International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 09, September 2016
www.ijates.com
ijates
ISSN 2348 - 7550

A SPECIAL CASE OF ECOLOGICAL COMMENSALISM- PHASE PLANE ANALYSIS

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ABSTRACT

The paper is aimed to discuss the stability nature of Ecological Commensalism with two species (Commensal and Host species). Commensal species is considered with unlimited resources. This model is constructed by a couple of first order non linear differential equations. The behavior of this model is established with Phase plane analysis.

Keywords: Commensalism, Stability and Threshold Diagrams.

I. INTRODUCTION

Many considerable solutions for many complex situations in nature are derived by Mathematical Modeling. Initially the thorough concept was discussed by by Lotka [10] and Volterra [15].Most of the advanced concepts of modeling have been investigated by Meyer [11], Cushing [5], Gause [7], Paul Colinvaux [12], Haberman [8], Pielou [13], Thompson [14], Freedman [6], Kapur [9] etc. Later Pattabhi Ramacharyulu , Acharyulu [1-4] concentrated the nature of different types of Ecology. The peculiar behavior of unlimited commensal species in this model is observed with Phase plane analysis.

II. NOTATIONS ADOPTED

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) : The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Commensal coefficient.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{12} are assumed to be non-negative constants.

III. BASIC EQUATIONS

The basic equations are given as

$$\frac{dN_1}{dt} = a_1N_1 + a_{12}N_1N_2 \quad (1)$$

$$\frac{dN_2}{dt} = a_2N_2 \quad \text{With initial conditions } N1(0) = c_1 \text{ and } N_2(0) = c_2 \quad (2)$$

Here, fully washed out state is only occurred.

The corresponding equilibrium point is $\bar{N}_1 = 0;\ \bar{N}_2 = 0$

By the concept of linearization,

$$\frac{dU_1}{dt} = a_1U_1 \text{ and } \frac{dU_2}{dt} = a_2U_2 \qquad (3)$$

and the characteristic equation is $(\lambda - a_1)(\lambda - a_2) = 0$

the roots of which are $\lambda = a_1$, $\lambda = a_2$ i.e. both the roots are negative,

Hence the steady state is **unstable**.

The solutions are obtained as $U_1 = U_{10}\ e^{a_1 t}$ and $U_2 = U_{20}\ e^{a_2 t}$. (4)

Now , the nature of this model is discussed with Phase plane analysis with the considered conditions.

Case (i): When a_1= 0.5, $\mathbf{a_{12}=0.2}$ and a_2=0.5, The Null clines and Trajectories are shown in the Fig.1(A)and Fig.1(B) respectively.

In this case, The Eigen values are 0.5 and 0.5 with the Eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

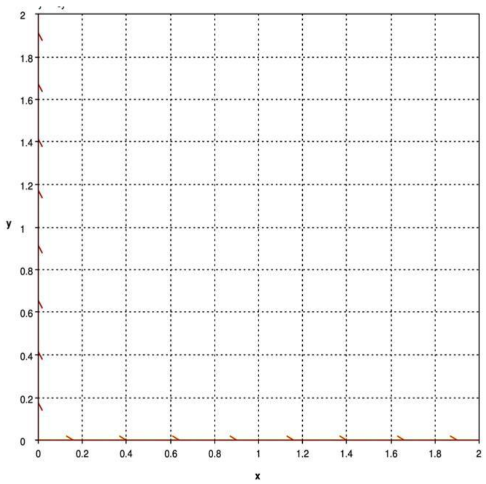

Fig.1 (A)

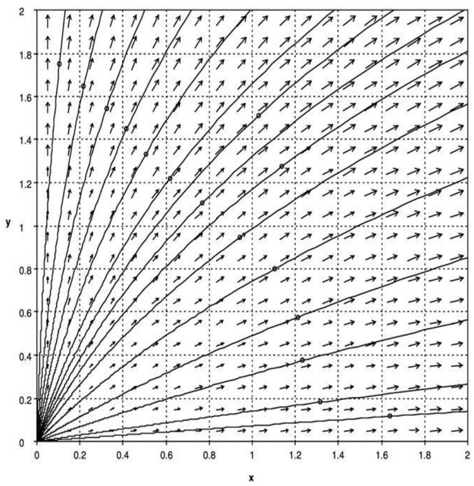

Fig.1 (B)

Case (ii): When a_1= 0.5, $\mathbf{a_{12}=0.4}$ and a2=0.5, The Null clines and Trajectories are shown in the Fig.2 (A) and Fig.2 (B) respectively.

