# COHERENTLY COUPLED PYROELECTRIC PHOTOVOLTAIC SOLITONS IN PHOTOREFRACTIVE MEDIA

**[1]Aavishkar Katti, [2]R.A. Yadav**

*[1,2]Department of Physics, Institute of Science, Banaras Hindu University, Varanasi(U.P), India*

**ABSTRACT**

*Coherently coupled soliton pairs travelling collinearly in photorefractive media having both the photovoltaic effect and pyroelectric effect are predicted in both dark and bright realizations. The effect of the mutual phase difference and the beam intensity ratio on the soliton pair is investigated. These pairs can be established provided that the incident beams are of same polarization, and mutually coherent with a fixed phase difference.*

***Keywords-Optical spatial solitons, Photorefractive effect, Pyroelectric effect***

## I. Introduction

Photorefractive spatial solitons have been at the forefront of current research because of their possible applications for all-optical switching and routing. The Photorefractive effect can be explained as a refractive index change through the electro-optic effect driven by the buildup of space–charge field.Till now, three types of steady-state photorefractive spatial solitons have been observed and investigated in detail i.e. screening solitons [1,2], photovoltaic solitons [3,4]and screening–photovoltaic solitons [5,6]

For the generation of screening solitons, we need the application of an external bias field. The external electric field is screened non-uniformly resulting in an index waveguide and hence, self trapping is observed. The photovoltaic solitons can exist in unbiased photorefractive crystals with appreciable photovoltaic effects. In the case of photovoltaics, the photovoltaic field generated acts to form a non uniform space charge field and hence, the index waveguide. Screening–photovoltaic solitons result from the combination of the external bias field and photovoltaic effect. Apart from the external electric field and the photovoltaic field, the pyroelectric field can also induce a space–charge field [7–9].

In a ferroelectric crystal at equilibrium the net internal field inside the crystal is zero as the field due to spontaneous polarisation is compensated by charge distribution on the crystal faces. However, a temperature change induces spontaneous polarisation variation and hence an electric field $E_{py}$. This field is not immediately compensated and a drift current can consequently be set up as if an external voltage is applied to the crystal. This field is locally screened due to the photorefractive effect and hence a self trapped beam,i.e a soliton results.Very recently, the pyroliton i.e. pyroelectric spatial solitons originating from the pyroelectric field in photorefractive crystals have been predicted and observed [10,11].

Now, if two mutually incoherent collinear propagating soliton beams have the same frequency and polarization, they create an effective refractive index modulation due to the combination of their intensities. A coupled pair results. Incoherently coupled spatial solitons were first investigated by Christodoulides et al.[12,13 ] many years ago. Since then, incoherently coupled soliton pairs in various realizations have been studied in photorefractive media[14-17]. Incoherently coupled multi component solitons or soliton families have also been investigated[18-20]. Recently, bright-bright, dark-dark and grey-grey incoherently coupled pyroelectric photovoltaic soliton pairs have been predicted[21].

As opposed to this, a coherently coupled soliton pair can also form and travel collinearly. This type of a soliton pair is similar to the incoherently coupled solitons except for the fact that now the two beams have a fixed phase difference between them. These collinearly propagating coherently coupled solitons have been predicted and studied recently in centro-symmetric photorefractive media[22] and novel photorefractive media having both linear and the quadratic electro-optic effect[23].

To our knowledge, nobody has yet studied the studied the collinearly propagating coherently coupled pyroelectric soliton pairs in photovoltaic photorefractive media. In this paper, we discuss the existence and properties of collinearly propagating bright-bright and dark-dark pyroelectric photovoltaic coherently coupled soliton pair.

