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PHASE PLANE ANALYSIS ON AMMENSALISM WITH MORTAL ENEMY SPECIES

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ABSTRACT

The nature of Ammensalism with Mortal Enemy Species is studied with Phase plane analysis in this paper. Enemy species is intensified with unlimited resources along with mortality. This model is designed by a couple of first order non linear differential equations. The behavior of this model is discussed in various cases with the illustrations of null clines, trajectories and solution curves.

Keywords: Ammensalism, Stability and Threshold Diagrams.

I. INTRODUCTION

Many computational principles are established in various mathematical models of the diverse techniques. They have been adopted in mathematical modeling to illustrate the nature and behavior of mathematical models. Useful monograph by Kapur [9] deals with diverse topics on mathematical modeling in biological and medical sciences. The vast information with the basic ideas are available in the Research work of Meyer [11], Cushing [5], Gause [7], Paul Colinvaux [12], Haberman [8], Pielou [13], Thompson [14], Freedman [6] etc. with a wide spectrum of areas of knowledge and function. The Basic concepts were deeply explained by Lotka [10] and Volterra [15]. Acharyulu [1-4] investigated many models of Ammensalism in peculiar situations.

II. NOTATIONS ADOPTED

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) : The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Ammensal coefficient.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{12} are assumed to be non-negative constants.

III. BASIC EQUATIONS

The basic equations are given as

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 - a_{12}N_1N_2 \qquad (1)$$

$\frac{dN_2}{dt} = -a_2 N_2$ with initial conditions $N_1(0)=\mathbf{c}_1$ and $N_2(0)=c_2$ (2)

In this model, Fully washed out state and Enemy washed out state are occurred.

The corresponding equilibrium points are (i). $\bar{N}_1 = 0$; $\bar{N}_2 = 0$ (ii). (i). $\bar{N}_1 = K_1$; $\bar{N}_2 = 0$

By the concept of linearization ,

$$\frac{dU_1}{dt} = a_1 U_1 \quad \text{and} \quad \frac{dU_2}{dt} = -a_2 U_2 \qquad (3)$$

and the characteristic equation is $(\lambda - a_1)(\lambda + a_2) = 0$

the roots of which are $\lambda = a_1$, $\lambda = -a_2$ i.e. One of the roots is positive.

Hence the steady state is **unstable**.

The solutions are obtained as $U_1 = U_{10}\, e^{a_1 t}$ and $U_2 = U_{20}\, e^{-a_2 t}$. (4)

Now , the nature of this model is discussed with Phase plane analysis with the considered conditions.

Case (i): When a_1=0.456, a_{11}=0.576, a_{12}=0.254and a_2=0.689, The Null clines, Trajectories and Solution curves are drawn in the Fig.1(A), Fig.1(B), & Fig.1(C) respectively.

In this case, The Eigen values are -0.456 and -0.678 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is $\begin{pmatrix} -0.456 & -0.200390 \\ 0 & -0.678 \end{pmatrix}$

The equilibrium point of this system exists at (0.78893, 0).Both eigen values are negative. Hence, it is stable.