In this case, The Eigen values are 1 and 0.5 with the Eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

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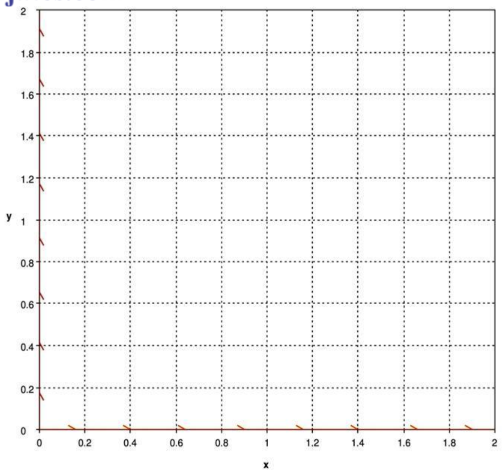

Fig.2 (A)

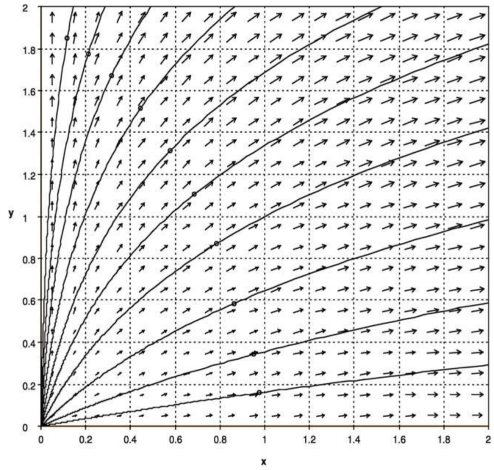

Fig.2 (B)

Case (iii).: When $a_1= 0.5$, $\mathbf{a_{12}=0.6}$ and $a_2=0.5$, The Null clines and Trajectories are shown in the Fig.3(A)and Fig.3(B) respectively.

In this case, The Eigen values are 0.5 and 0.5 with the Eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

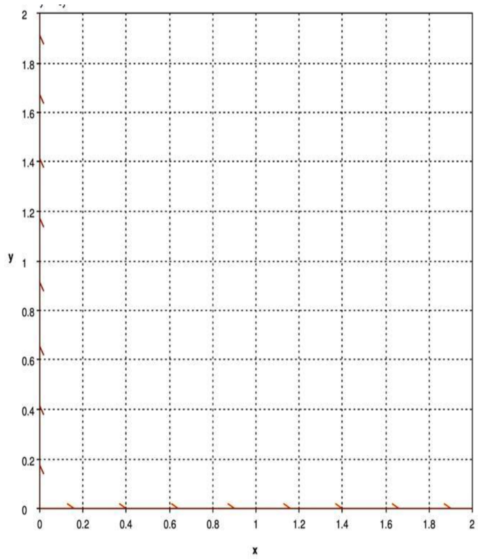

Fig.3 (A)