## II. Theoretical Model

We consider two mutually coherent optical beams which propagate along the z axis. We assume diffraction only in the x direction. The soliton beams are polarized in the x direction and the external electric field is also applied in the same direction.The incident beams are expressed as slowly varying envelopes $\boldsymbol{E_1} = \hat{x}A_1(x,z)exp(ikz)$, and$\boldsymbol{E_2} = \hat{x}A_2(x,z)exp(ikz)$ where $k = k_0 n_e$ , $n_e$ is the unperturbed refractive index and $\lambda_0$ is the free space wavelength. Under the above conditions, $\mathbf{E}=\mathbf{E_1}+\mathbf{E_2}$ satisfies the Helmholtz equation,

$$\nabla^2 \boldsymbol{E} + (k_0 n_e')^2 \boldsymbol{E} = 0 \qquad (1)$$

Substituting the form of **E** in (1), and noting that the perturbed refractive index $n_e'^2 = n_e^2 - n_e^4 r_{eff} E_{sc}$ [24], we get,

$$\left(i\frac{\partial}{\partial z} + \frac{1}{2k}\frac{\partial^2}{\partial x^2} + \frac{k}{n_e}\Delta n\right)A_j(x,z) = 0;\ j = 1,2 \qquad (2)$$

$$\Delta n = -\frac{1}{2}n_e^3 r_{eff} E_{sc} \qquad (3)$$

where $E_{sc}$ is the space charge field in the medium resulting from the photovoltaic drift and the pyroelectric field. $r_{eff}$is the effective (linear) electro-optic coefficient.

$E_{sc}$, which is the space charge field, consists of two parts, one is the photovoltaic space charge field and the other is the pyroelectric space charge field[7-9,25]:

$$E_{sc} = E_{phsc} + E_{pysc} \quad (4)$$

The total space charge field has been derived elsewhere[25]:

$$E_{sc} \approx -E_p \frac{I}{I+I_d} - E_{py} \frac{t_p}{2} \frac{\sigma_{ph}}{\varepsilon_0 \varepsilon_r} \quad (5)$$

where $I$ is the intensity of the optical beam, $\sigma_{ph}$ is the photoconductivity, $t_p$ is the pulse duration,$\varepsilon_0$ is the permittivity in vacuum, $\varepsilon_r$ is the dielectric constant of the material and $I_d$ is the dark irradiance. The amplitude of $E_p$is depends on the light polarization and the sign is determined by the photovoltaic coefficient. $E_{pysc}$is the space–charge field arising due to the pyroelectric field. $E_{py}$results from the change in temperature. This change in temperature is caused by the beam illumination itself in this case. $E_{sc}$ can be expressed approximately as [10-11,25]:

$$E_{sc} \approx -E_p \frac{I}{I+I_d} - E_{py} \frac{\sigma I}{I_d} \quad (6)$$

where$\sigma$ is a parameter related to the crystal such that $\frac{\sigma I}{I_d} < 1$.The approximation is valid since the pyroelectric effect induced by a pulse of light is similar to that induced by a temperature change and the photoconductivity is directly proportional to the intensity of the beam.

The total optical power density for the two mutually coherent beams is:

$$I = \frac{n_e}{2\eta_0} (|A_1 + A_2|^2) \quad (7)$$

with $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$ .

Substituting $E_{sc}$ and $\Delta n$ in (1) and in terms of dimensionless variables, one gets the following equations:

$$iU_\xi + \frac{1}{2} U_{ss} + \beta(|U+V|^2)U + \frac{\alpha(|U+V|^2)}{(1+|U+V|^2)} U = 0 \quad (8)$$

$$iV_\xi + \frac{1}{2} V_{ss} + \beta(|U+V|^2)V + \frac{\alpha(|U+V|^2)}{(1+|U+V|^2)} V = 0 \quad (9)$$

where we have written , $A_1 = (2\eta_0 I_d / n_e)^{1/2} U$ and $A_2 = (2\eta_0 I_d / n_e)^{1/2} V$ , $\xi = z/kx_0^2$ , $s = x/x_0$

and $U_\xi = \frac{\partial U}{\partial \xi}$ , $U_{ss} = \frac{\partial^2 U}{\partial s^2}$ , $\rho = I_\infty / I_d$ , $\beta = \sigma\tau E_{py}$ , $\tau = (k_0 x_0)^2 n_e^4 r_{eff}/2$ , $\alpha = \tau E_p$