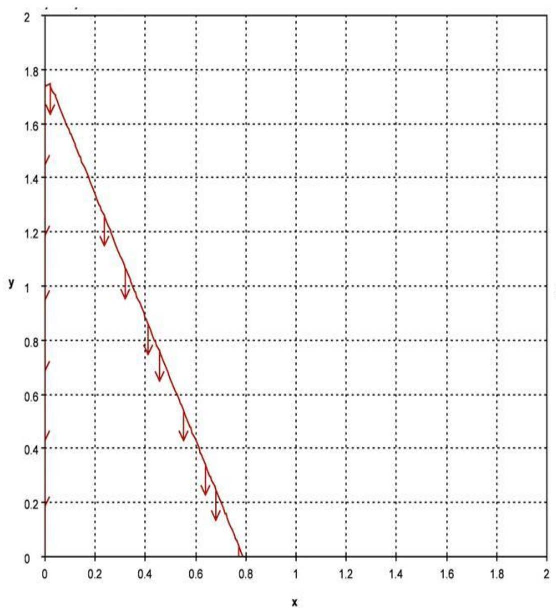

Fig.1 (A): Null clines

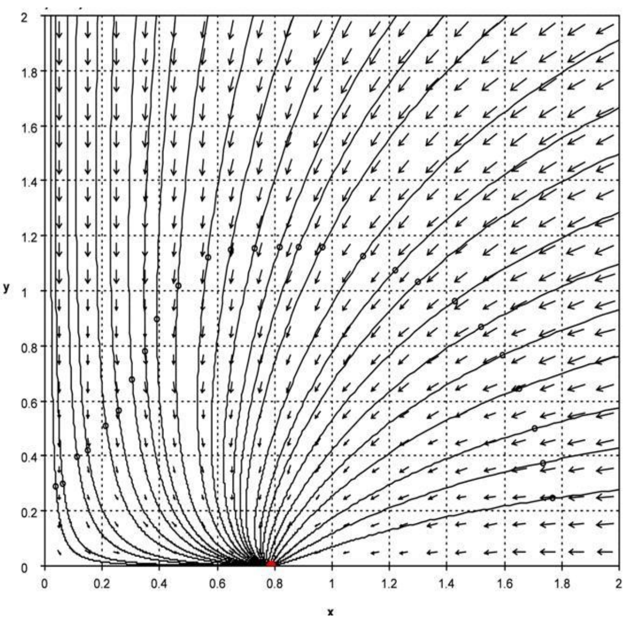

Fig.1(B): Threshold Diagram

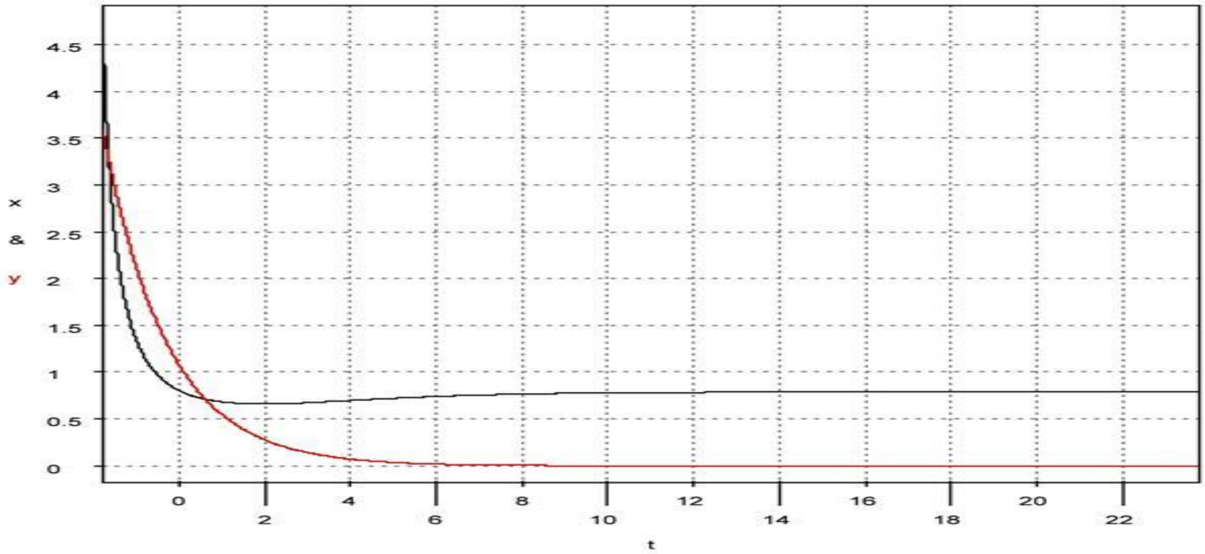

Fig.1(C): Solution Curves

Case (ii): When a_1=0.456, a_{11}=0.576, a_{12}=0.254and **a_2=2.678**, The Null clines, Trajectories and Solution curves are drawn in the Fig.2(A), Fig.2(B), & Fig.2(C) respectively.

