c_1$$
$$-a_{12}c_1c_2(a_2)][(a_1 - a_{12}c_2)^2] + (a_1 - a_{12}c_2)\{2(a_1 - a_{12}c_2)c_1a_{12}(-a_2) - a_{12}c_1c_2a_2{}^2\}$$
$$+[a_2{}^2c_2][a_{12}c_1(-a_2) - 3a_{12}c_1(a_1 - a_{12}c_2)] + (a_2)c_2\{(a_1 - a_{12}c_2)c_1[3(a_1 - a_{12}c_2)a_{12}]$$
$$+a_2c_2(3a_{12}^2c_1)\}\frac{t^4}{24}$$

$$\therefore\ N_2(t) = +a_2c_2t + [a_2{}^2c_2]\frac{t^2}{2} + [a_2{}^3c_2]\frac{t^3}{6} + [a_2{}^4c_2]\frac{t^4}{24}$$

V. CONCLUSIONS

A special mathematical model of Ammensalism with unlimited resources is formed by a couple of first order nonlinear differential equations. A series solution of ecological Ammensalism is obtained by Homotopy Perturbation Method.

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