## III. Results and Discussion

### 3.1 Bright-Bright coherently coupled soliton pair

In case of a bright-brightcoherently coupled soliton pair, we express the solutions as:

$$U = r_1^{1/2} f(s) \exp(i\mu\xi + \varphi_1) \quad (10)$$

$$V = r_2^{1/2} f(s)\, exp(i\mu\xi + \varphi_2) \quad (11)$$

where $r = \frac{I_{max}}{I_d}$ where $I_{max} = I(0)$= maximum soliton beam intensity, and μ is nonlinear shift of the respective propagation constant, and *f(s)* is a normalized bounded function which satisfies $0 \leq f(s) \leq 1$ and $f(\pm\infty) = 0$ , $f'(0) = 0, f'(\pm\infty) = 0, f''(\pm\infty) = 0, f(0) = 1$. $\varphi_1$ and $\varphi_2$ are the respective phases of the two soliton beams. Substituting (11) and (10) in Eqs. (8) and (9),we get:

$$f'' = 2\mu f - 2\beta(rf^2)f - 2\alpha\frac{(rf^2)f}{1+rf^2} \qquad (12)$$

where $f'' = \frac{d^2f}{ds^2}$ and where $r = r_1 + r_2 + 2\sqrt{r_1}\sqrt{r_2}\cos(\varphi_1 - \varphi_2)$

Integrating once, we get:

$$f'^2 = 2\mu f^2 - \beta r f^4 - 2\alpha(f^2 - \frac{1}{r}(\log(1 + rf^2)) + 2c \qquad (13)$$

where *c* is the constant of integration.

Using the boundary conditions, $f(\pm\infty) = 0$ and $f'(\pm\infty) = 0$ in (13), we have:

$$c = 0 \qquad (14)$$

Using the boundary conditions, $f(0) = 1$ and $f'(0) = 0$ in (13), we get:

$$\mu = \beta\frac{r}{2} + \alpha(1 - \frac{1}{r}(\log(1 + r)) \qquad (15)$$

Substituting (14) and (15) in (13), and integrating again:

$$s = \pm\int_f^1 \frac{df}{\left[\beta r f^2(1-f^2) + \frac{2\alpha}{r}[\log(1+rf^2) - f^2\log(1+r)]\right]^{1/2}} \qquad (16)$$

(16) gives us the soliton envelope.

For illustration purpose, we consider the parameters as in Table 1 for a $LiNbO_3$ crystal.[10,24-25]. Using these, we get $\beta = 46.07, \alpha = -37.25$.Note that $E_{ph}$ is negative in case of $LiNbO_3$ which results in a negative refractive index change and hence self-defocussing, whereas in the bright soliton case, we need a positive refractive index change and self-focussing. That there still forms a bright soliton is explained by the fact that pyroelectric space charge field $E_{pysc}$ causes self focussing which far exceeds the magnitude of the self-defocussing induced by the photovoltaic effect.[10]

**Table.1. The values of parameters used in our calculations for the bright soliton pair.[10,24-25]**

| Parameter | Value |
|---|---|
| Wavelength $\lambda$ | 0.532 nm |
| $x_0$ | 20 μm |
| Photovoltaic Field $E_{ph}$ | $-1.9 \times 10^6$ V/m |
| Pyroelectric field $E_{py}$ | $4.7 \times 10^6$ V/m |
| $r$ | 1 |
| Unperturbed Refractive Index $n_e$ | 2.2 |

International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 09, September 2016
www.ijates.com
ijates
ISSN 2348 - 7550

| | |
|---|---|
| Electro Optic Coefficient $r_{eff}$ | 30 x $10^{-12}$ $mV^{-1}$ |
| $\sigma$ | 0.5 |

The normalized intensities of the coherently coupled soliton pair are shown in Fig.-1.