In this case, The Eigen values are -0.456 and -2.678 with the eigen vectors (1, -2.77019E-16) & (0,1) and the Jacobean matrix is $\begin{pmatrix} -0.456 & -0.20039 \\ 0 & -2.6789 \end{pmatrix}$

The equilibrium point of this system exists at (0.78893, 0).Both eigen values are negative. Hence, it is stable

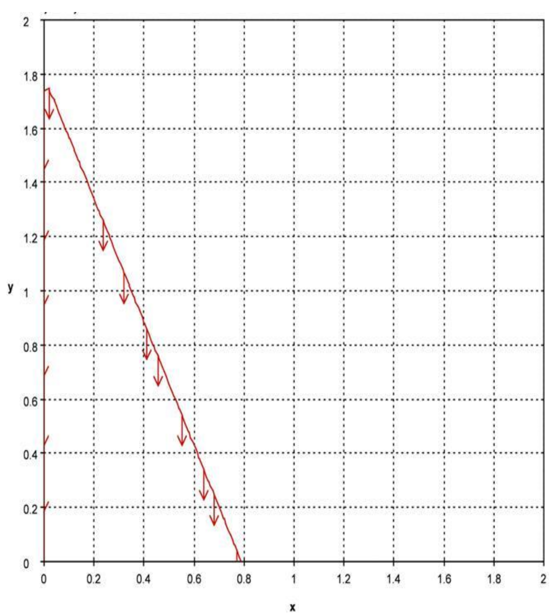

Fig.2 (A): Null clines

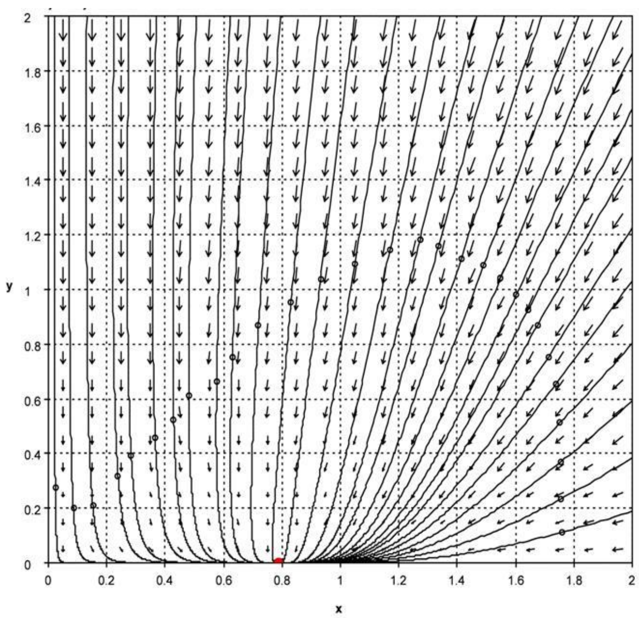

Fig.2 (B): Threshold Diagram

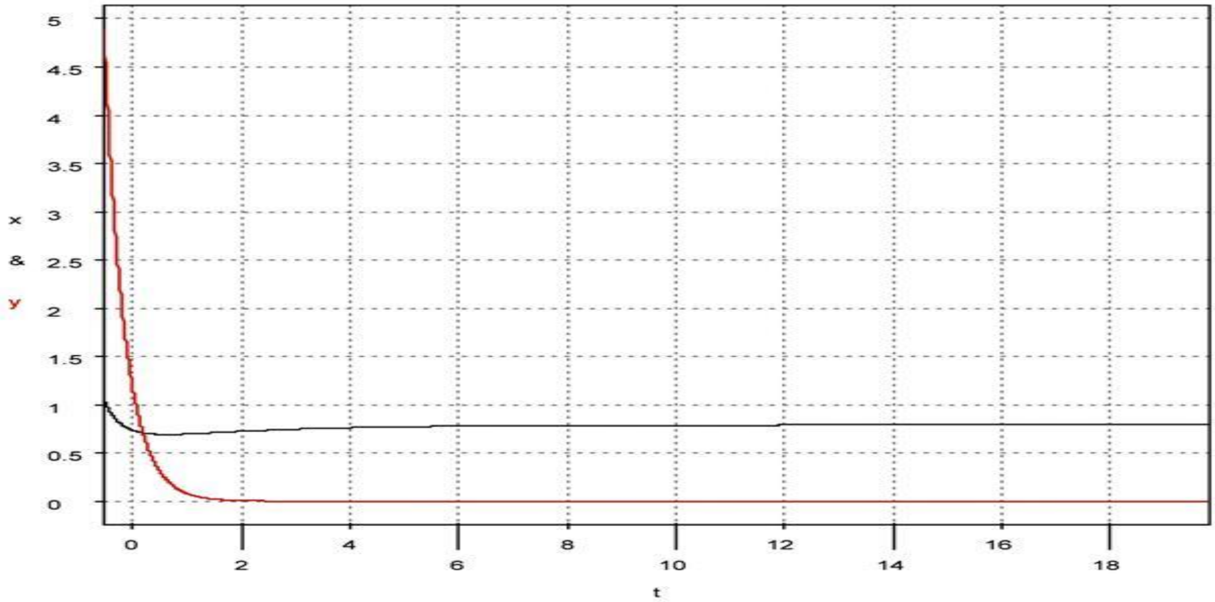

Fig.2(C): Solution Curves

Case (iii): When a_1=0.456, a_{11}=0.576, a_{12}=0.254and **a_2=4.678**, The Null clines, Trajectories and Solution curves are drawn in the Fig.3(A), Fig.3(B), & Fig.3(C) respectively.

In this case, The Eigen values are 0.12192 and -4.678 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is $\begin{pmatrix} 0.12192 & 0 \\ 0 & -4.678 \end{pmatrix}$

The Saddle point of this system exists at (0, 0).One of the eigen values is non- negative. Hence, it is unstable.