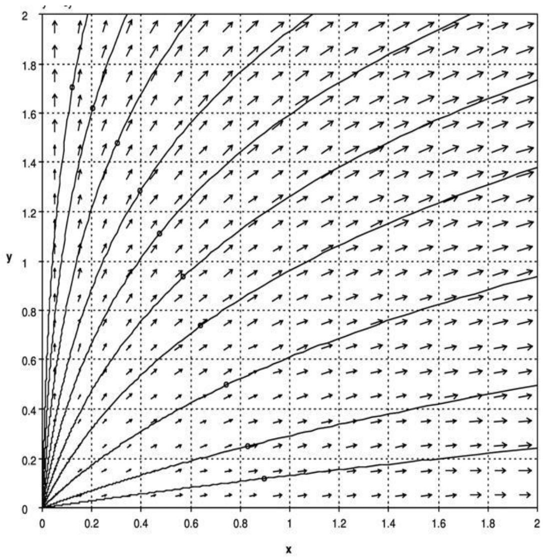

Fig.3 (B)

Case (iv): When $a_1= 0.5$, $\mathbf{a_{12}=0.8}$ and $a_2=0.5$, The Null clines and Trajectories are shown in the Fig.4(A)and Fig.4(B) respectively.

In this case, The Eigen values are 0.5 and 0.5 with the Eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

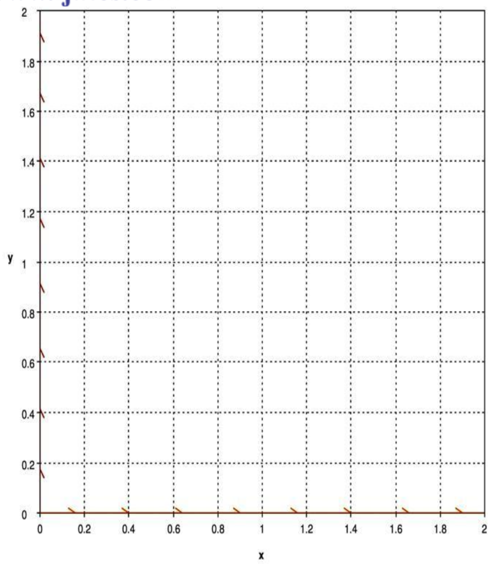

Fig.4 (A)

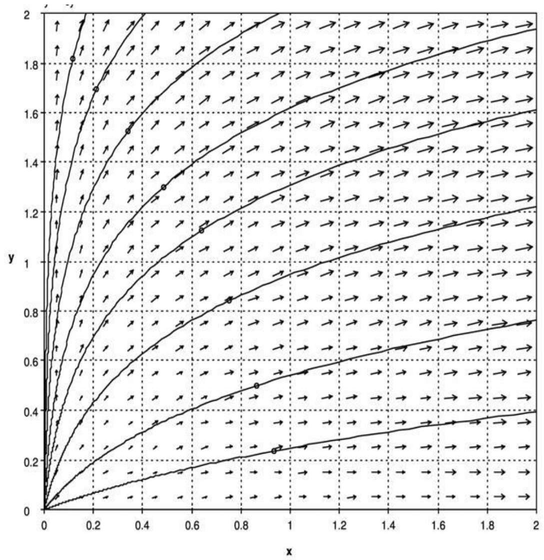

Fig.4 (B)

Case (v): When $a_1= 0.5$, $\mathbf{a_{12}=1.0}$ and $a_2=0.5$, The Null clines and Trajectories are shown in the Fig.5(A)and Fig.5(B) respectively.

In this case, The Eigen values are 0.5 and 0.5 with the Eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

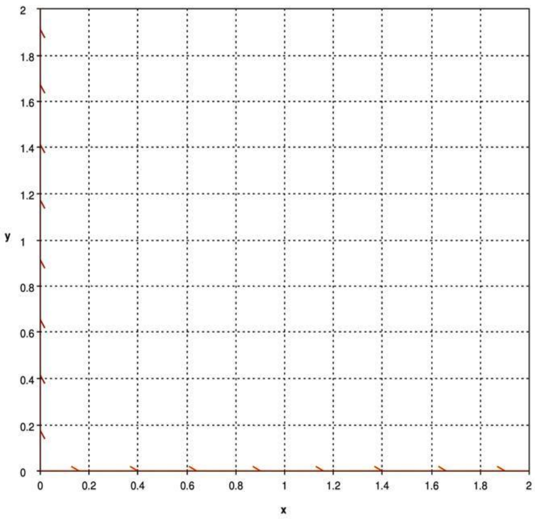

Fig.5 (A)