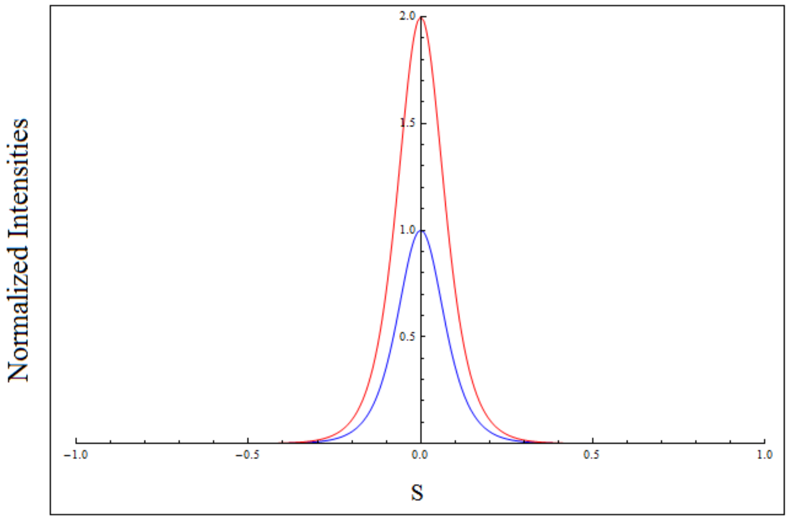


**Fig.1. The normalized intensities of the coherently coupled soliton pair when $\Delta\varphi = \pi/2$ and $\frac{r_1}{r_2} = 0.5$, $r_2 = 2$**

Also, we investigate the relationship between the mutual phase difference$\varphi_1 - \varphi_2 = \Delta\varphi$ of the two solitons and their FWHM. The dependence is shown in Fig.2. We can see that the FWHM first decreases slightly with the increase in phase difference and after that, it increases with the increase in the phase difference between the coherently coupled soliton pair. The decrease in FWHM happens till $\Delta\varphi \approx {}^{\pi}/_{12}$. The increase becomes very rapid as the values of $\Delta\varphi$ become more and more greater than$2\pi/3$ till $\Delta\varphi = \pi$. This implies that as the phase difference increases more than ${}^{\pi}/_{12}$, the self focussing is weakenedand the nonlinearity becomes weaker.

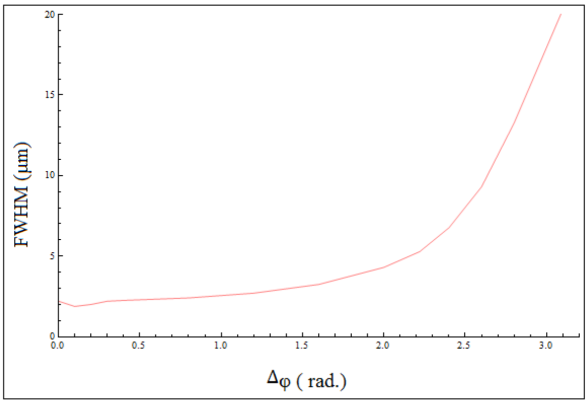


**Fig.2. The dependence of FWHM on the mutual phase difference $\Delta\varphi$ when $\frac{r_1}{r_2} = 0.5$, $r_2 = 2$**

International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 09, September 2016
www.ijates.com
ijates
ISSN 2348 - 7550

We also find that the ratio $\frac{r_1}{r_2}$ affects the self trapping mechanism and hence the FWHM.Fig.3 shows the dependence of FWHM on the beam intensity ratio $\frac{r_1}{r_2}$. The FWHM decreases as the beam intensity ratio increases. This implies that self focussing is enhanced as one of the beams' intensity becomes closer in magnitude to the intensity of the other.