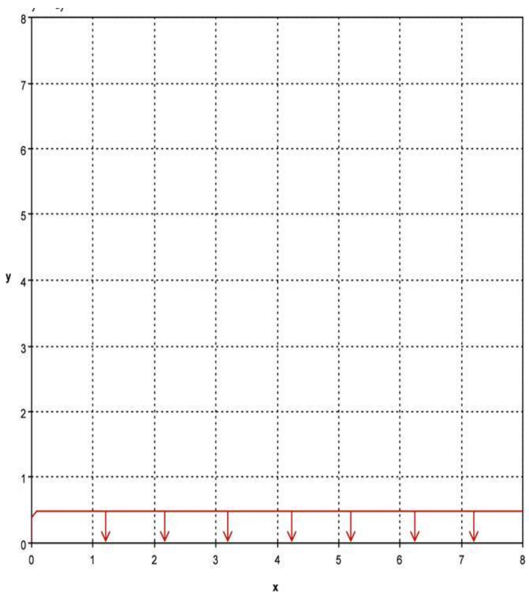

Fig.3 (A): Null clines

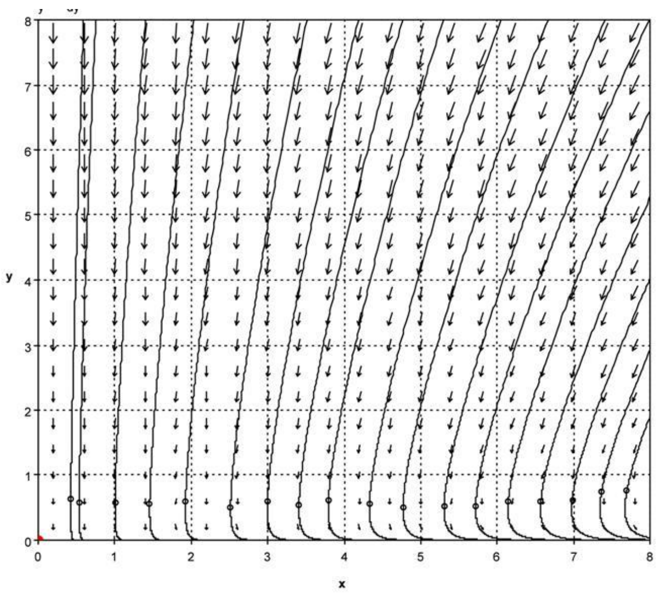

Fig.3 (B):Threshold Diagram

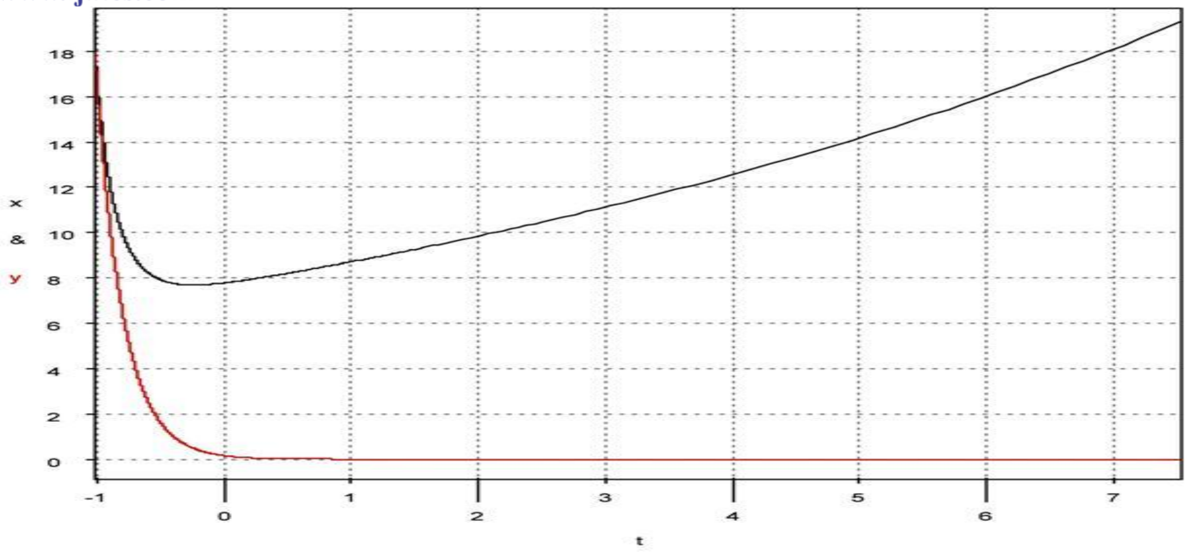

Fig.3(C): Solution Curves

Case (iv): When a_1=0.456, a_{11}=0.576, a_{12}=0.254and **a_2=6.678**, The Null clines, Trajectories and Solution curves are drawn in the Fig.4(A), Fig.4(B), & Fig.4(C) respectively.

In this case, The Eigen values are 0.12192 and −6.678 with the eigen vectors (1,0) & (0,1) and the Jacobean matrix is $\begin{pmatrix} 0.12192 & 0 \\ 0 & -6.678 \end{pmatrix}$

The Saddle point of this system exists at (0, 0).One of the eigen values is Positive. Hence, it is unstable .