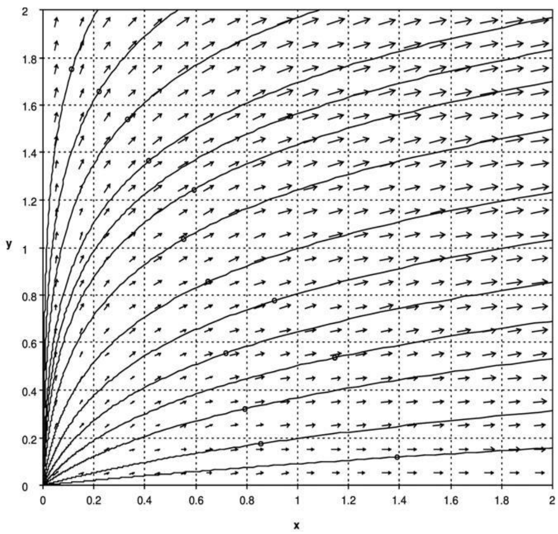

Fig.5 (B)

Case (vi): When $a_1= 0.5$, $\mathbf{a_{12}=1.2}$ and a2=0.5, The Null clines and Trajectories are shown in the Fig.6(A)and Fig.6(B) respectively.

In this case, The Eigen values are 0.5 and 0.5 with the Eigen vectors (1,0) & (0,1) and the Jacobean matrix is

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

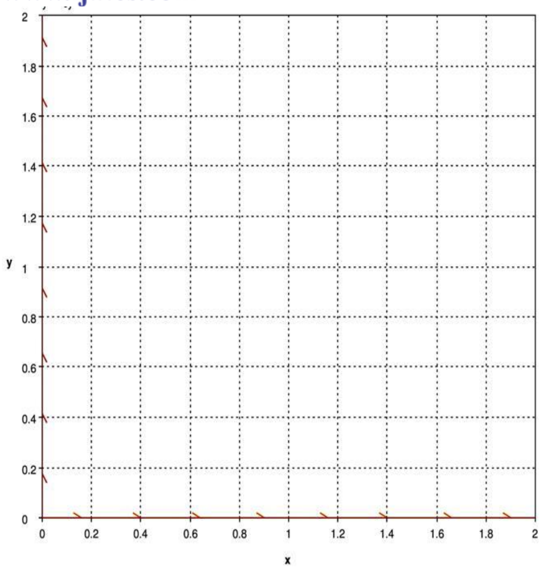

Fig.6 (A)

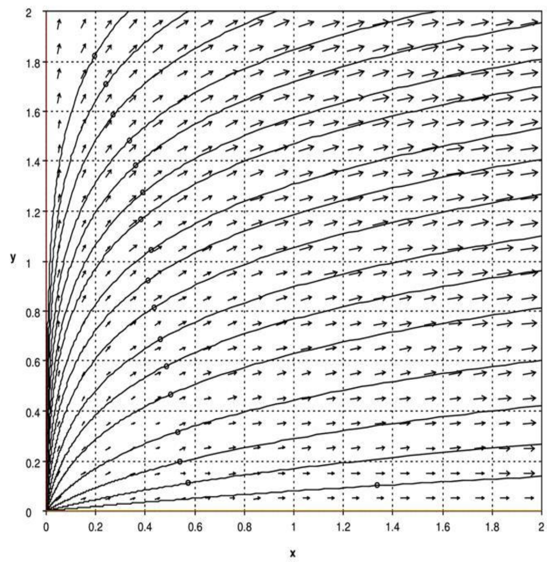

Fig.6 (B)

IV. CONCLUSIONS

The nature of unlimited commensal model is unstable. The nature can be altered by increasing the growth rate of Commensal coefficient with the fixed growth rates of both the species. No considerable influence is identified by growth rates of Commensal and Host species**.**

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