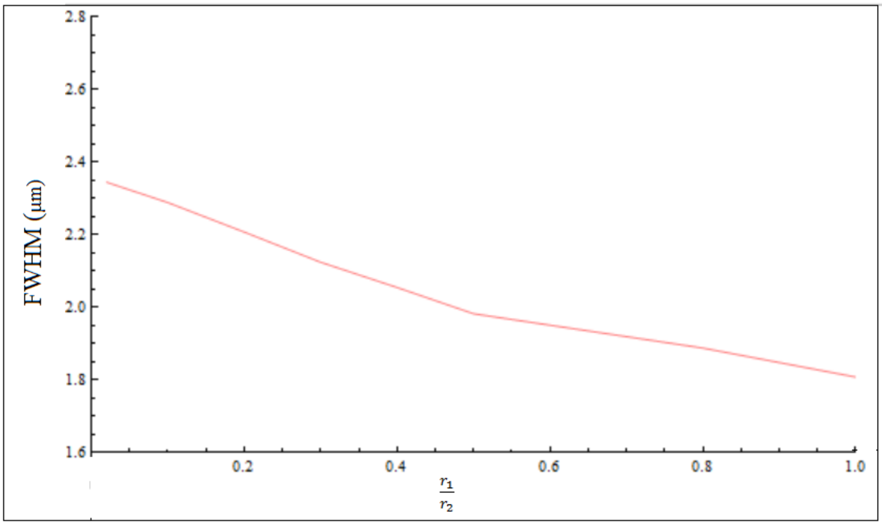


**Fig.3. The dependence of FWHM on the beam intensity ratio $\frac{r_1}{r_2}$ when $\Delta\varphi = \frac{\pi}{4}$, $r_2 = 2$**

### 3.2 Dark-Dark coherently coupled soliton pair

In case of a dark-dark incoherently coupled soliton pair, we express the solutions as:

$$U = \rho_1^{1/2} g(s) \exp(i\mu\xi + \varphi_1) \quad (17)$$

$$V = \rho_2^{1/2} g(s) \exp(i\mu\xi + \varphi_2) \quad (18)$$

where $\rho = I_\infty / I_d$ where $I_\infty = I(x \to \pm\infty)$= Intensity at constant illumination, μis the nonlinear shift of the respective propagation constants, and g(s) is a normalized bounded function which satisfies $0 \leq g(s) \leq 1$ and $g(\pm\infty) = \pm 1$, $\dot{g}(\pm\infty) = 0, \ddot{g}(\pm\infty) = 0, g(0) = 0$. $\varphi_1$ and $\varphi_2$ are the respective phases of the two soliton beams.

Substituting (17) and (18) in Eq. (8) and (9),we get:

$$\ddot{g} = 2\mu g - 2\beta(\rho g^2)g - 2\alpha \frac{(\rho g^2)g}{1 + \rho g^2} \quad (19)$$

where $\ddot{g} = \frac{d^2 g}{ds^2}$ and where $\rho = \rho_1 + \rho_2 + 2\sqrt{\rho_1}\sqrt{\rho_2}\cos(\varphi_1 - \varphi_2)$

Integrating once, we get:

$$\dot{g} = 2\mu g^2 - \beta\rho g^4 - 2\alpha g^2 + \frac{2\alpha}{\rho}(log(1+\rho g^2)) + 2c \tag{20}$$

where *c* is the constant of integration.

Using the boundary conditions, $g(\pm\infty) = \pm 1$ and $\ddot{g}(\pm\infty) = 0$ in (19), we have:

$$\mu = \beta\rho + \alpha\left(\frac{\rho}{1+\rho}\right) \tag{21}$$

Using the boundary conditions, $g(\infty) = 1$ and $\dot{g}(\infty) = 0$ in (20), we get:

$$c = -\mu + \beta\frac{\rho}{2} + \alpha(1 - \frac{1}{\rho}log(1+\rho)) \tag{22}$$

Substituting (21) and (22) in (20), and integrating again:

$$s = \pm\int_g^0 \frac{dg}{\left[-\beta\rho(g^2-1)^2 - 2\alpha\left[\frac{g^2-1}{1+\rho} - \frac{1}{\rho}\log\left(\frac{1+\rho g^2}{1+\rho}\right)\right]\right]^{\frac{1}{2}}} \tag{23}$$

**Table.2. The values of parameters used in our calculations for the dark soliton pair.**

| **Parameter** | **Value** |
|---|---|
| Wavelength $\lambda$ | 0.532 nm |
| $x_0$ | 20 μm |
| Photovoltaic Field $E_{ph}$ | -1.9 x $10^6$ V/m |
| Pyroelectric field $E_{py}$ | -4.7 x $10^6$ V/m |
| $\rho$ | 1 |
| Unperturbed Refractive Index $n_e$ | 2.2 |
| Electro Optic Coefficient $r_{eff}$ | 30 x $10^{-12}$ $mV^{-1}$ |
| $\sigma$ | 0.5 |

(23) gives us the soliton envelope. Dark solitons require the refractive index perturbation to be negative, so we take $E_{py}$ to be negative in this case, i.e, the pyroelectric coefficient is assumed to be positive now. Hence the photovoltaic effect is complemented by the pyroelectric effect for the formation of dark soliton pair in such a scenario.