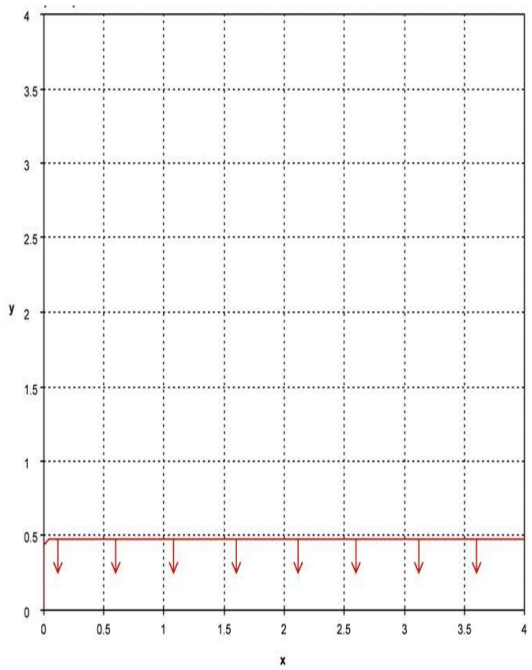

Fig.4 (A): Null clines

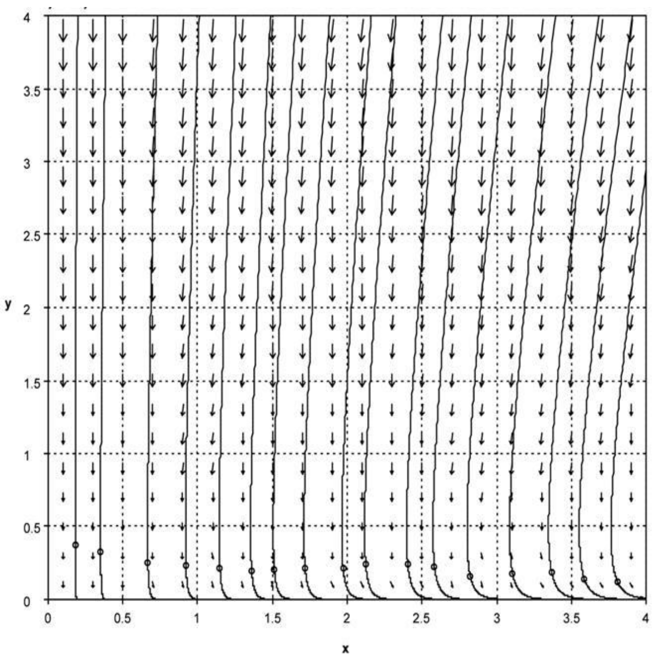

Fig.4 (B): Threshold Diagram

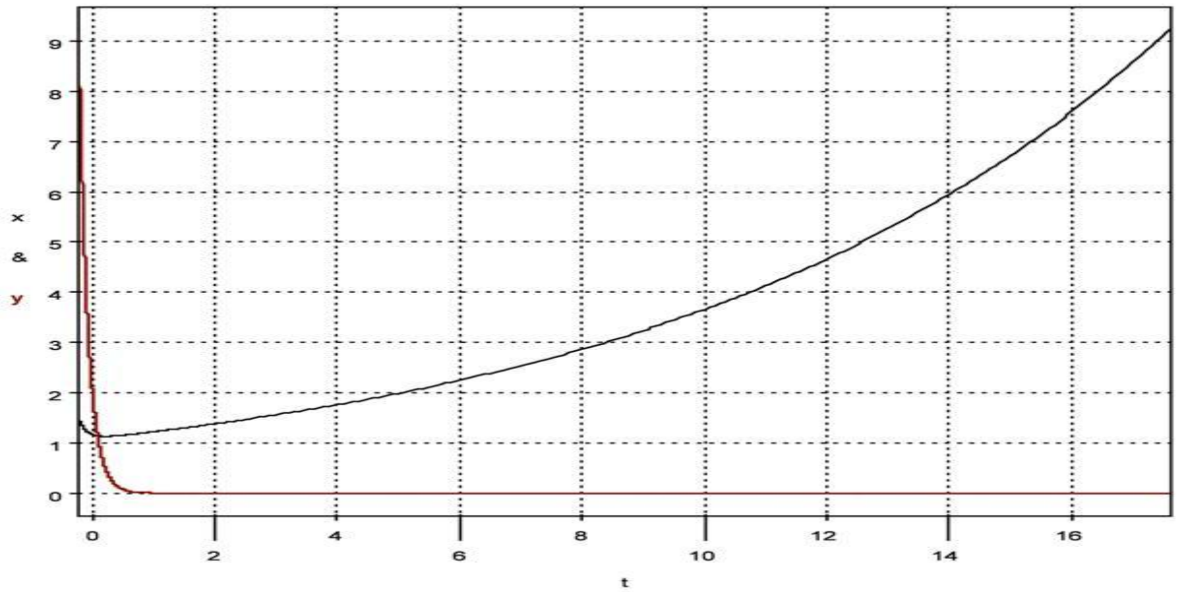

Fig.4(C): Solution Curves

IV. CONCLUSIONS

The general behavior of Ecological Ammensalism with mortal enemy species is identified as unstable. The stability nature can be obtained at the low growth rate of enemy Species with the constant growth rate of Ammensal species. No influence of the Ammensal coefficient is traced at any situation. The Ammensal and enemy species initially decline through out and it is evident that both the species asymptotically converge to the equlibriumpoint.While increasing the growth rate of enemy species,Ammensal Species flourishes gradually and no significant change in the growth rate of enemy species is occurred.

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