For illustration purpose, we consider the parameters as in Table 2. Using these, we get $\beta = -46.07, \alpha = -37.06$. The normalized intensities of the coherently coupled soliton pair are shown in Fig.4.

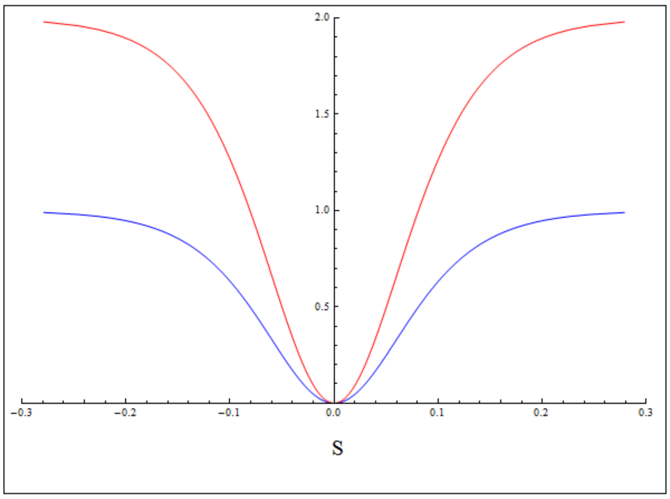


Fig.4. The normalized intensities of the coherently coupled soliton pair when $\Delta\varphi = \pi/4$ and $\frac{\rho_1}{\rho_2} = 0.5$, $r_2 = 2$

As in the bright case, the mutual phase difference $\Delta\varphi = \varphi_1 - \varphi_2$ between the two coupled solitons affects the self defocussing and hence the FWHM of the soliton. The dependence of the soliton width on the mutual phase difference is shown in Fig.4. The FWHM is nearly constant till $\Delta\varphi = \pi/6$. As the phase difference increases more than $\pi/6$, the FWHM of the soliton pair increases which implies a weakening of the self trapping. Here also, the FWHM increases rapidly as the phase difference $\Delta\varphi$ becomes more and more greater than $2\pi/3$ till $\Delta\varphi = \pi$.

Fig.6 shows the dependence of FWHM on the beam intensity ratio $\frac{\rho_1}{\rho_2}$. The FWHM decreases as the beam intensity ratio increases. This implies that self defocussing or self trapping is enhanced as one of the beams' intensity becomes closer in magnitude to the intensity of the other.

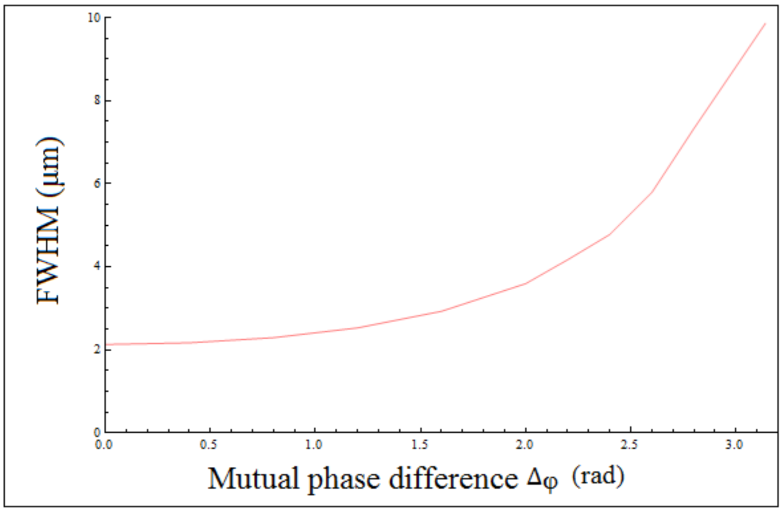


**Fig.5.The dependence of FWHM on the mutual phase difference $\Delta\varphi$ when $\frac{\rho_1}{\rho_2} = 0.5$, $r_2 = 2$**

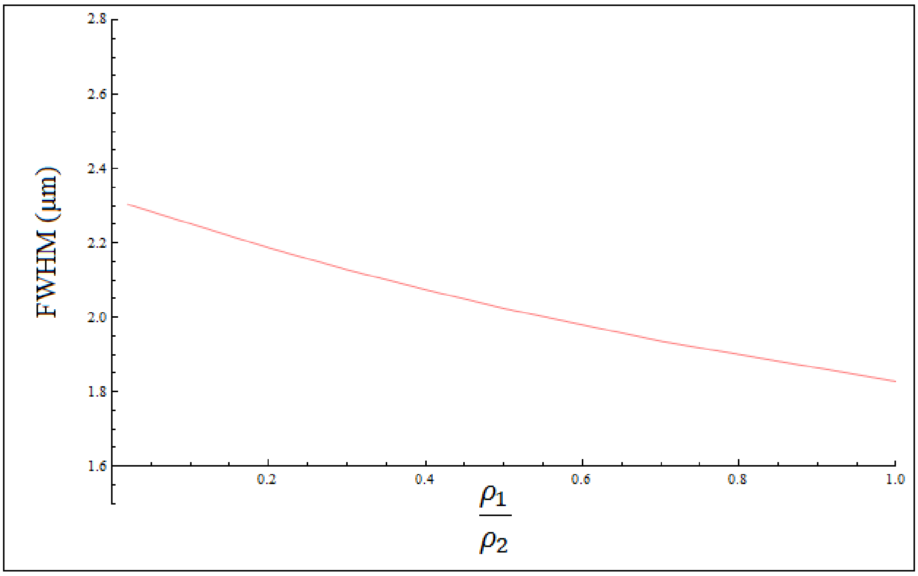


**Fig.6. The dependence of FWHM on the beam intensity ratio $\frac{\rho_1}{\rho_2}$ when $\Delta\varphi = \frac{\pi}{4}$, $r_2 = 2$**

## IV. CONCLUSIONS

We have proved the existence of collinearly propagating coherently coupled bright-bright and dark-dark soliton pairs due to the pyroelectric photovoltaic effect in photorefractive media. We find that the mutual phase difference and the beam intensity ratio affects the self trapping in both, the bright and dark cases. For very low values of the phase difference, an increase in the mutual phase difference causes a slight enhancement of self focussing and hence a negligible decrease in FWHM of the bright soliton pair. For all other values of the phase difference, there is weakening of self focussing in this case and hence an increase in the FWHM of the soliton pair. In case of the dark soliton pair, the FWHM remains nearly constant till the mutual phase difference reaches $\frac{\pi}{6}$ after which there is an increase in the FWHM and hence, a weakening of self defocussing. An increase in the beam intensity ratio of the two coupled solitons causes an enhancement in the self focussing or self defocussing and hence a decrease in the FWHM of the soliton pair for both the bright and dark soliton pairs.

## V. ACKNOWLEDGEMENTS

Aavishkar Katti acknowledges financial assistance through the Research Fellowship awarded by University Grants Commission, New Delhi.

**REFERENCES**

[1]Christodoulides, Demetrios N., and M. I. Carvalho, Bright, dark, and gray spatial soliton states in photorefractive media, *JOSA B, 12(9)*, 1995, 1628-1633.

[2] Kos, Konstantine, et al.,One-dimensional steady-state photorefractive screening solitons, *Physical Review E, 53(5)*, 1996, R4330.

[3] Valley, George C., et al., Dark and bright photovoltaic spatial solitons, *Physical Review A 50(6*),1994, R4457.

[4] She, Wei-Long, et al., Formation of photovoltaic bright spatial soliton in photorefractive LiNbO3 crystal by a defocused laser beam induced by a background laser beam, *JOSA B, 23(10)*, 2006, 2121-2126.

[5] Jinsong, Liu, and Lu Keqing., Screening-photovoltaic spatial solitons in biased photovoltaic–photorefractive crystals and their self-deflection, *JOSA B, 16(4)*, 1999, 550-555.

[6] Fazio, E., et al., Screening-photovoltaic bright solitons in lithium niobate and associated single-mode waveguides, *Applied physics letters 85(12)* 2004, 2193-2195.

[7] Buse, K., and K. H. Ringhofer, Pyroelectric drive for light-induced charge transport in the photorefractive process, *Applied Physics A, 57(2)*, 1993, 161-165.

[8] Buse, Karsten, Light-induced charge transport processes in photorefractive crystals I: Models and experimental methods, *Applied Physics B: Lasers and Optics*, *64(3)*, 1997, 273-291.

[9] Buse, K., R. Pankrath, and E. Krätzig, Pyroelectrically induced photorefractive effect in Sr 0.61 Ba 0.39 Nb 2 O 6: Ce, *Optics letters* 19(4), 1994, 260-262.

[10] Safioui, Jassem, Fabrice Devaux, and Mathieu Chauvet, Pyroliton: pyroelectric spatial soliton, *Optics express*, *17(24)*, 2009, 22209-22216.

[11] Safioui, Jassem, et al., Surface-wave pyroelectric photorefractive solitons, *Optics letters 35(8),* 2010, 1254-1256.

[12]Christodoulides, D. N., et al., Incoherently coupled soliton pairs in biased photorefractive crystals, *Applied physics letters, 68(13)*, 1996, 1763-1765.

[13] Chen, Zhigang, et al., Observation of incoherently coupled photorefractive spatial soliton pairs, *Optics letters, 21(18)*, 1996, 1436-1438.

[14] Chen, Zhigang, et al., Incoherently coupled dark–bright photorefractive solitons, *Optics letters* 21(22), 1996, 1821-1823.

[15] Hou, Chunfeng, et al., Incoherently coupled grey-grey screening-photovoltaic soliton pairs in biased photovoltaic-photorefractive crystals, *Optik-International Journal for Light and Electron Optics* 112(1), 2001, 17-20.

[16] Keqing, Lu, et al., Incoherently coupled steady-state soliton pairs in biased photorefractive-photovoltaic materials." *Physical Review E, 64(5),* 2001, 056603.

[17] Hou, Chun-Feng, Chun-Guang Du, and Shi-Qun Li., Incoherently Coupled Bright-Dark Soliton Pairs in Biased Centrosymmetric Photorefractive Media*, Chinese Physics Letters 18,* 2001, 1607-1609.

[18] Chun-feng, Hou, et al., Incoherently coupled screening-photovoltaic soliton families in biased photovoltaic photorefractive crystals., *Chinese Physics, 10(4),*2001, 310.

[19] Hou, C., et al., Incoherently coupled bright–dark hybrid soliton families in biased photovoltaic–photorefractive crystals, *Applied Physics B*, *72(2)*,2001, 191-194.

[20] Hong-Cheng, Wang, and She Wei-Long., Incoherently coupled grey photovoltaic spatial soliton families, *Chinese Physics Letters*, *22(1)*, 2005, 128.

[21] SU, Yan-li, Qi-chang JIANG, and Xuan-mang JI., Incoherently Coupled Photorefractive Spatial Soliton Pairs Based on the Combination of Pyroelectric and Photovoltaic Effect, *Acta Photonica Sinica*, *3*,2014, 014.

[22] Hao, Lili, et al. Coherently coupled bright-bright screening soliton pairs in biased centrosymmetric photorefractive crystals. *Optik-International Journal for Light and Electron Optics*, *127(15)*, 2016, 5928-5934.

[23] Hao, Lili, et al., Coherently coupled spatial soliton pairs in biased photorefractive crystals with both the linear and quadratic electro-optic effects, *Optik-International Journal for Light and Electron Optics 127(10)*, 2016, 4339-4344.

[24]Peter Günter, Jean Pierre Huignard, *Photorefractive Materials and Their Applications 1* (New York: Springer-Verlag, 2006)

[25]Jiang, Qichang, Yanli Su, and Xuanmang Ji.,Pyroelectric photovoltaic spatial solitons in unbiased photorefractive crystals., *Physics Letters A*, *376(45)*, 2012, 3085-3087